Topics in Reactions with Exotic Nuclei:Elastic Scattering and Fusion

Excercises

1. It is known that the atmosphere of the Saturn moon Titan is mostly methane CH_4, or, more popularly, natural gas. Its index of refraction is $N_m = 1.286$ (to be compared to water: $N_w = 1.33$). The temperature on Titan is 179^o C below zero. Nevertheless, methane remians a liquid even in this frost. Show that the primary rainbow angle in this case is 49^o, compared to 42.5^o for water. The order of colors would be the same in both cases. Since Titan has an orange sky, there is a background orange color behind the rainbow colors, making the observation of the blue-red primary bow more difficult for an astronaut on Titan.

2. Consider the case of pure diffractive scattering within the sharp cutoff approximation for the S-matrix,

$$S_l = |S_l| e^{2i\delta_l}, \tag{1}$$

   with, $\delta_l = 0$, and $|S_l| = \Theta(l - L)$.

   Show that evaluating the partial-wave sum series for the scattering amplitude one gets,

$$f(\theta) = \frac{L}{2ik}\left[\frac{P_L(\cos(\theta)) - P_{L-1}(\cos(\theta))}{1 - \cos(\theta}\right] \tag{2}$$

3. Evaluate the total cross section using the elastic scattering amplitude with absorption.

$$\sigma_t = \sigma_a + \int \frac{d\sigma(\theta)}{d\Omega} \tag{3}$$

Show that the optical theorem,

$$\sigma_t = \frac{4\pi}{k} Im[f(\theta = 0)] \tag{4}$$

is still valid.

Also show that the absorption cross section σ_a is given by,

$$\sigma_t = \frac{\pi}{k^2} \sum_{l=0}^{\infty} (2l+1)(1 - |S_l|^2 = \frac{\pi}{k^2} \sum_{l=0}^{\infty} T_l \tag{5}$$

where $T_l = 1 - |S_l|^2$ is called the transmission coefficient.

4. Consider the case where σ_a represents fusion. Treat the problem as an effective barrier tunneling. Then the cross section is given by, using the WKB approximation,

$$\sigma_F = \frac{\pi}{k^2} \sum_{l=0}^{\infty} \frac{1}{1 + \exp\left[2\pi[E - \hbar^2 l(l+1)/2_B^2 - V_B]/\hbar\omega\right]} \tag{6}$$

where R_B, V_B and ω are the radius, height, and curvature of the barrier, treated as an inverted parabola. Derive an analytical formula for σ_F, by replacing the l-sum by an integral and show that the resulting expression (Wong) is,

$$\sigma_F = R_B^2 \frac{\hbar\omega}{2E} ln\left[1 + \exp\left(\frac{2\pi(E - V_B)}{\hbar\omega}\right)\right] \tag{7}$$

Discuss the limits $E \gg V_B$, and $E \ll V_B$