Nuclear Astrophysics with Radioactive Beams

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Part I

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Literature

Nuclear Reactions
C. Bertulani
Wiley Encyclopedia of Physics, 2009
arXiv: 0908:3275

Nuclear Astrophysics with Radioative Beams
C. Bertulani and A. Gade
Phys. Reports 485 (2010) 195

Nuclear Physics in a Nutshell
C. Bertulani
Princeton Press (2007)

Literature

Or just ask him

Why are we here?

Because

you might have good friends in the audience

Why are we here?

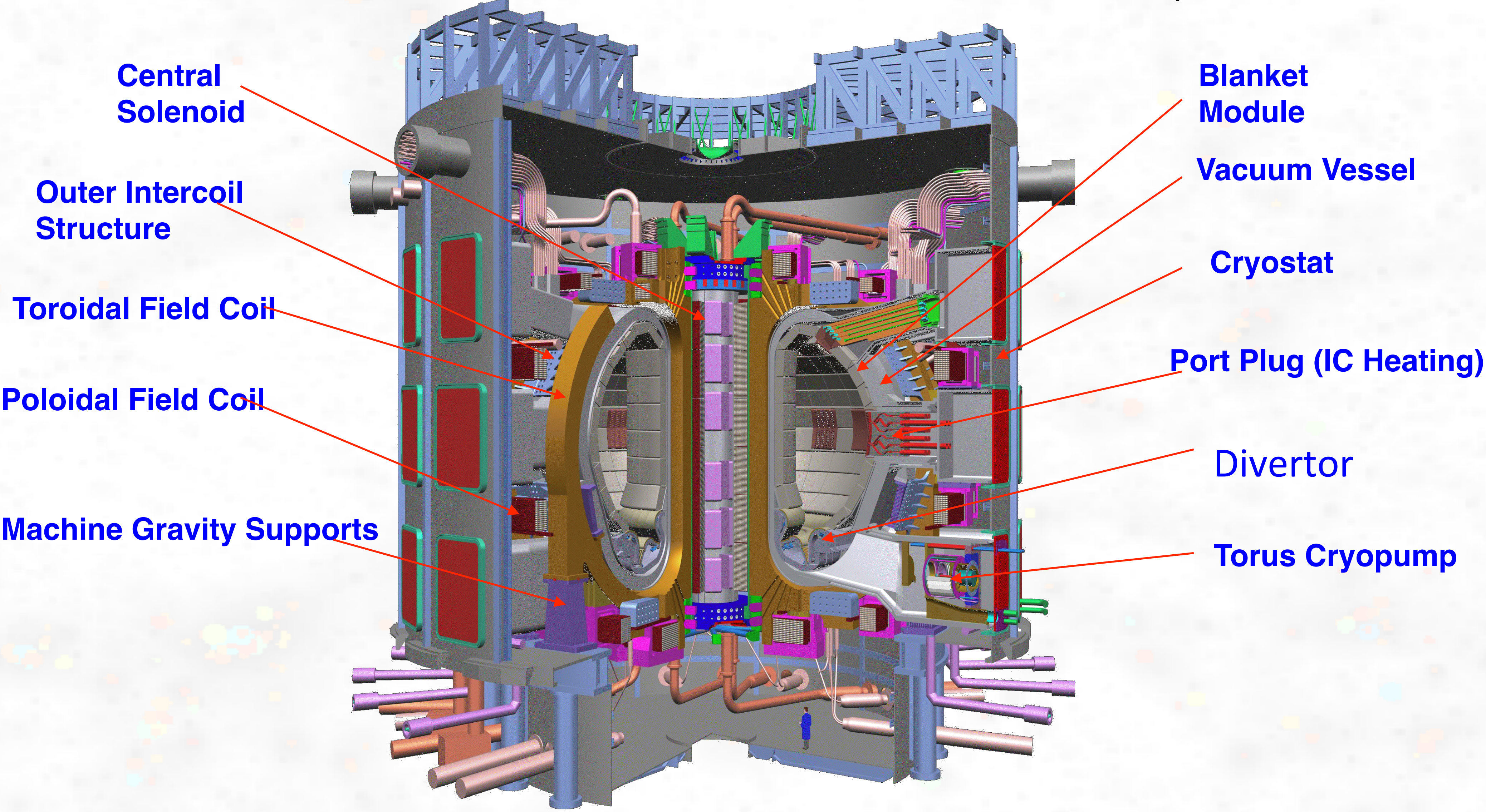
Because

Courtesy: Navin Alahari (GANIL)

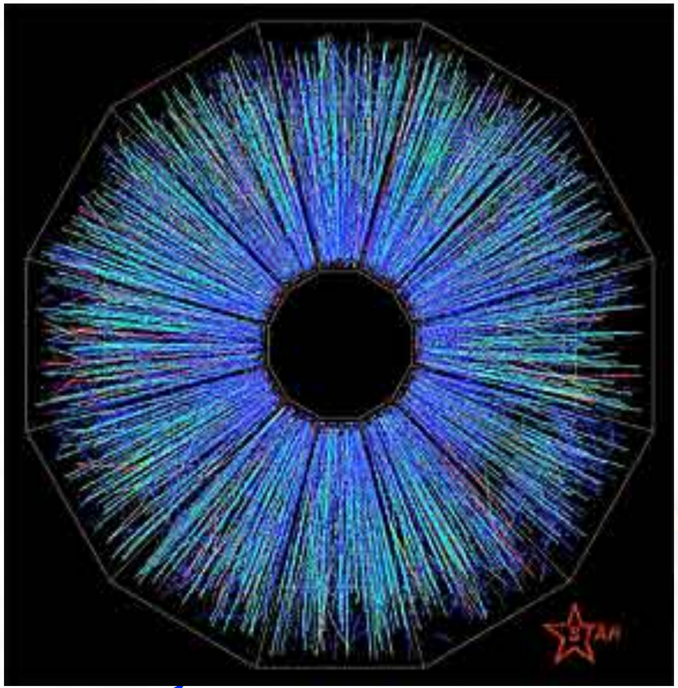
You can: Join a big collaboration

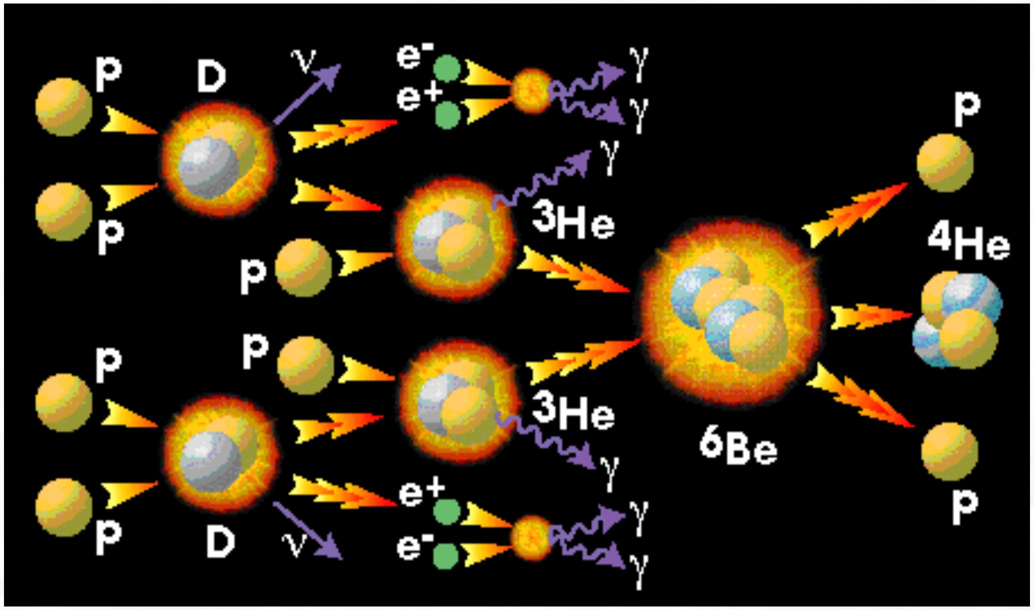
E.g., ITER Nuclear Fusion

- Lots of money (15 billion EUROS, as of 2011)
- If works, need additional ITERS in 2050 (maybe 2100)



Or, pick up a hard problem to solve





TeV/nucleon ← → **keV/nucleon**

???

Exotic stellar site

Quark matter in compact stars, Big Bang

Nuclear many-body problem:
the hardest problem of all physics!

- Interactions are complicated
- Nucleons = composite particles

???

Typical stellar site

Stellar evolution

What are the problems?

Part I

- Introduction
- Definitions
- Theory of astrophysics reactions

Stellar Modeling: nuclear reaction networks

Mass fraction of nuclear species	**X**
Abundance	**Y = X/A (A=mass number)**
Number density	**n = ρ N_AY (ρ=mass density, N_A=Avogadro)**

Astrophysical model (hydrodynamics, ….)

Temperature T and Density ρ

Network: System of differential equations:

$$\frac{dY_i}{dt} = \sum_j N^i_j \lambda_j Y_j + \sum_{jk} N^i_{jk} \rho N_A < \sigma v > Y_j Y_k + \ldots$$

1 body 2 body

Nuclear energy generation

Observations

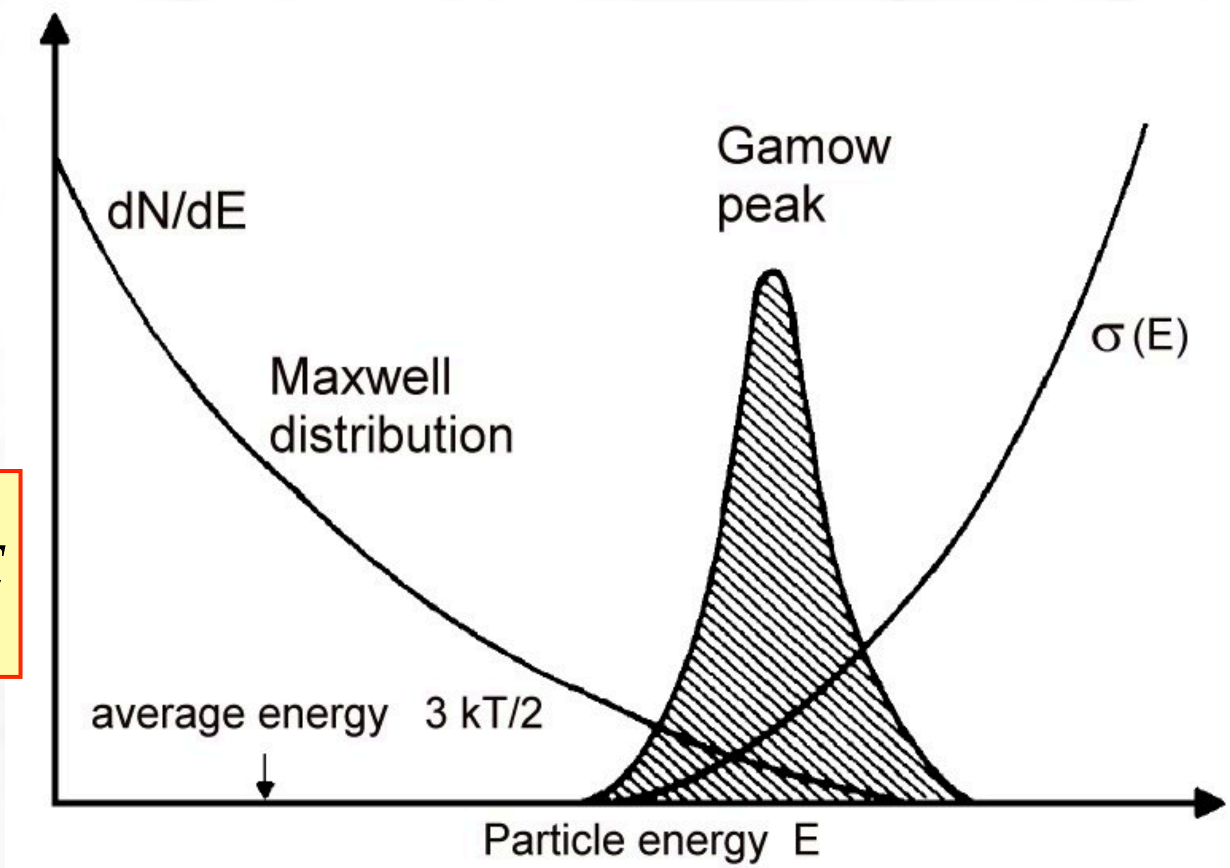
$N^i_{\ldots}$: number of nuclei of species I produced (positive) or destroyed (negative) per reaction

Reaction rates

$$\sigma(E) = \sum_{\ell} \sigma_{\ell}(E)$$

partial wave expansion

$$\langle \sigma v \rangle \sim \int \sigma(E) E \exp\left[-\frac{E}{kT}\right] dE$$



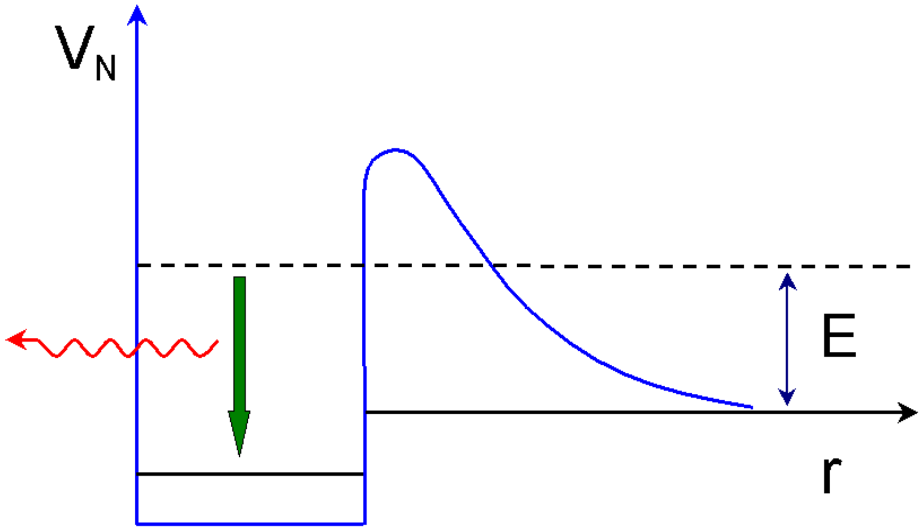
T = temperature
v = relative velocity
k_B = Boltzmann constant ~ (1/11.6) MeV/10^9 K

Always $E_0 \ll E_{coul}$ (coulomb barrier)
→ small number of partial waves (l=0 mainly)

P(E) ~ exp(-2πη) : Coulomb factor
(η = Sommerfeld parameter ~ $Z_1 Z_2 / E^{1/2}$)

S-factor

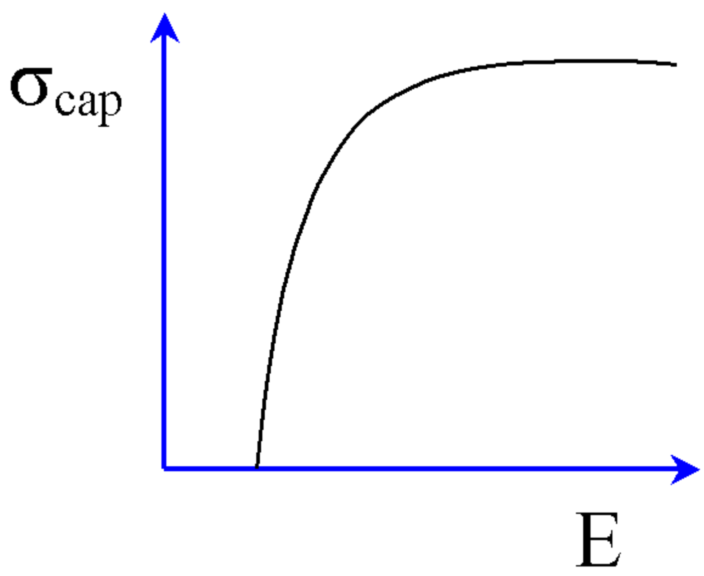
Charged particles



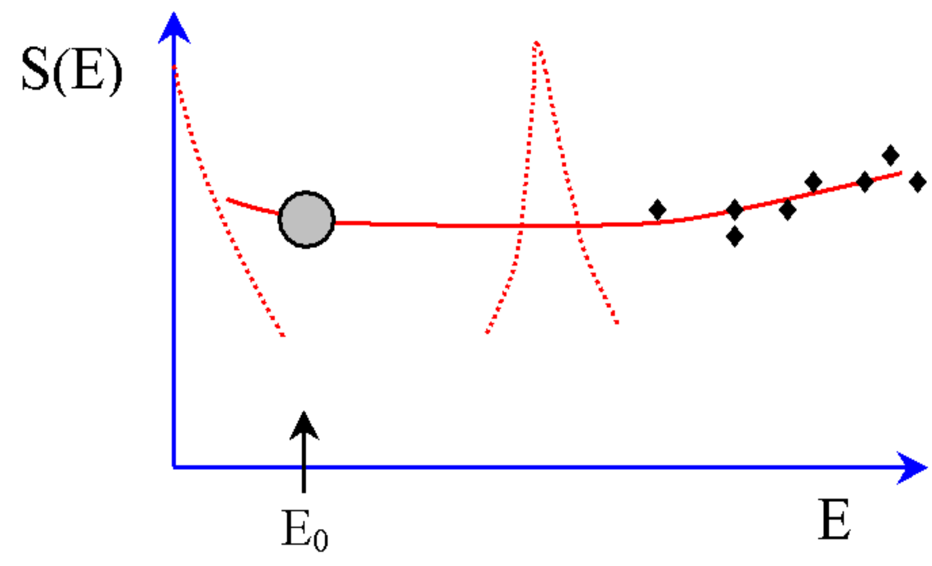
Astrophysical S-factor

$$\sigma(E) = \frac{1}{E} S(E) \exp\left[-2\pi \frac{Z_1 Z_2 e^2}{\hbar v}\right]$$

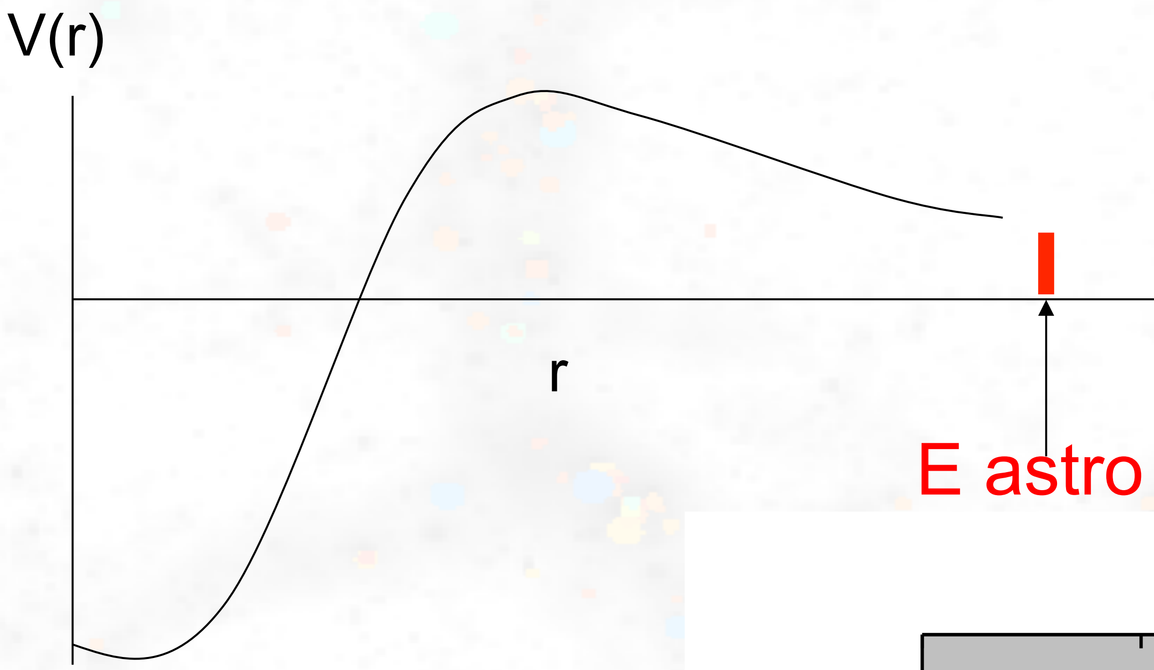
Steep energy dependence

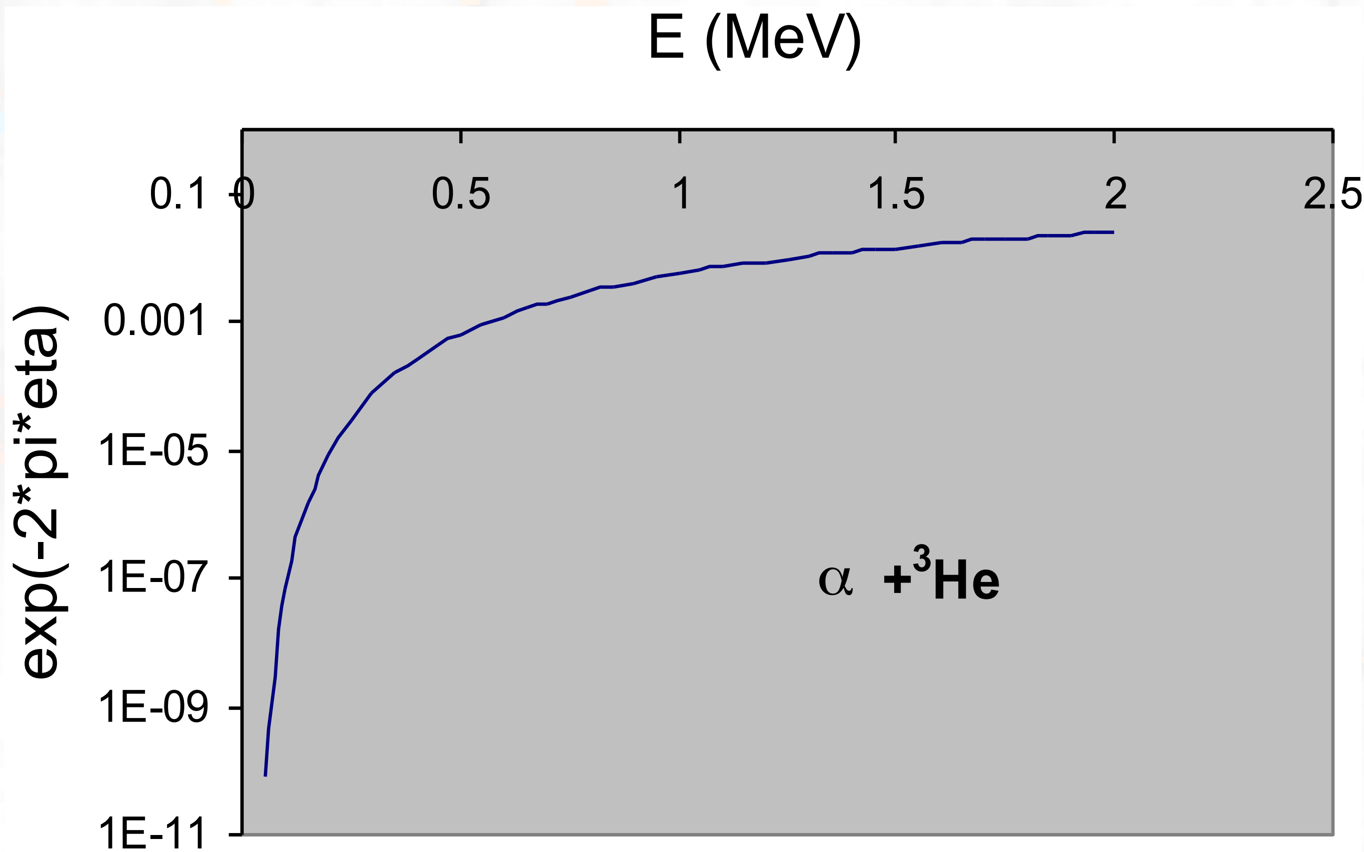


Extrapolation of data

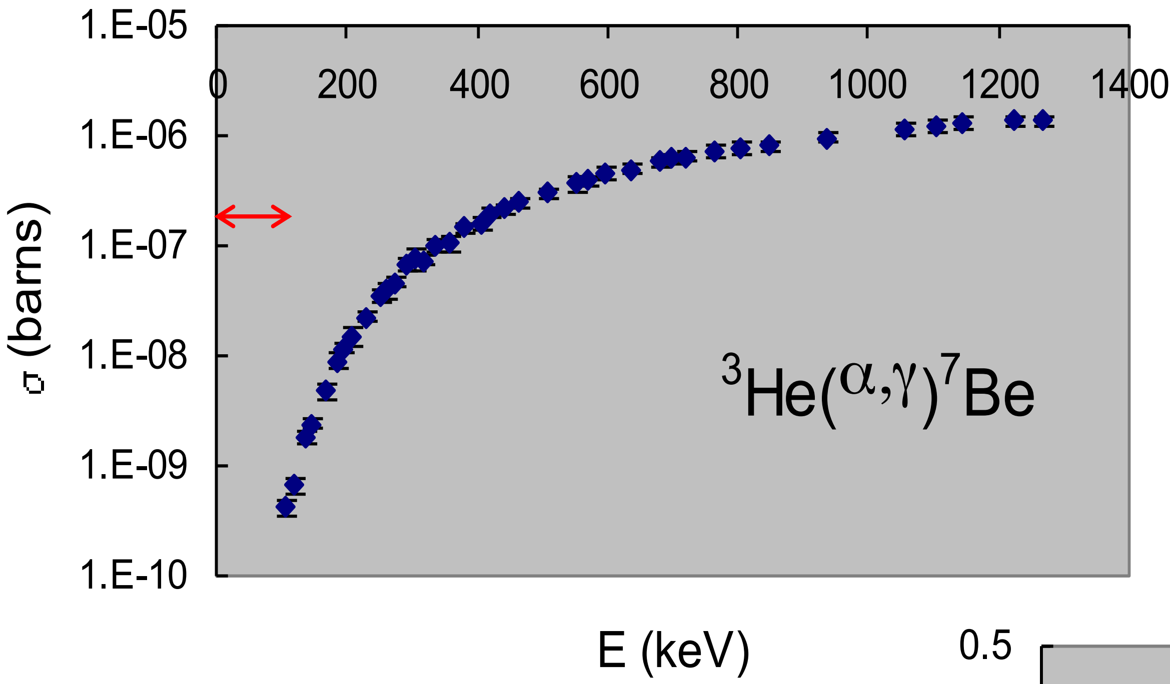


Barrier penetrability - theoretical examples





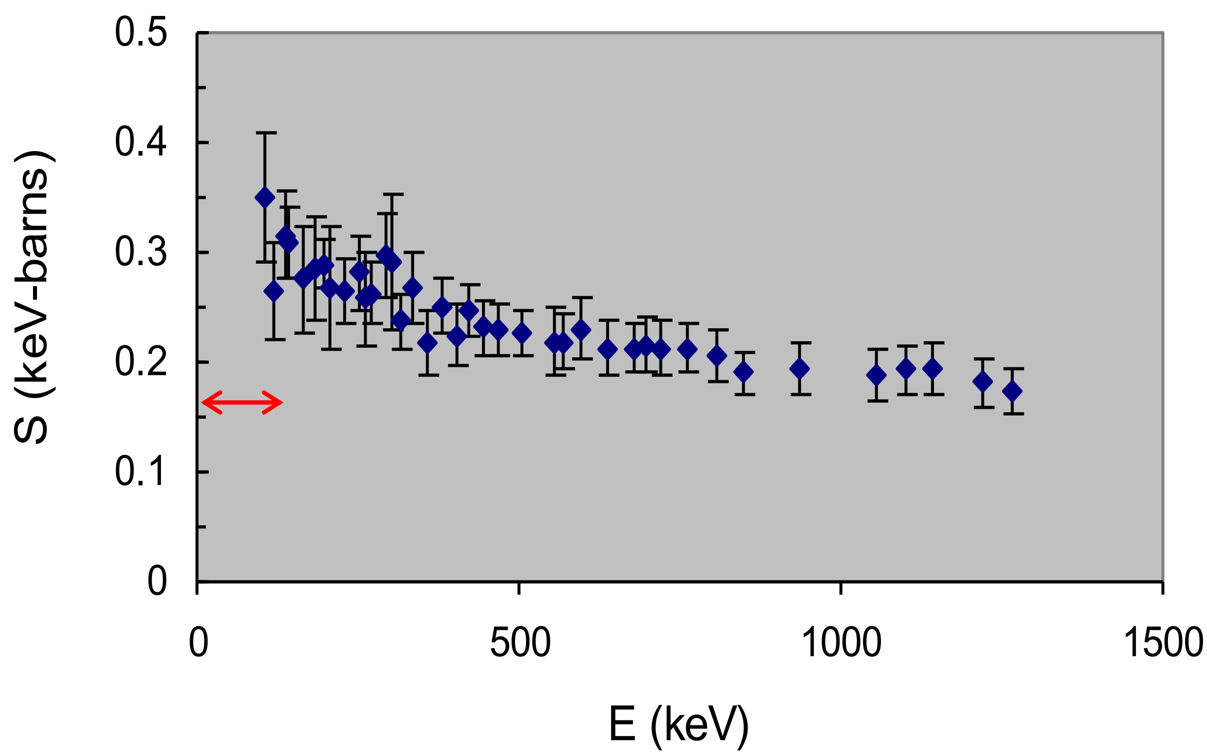
S-factor – experimental examples



S factor

$S(E) = \sigma(E).E.\ \exp(2\pi\eta)$

depends slowly on energy



Reaction rates - examples

1 MeV ~ 10^{10} K

$T_9 = 10^9$ K

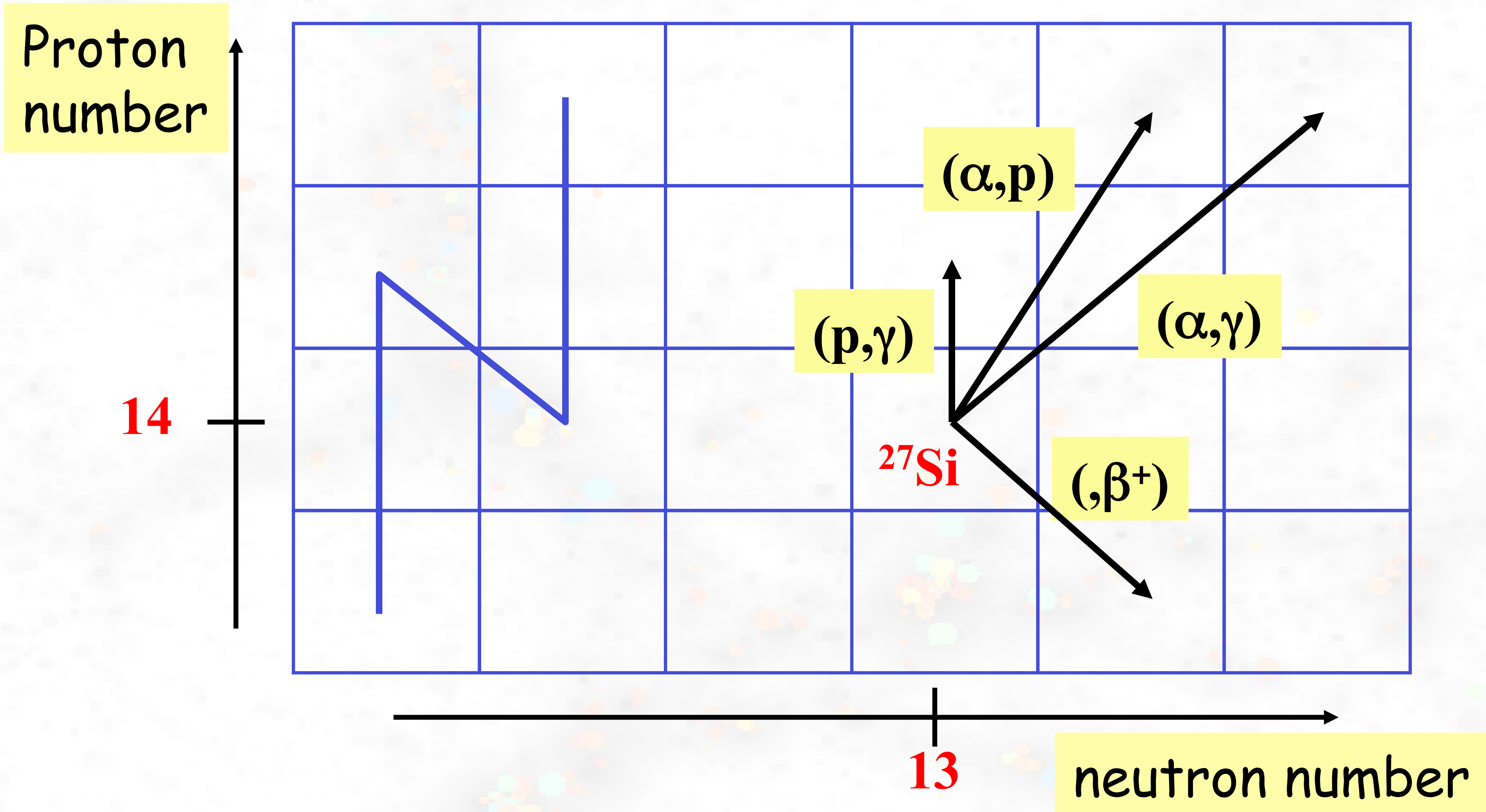
$$E_0 \sim 0.122\left(Z_i^2 Z_j^2 A\right)^{1/3} T_9^{2/3}\ \text{MeV}$$

Reaction	T (10^9 K)	E_0 (MeV)	E_{coul} (MeV)	$\sigma(E_0)/\sigma(E_{coul})$
d + p	0.015	0.006	0.3	10^{-4}
^{3}He + ^{3}He	0.015	0.021	1.2	10^{-13}
α + ^{12}C	0.2	0.3	3	10^{-11}
^{12}C + ^{12}C	1	2.4	7	10^{-10}

$$\langle \sigma v \rangle \sim \int \sigma(E)\, E \exp\left[-\frac{E}{kT}\right] dE$$

$$E_{Coul} \sim 1.2 \frac{Z_i Z_j}{A_i^{1/3} + A_j^{1/3}}\ \text{MeV}$$

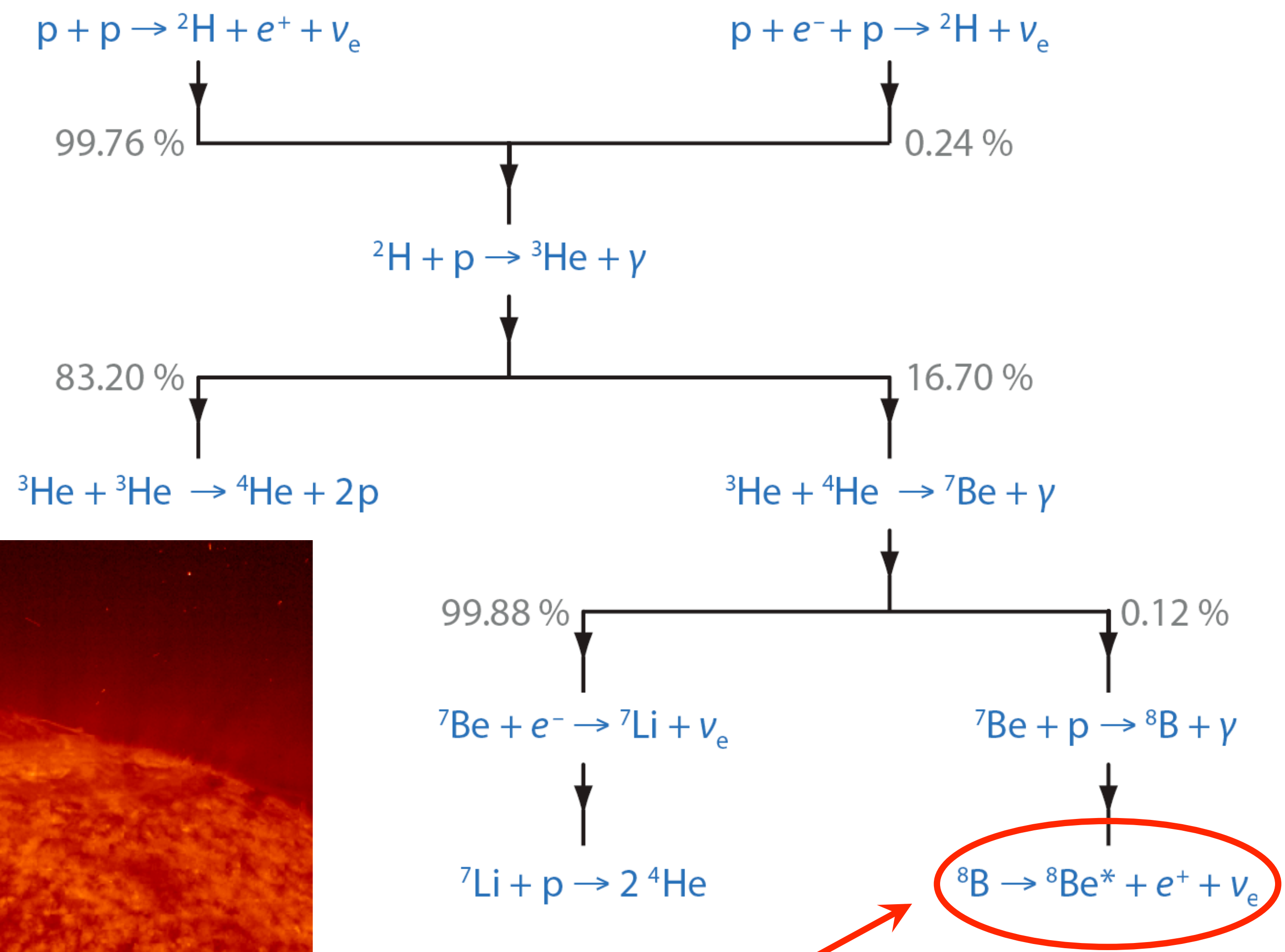
Reaction Flow

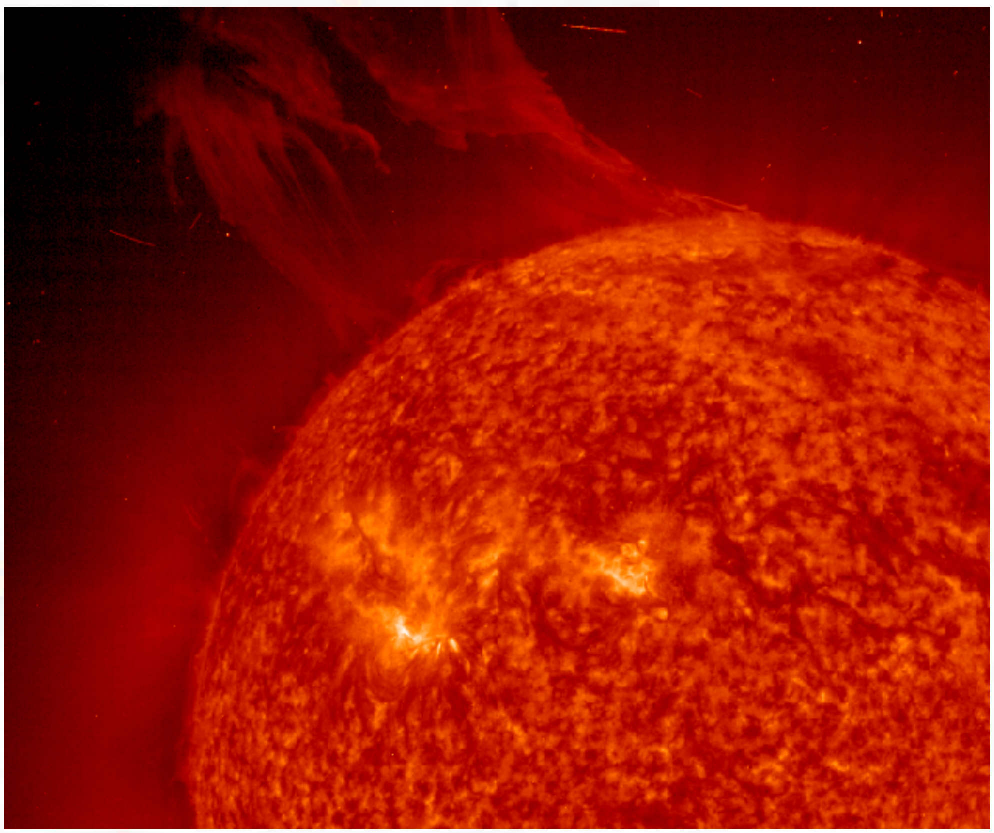


$$\frac{dY_i}{dt} = \sum_j N^i_j \, \lambda_j Y_j + \sum_{jk} N^i_{jk} \, \rho N_A < \sigma v > \; Y_j Y_k + \ldots$$

Reaction Chains - Ex: Our Sun

p-p chain

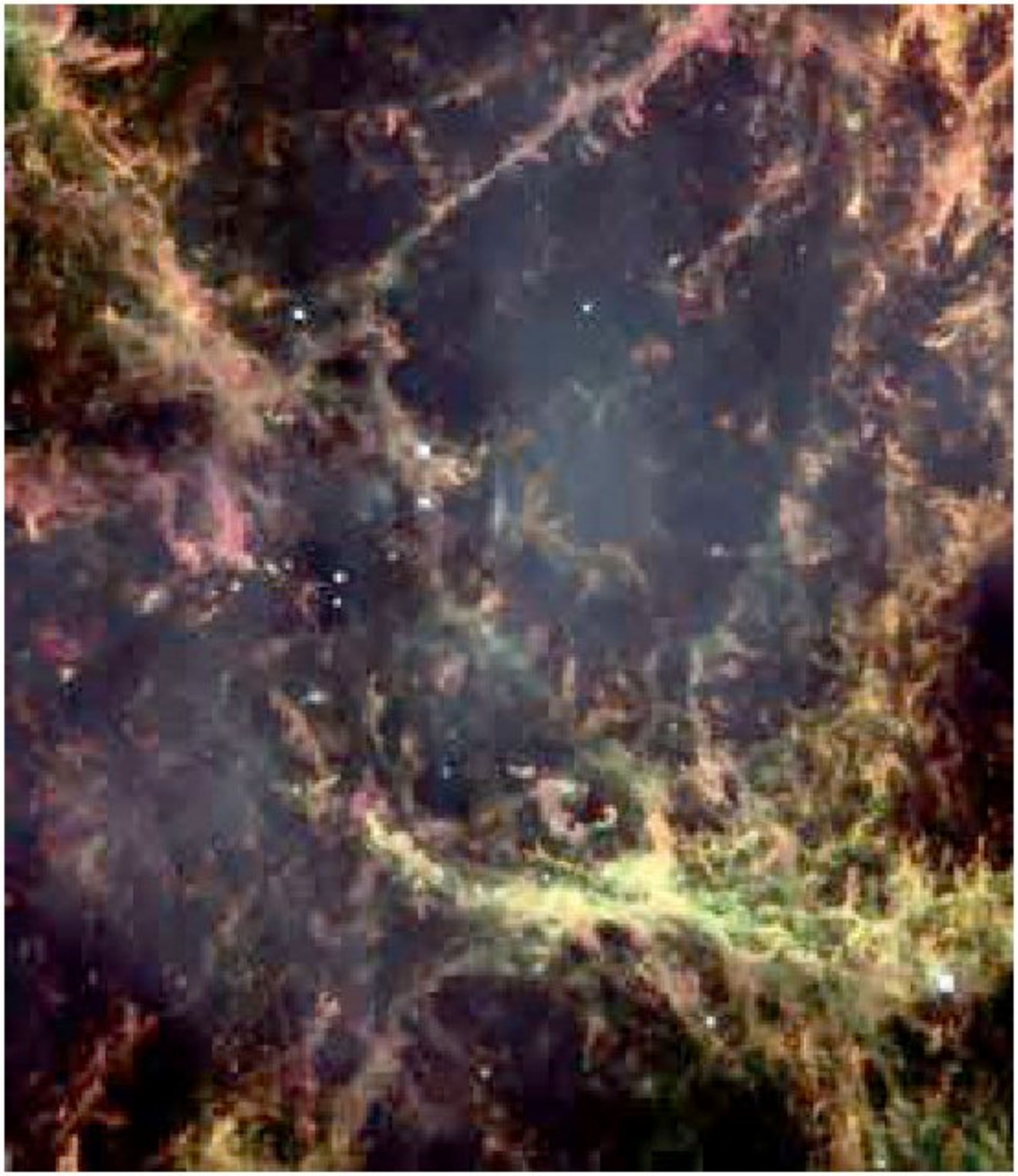


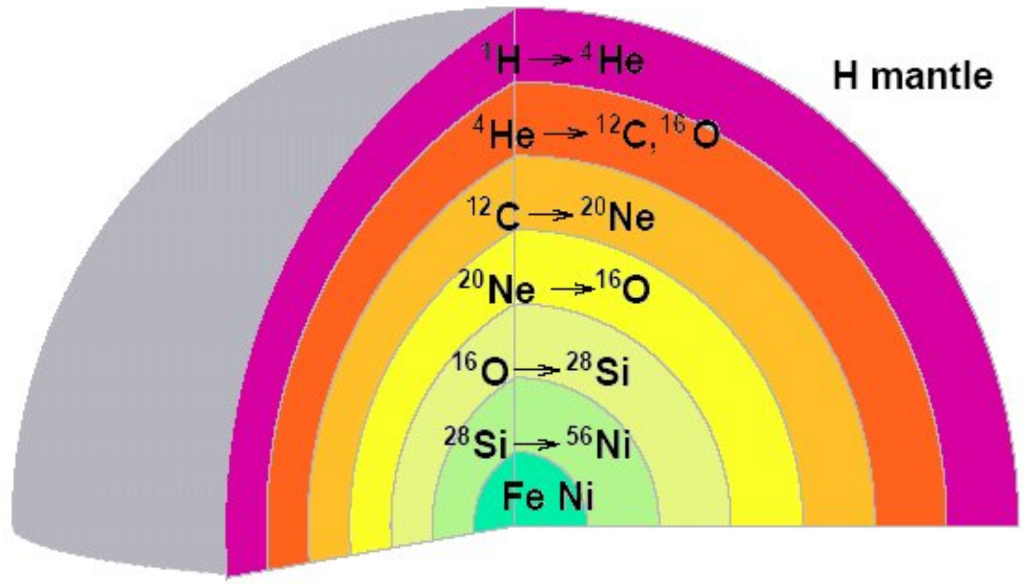


Solar neutrinos

$E_\nu < 15$ MeV

Massive stars





Triple-alpha capture

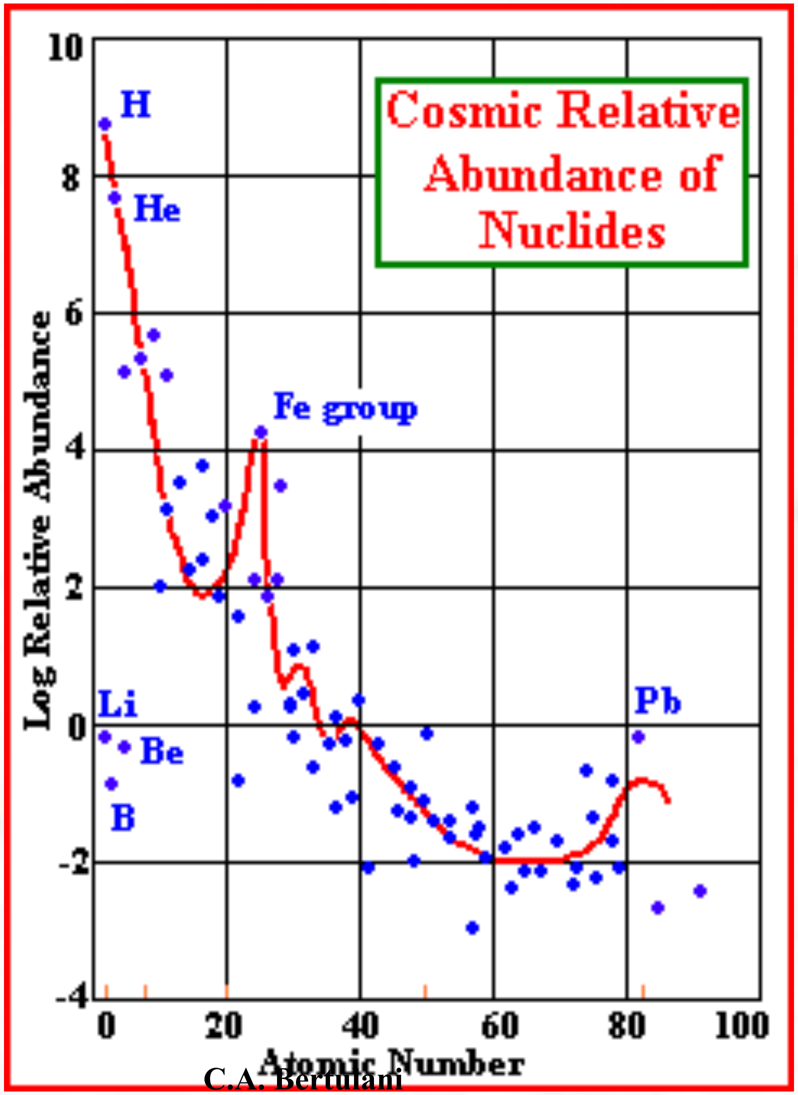
Helium burning: $^{12}C(\alpha,\gamma)^{16}O$

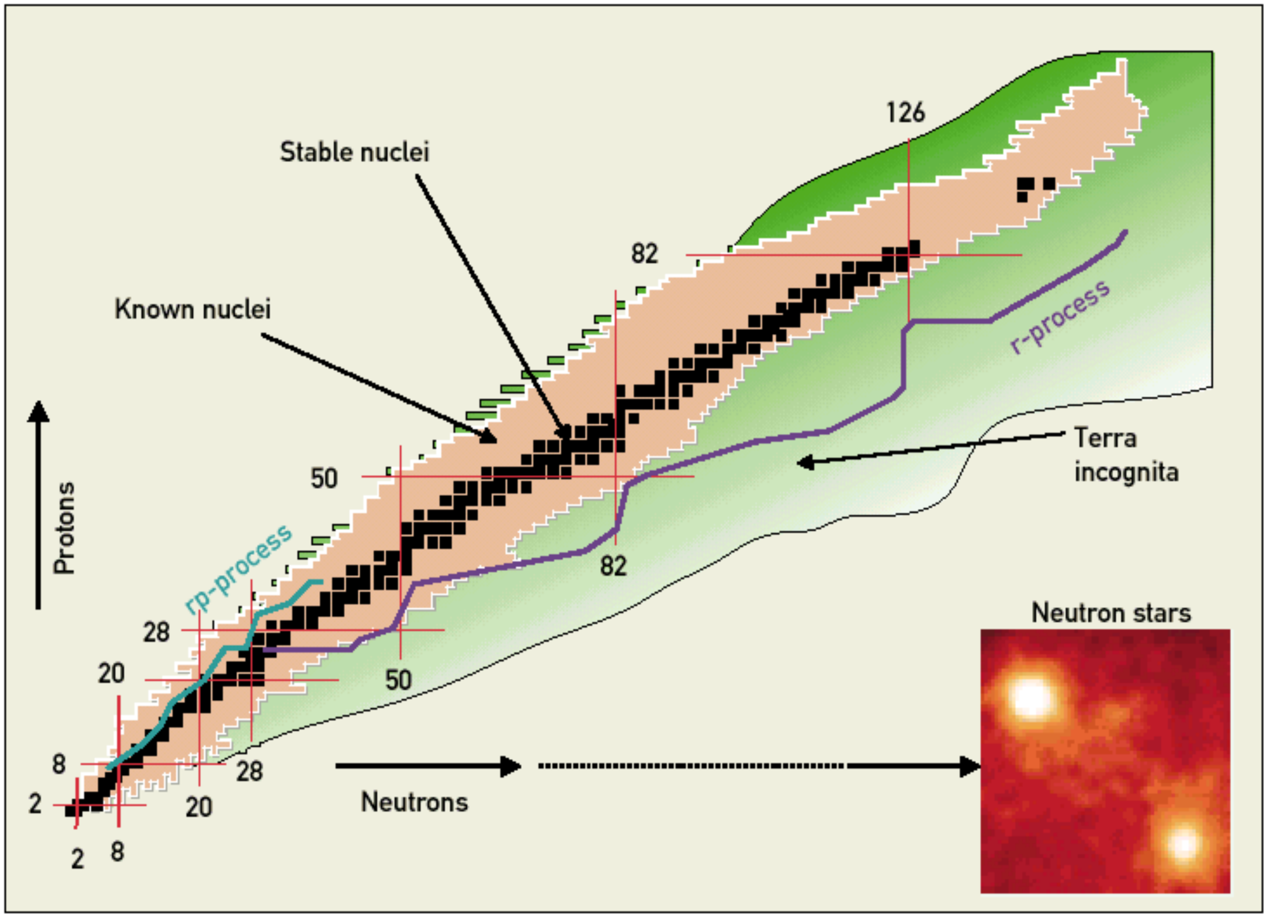
Supernovae remnants: black holes or neutron stars?

Nucleosynthesis

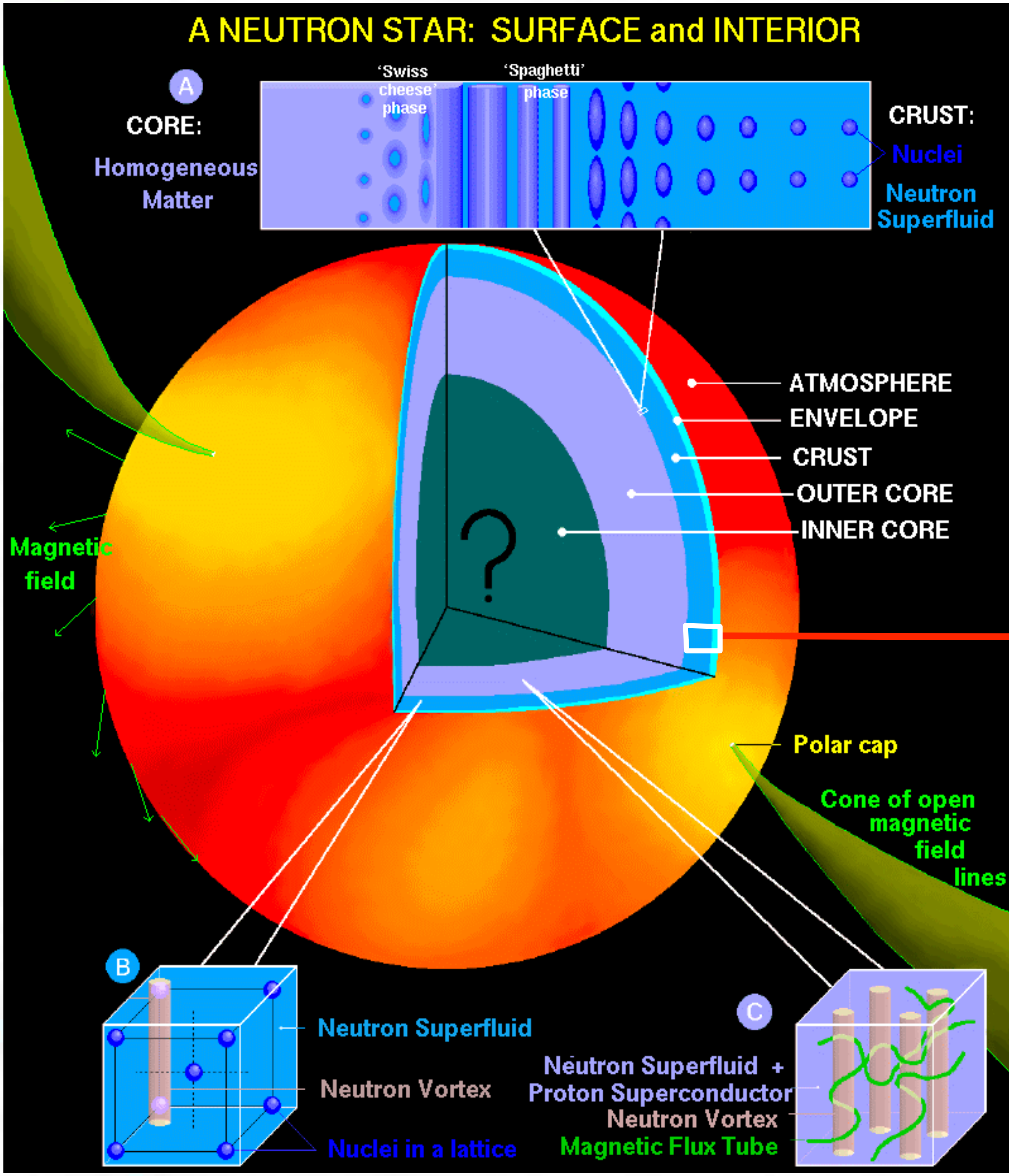
Rapid n (p) –capture followed by β-decay

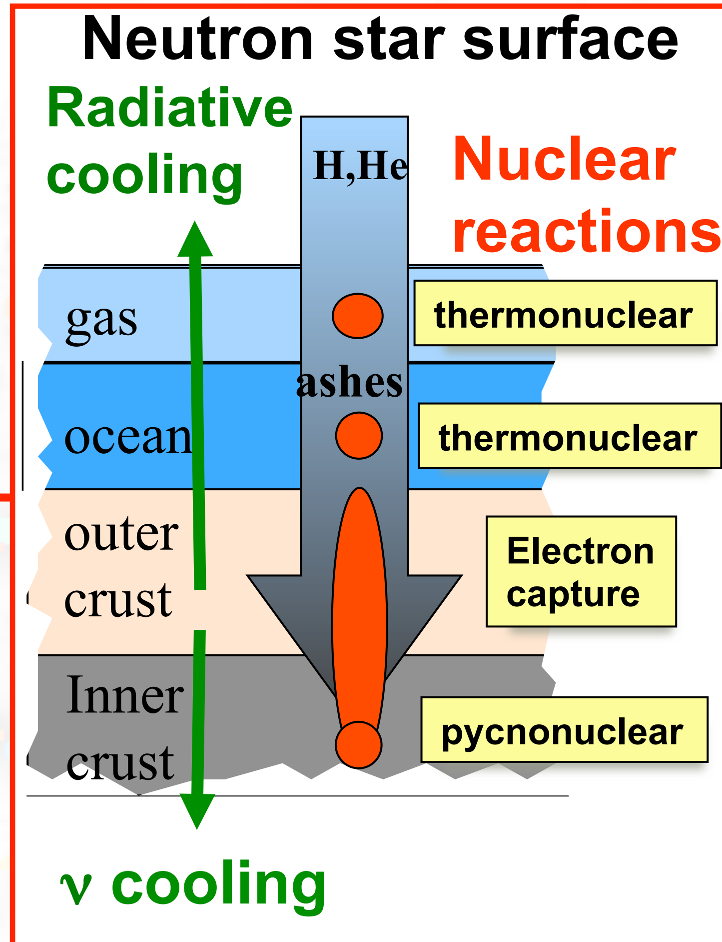
Numerous σ(n,γ) and σ(p,γ) needed





Neutron Stars





Courtesy: Dany Page

Theory of astrophysics reactions

Reaction Types

1. Fusion & Radiative capture reactions
$A + C \rightarrow B + \gamma$

2. Elastic collision : entrance channel=exit channel
$A + B \rightarrow A + B$: $Q = 0$

3. Inelastic collision ($Q \neq 0$)
$A + B \rightarrow A^* + B$ (A^* = excited state)
$A + B \rightarrow A + B^*$,
etc..

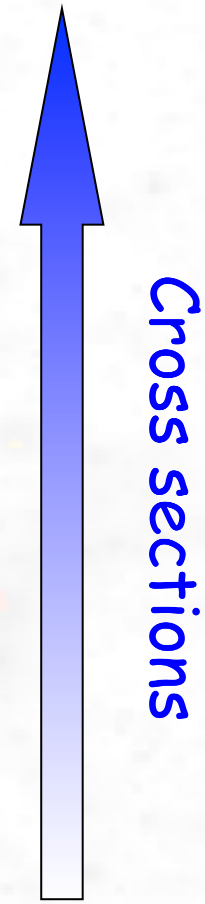
4. Breakup reactions
$A(= a + b) + C \rightarrow a + b + C$

5. Transfer reactions
$A + B \rightarrow C + D$

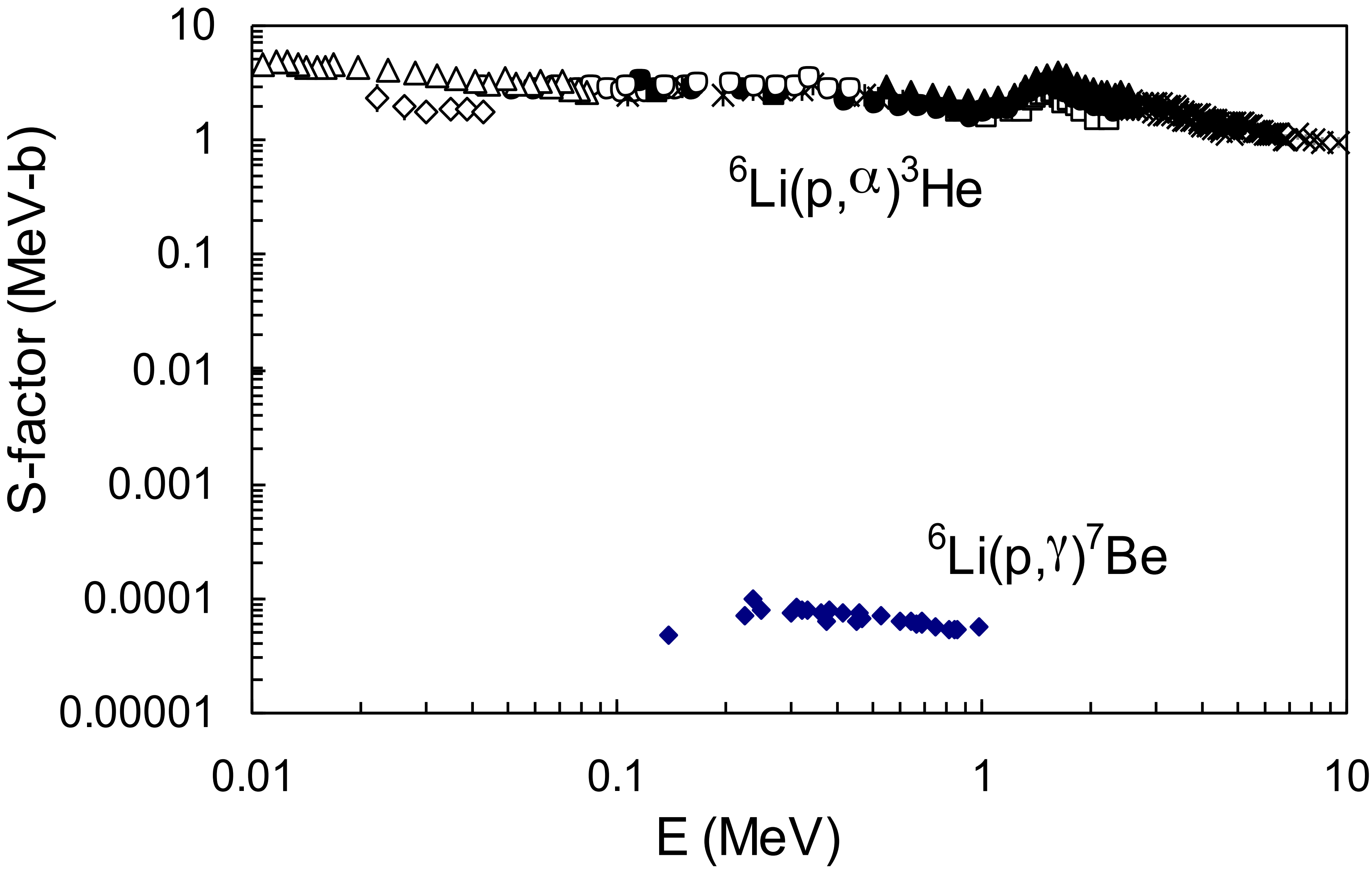
- *Transfer cross sections (**strong interaction**)*
  - Non resonant $^6Li(p,\alpha)^3He$
  - Resonant $^3He(d,p)\alpha$
  - Multiresonance $^{22}Ne(\alpha,n)^{25}Mg$

- *Capture cross sections (**electromag interact**)*
  - Non resonant $^6Li(p,\gamma)^7Be$
  - Resonant $^{12}C(p,\gamma)^{13}N$
  - Multiresonance $^{22}Ne(\alpha,\gamma)^{26}Mg$

- *Weak capture cross sections (**weak interaction**)*
  - Non resonant $p(p,e^+\nu)^2H$
    $^3He(p,e^+\nu)^4$



Reaction Types - Example



Cross sections

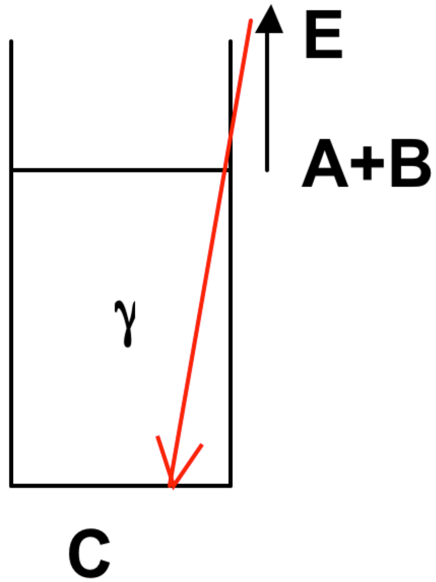
1. **Transfer** (*nuclear interaction*) - low *J* values at low energies

$$\sigma_{c\to c'}^{J\pi} = \frac{\pi}{k^2} | \delta_{cc'} - \mathcal{S}_{cc'}^{J\pi} |^2 \qquad \mathcal{S} = \text{collision matrix}$$

2. **Capture** (*electromagnetic interaction*):

$H = H_N + H_\gamma$, with H_γ = electromagnetic interaction

$$\sigma_{cap}(E) \sim |< \Psi^{J_f} | H_\gamma | \Psi^{J_i}(E) >|^2$$



H_γ is expanded in multipoles: electric (**M**$^{E\lambda}$) and magnetic (**M**$^{M\lambda}$)
⇒ matrix elements of the multipole operators (in general E1)

$$M_\mu^{E\lambda} = \sum_i e_i^{eff} r_i^\lambda Y_\lambda^\mu(\Omega_i) \qquad e_i^{eff} = \text{effective charges}$$

Theoretical models

1. **Cluster models:** assume a cluster structure of the system
   → collision + spectator
   Provide cross sections directly
   a. Potential model
   b. R-matrix method
   c. DWBA method
   d. RGM method (Resonating Group Method)

2. **Ab-initio models**
   a. Fermionic Molecular Dynamics
   b. No Core Shell Model
   c. Green's function Monte Carlo

Theoretical models

3. Non-cluster models: provide energies and widths of specific resonances

a. Shell model (essentially spectroscopy)

b. Hauser-Feshbach (high densities)

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4. Indirect methods

a. Trojan Horse

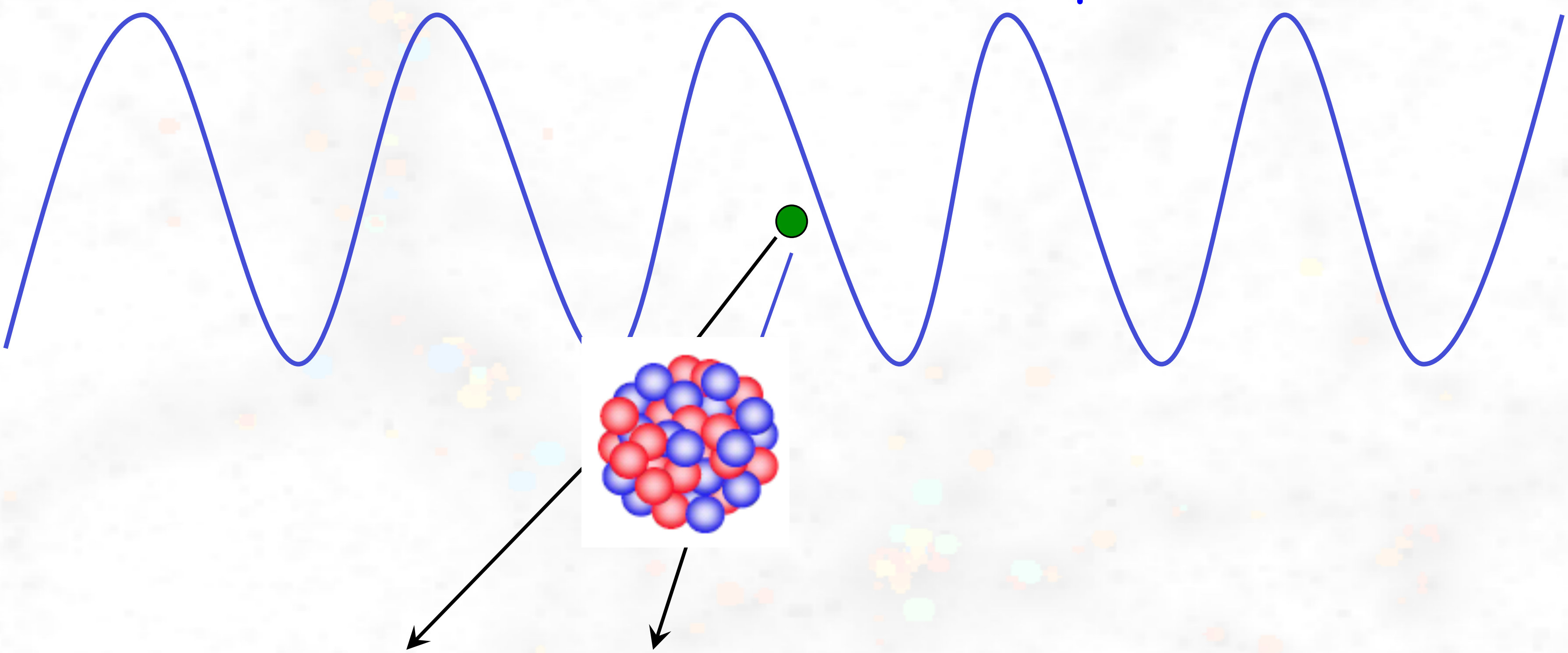
b. Coulomb dissociation

c. Asymptotic Normalization Constant

Why these? Later.

QCD Light – Effective Field Theories

probe: f(x)



$$f(x) = f(0) + p \cdot \left(\nabla f\right)\Big|_0 + Q \cdot \left(\nabla^2 f\right)\Big|_0 + \cdots$$

monopole:

dipole:

quadrupole:

controlled precision

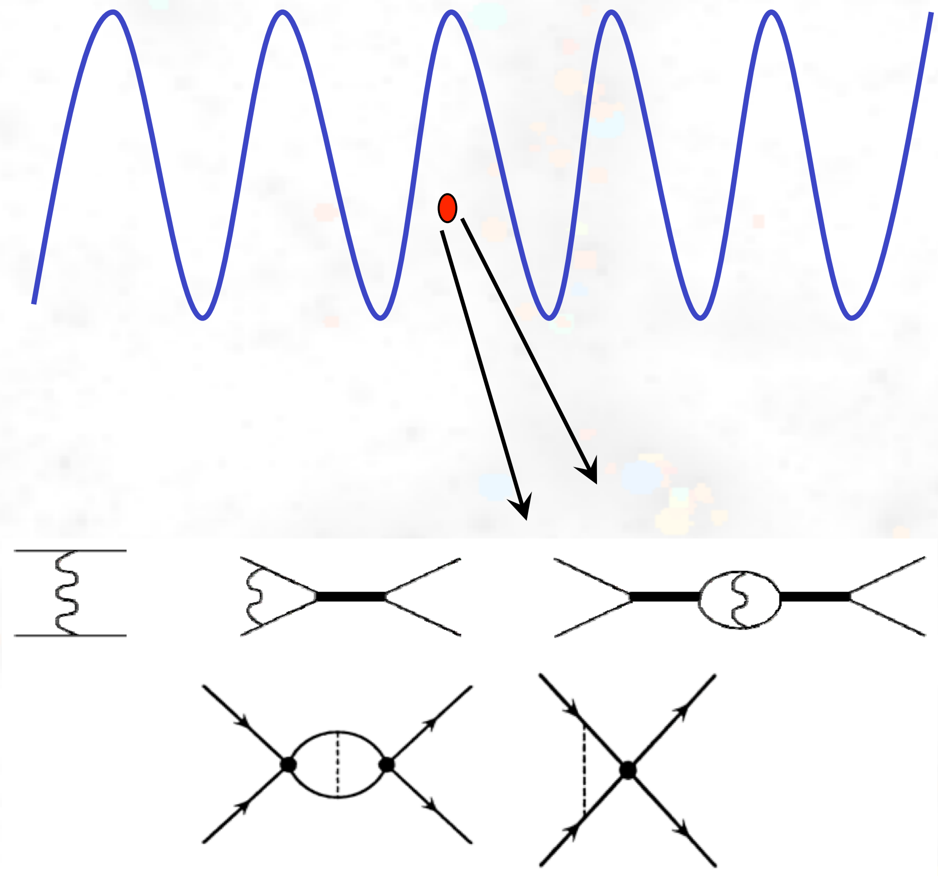
Effective Field Theories - Why?

Nuclear potential models are:

- phenomenological
- non fundamental
- non extrapolable (predictive)
- higher-order corrections (?)
- non controllable

→ Need to solve L_{QCD} for low-energy Nuclear Physics

Effective Field Theories

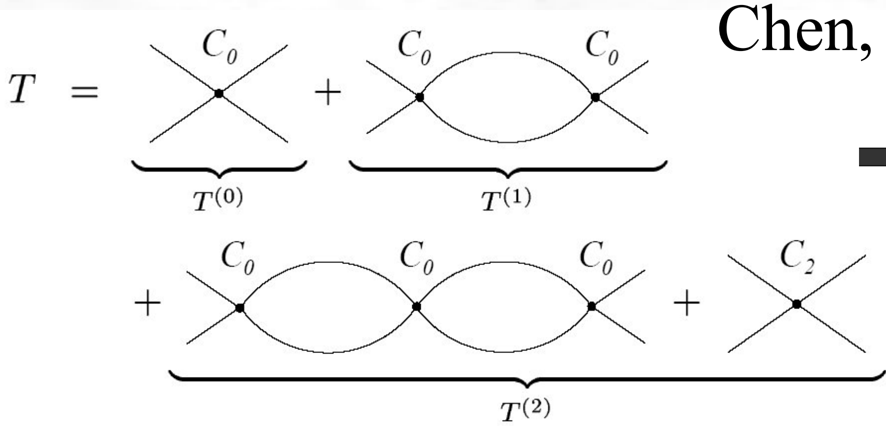


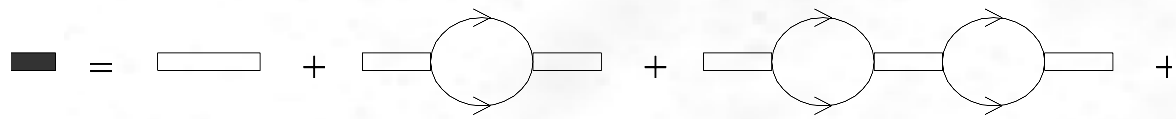
$$L_{EFT} = N^{+}\left(i\partial_0 + \frac{\nabla^2}{2m_N}\right)N + C_0\, N^{+}N\, N^{+}N$$
$$+ N^{+}\frac{\nabla^4}{8m_N^{\ 3}}N + C_2\, N^{+}N\, N^{+}\nabla^2 N$$
$$+ C_2'\, N^{+}\vec{\nabla}N \cdot N^{+}\vec{\nabla}N + \ldots$$

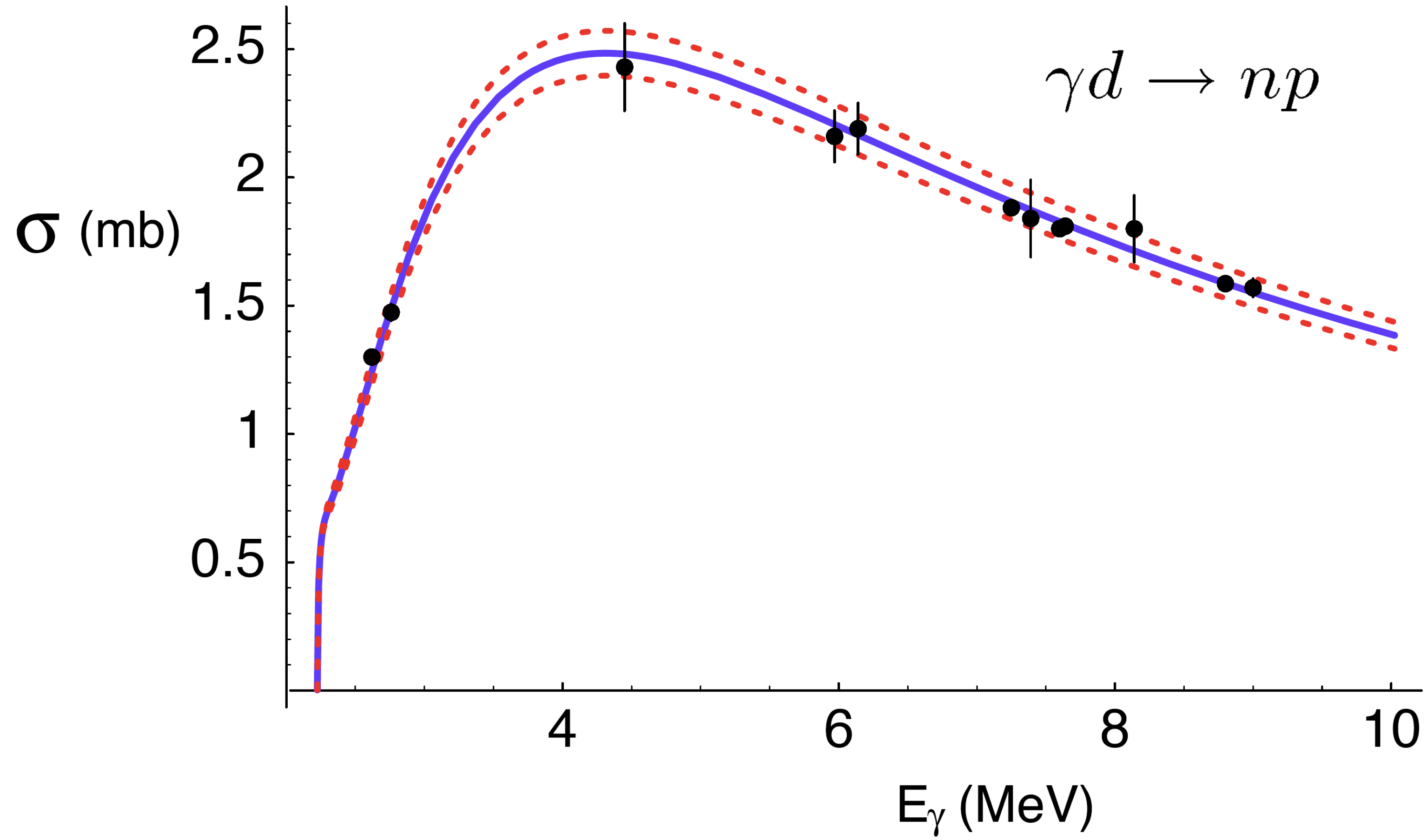
- Feynman diagrams
- particle exchange
- vacuum polarization
- loop integrals, divergences
- regularization, renormalization

Pion-less EFT - Example

Chen, Rupak, Savage, NPA 653 (1999) 386

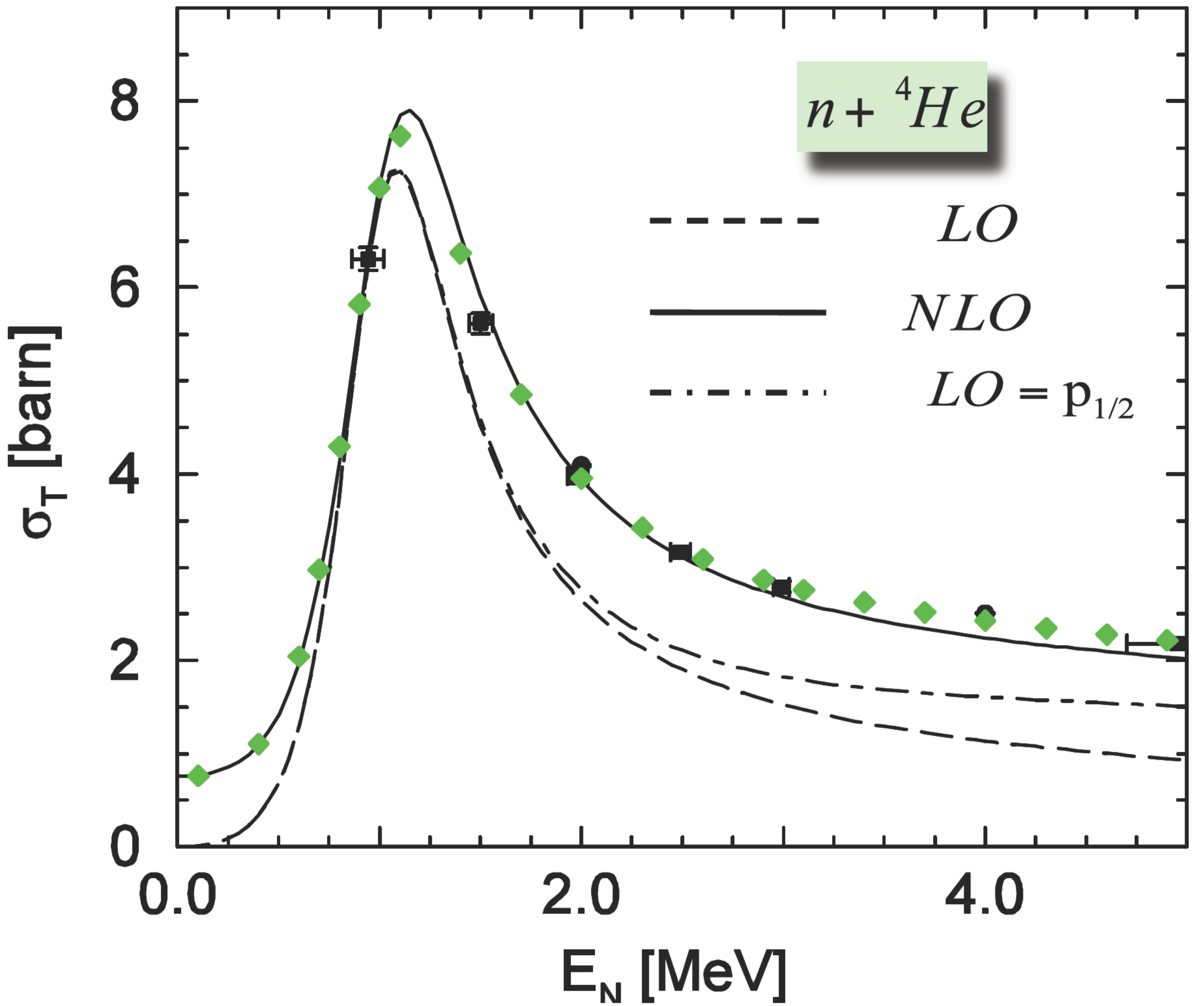




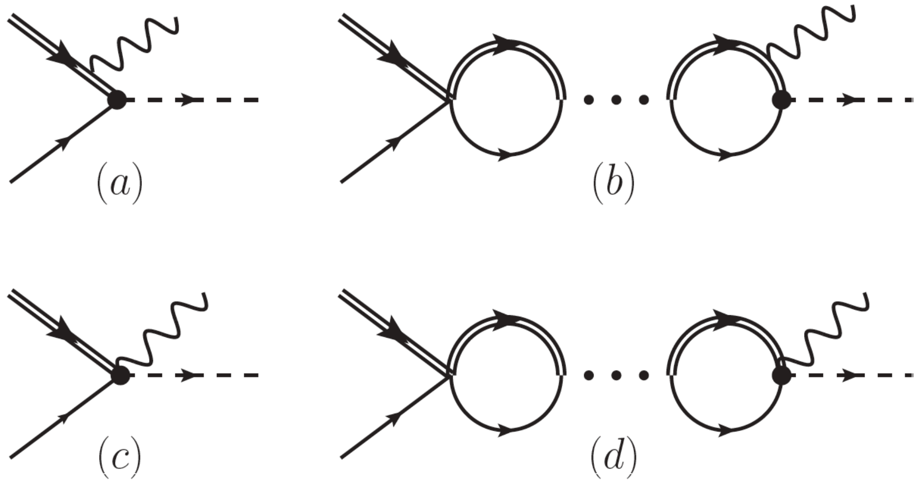


Pion-less EFT - Example

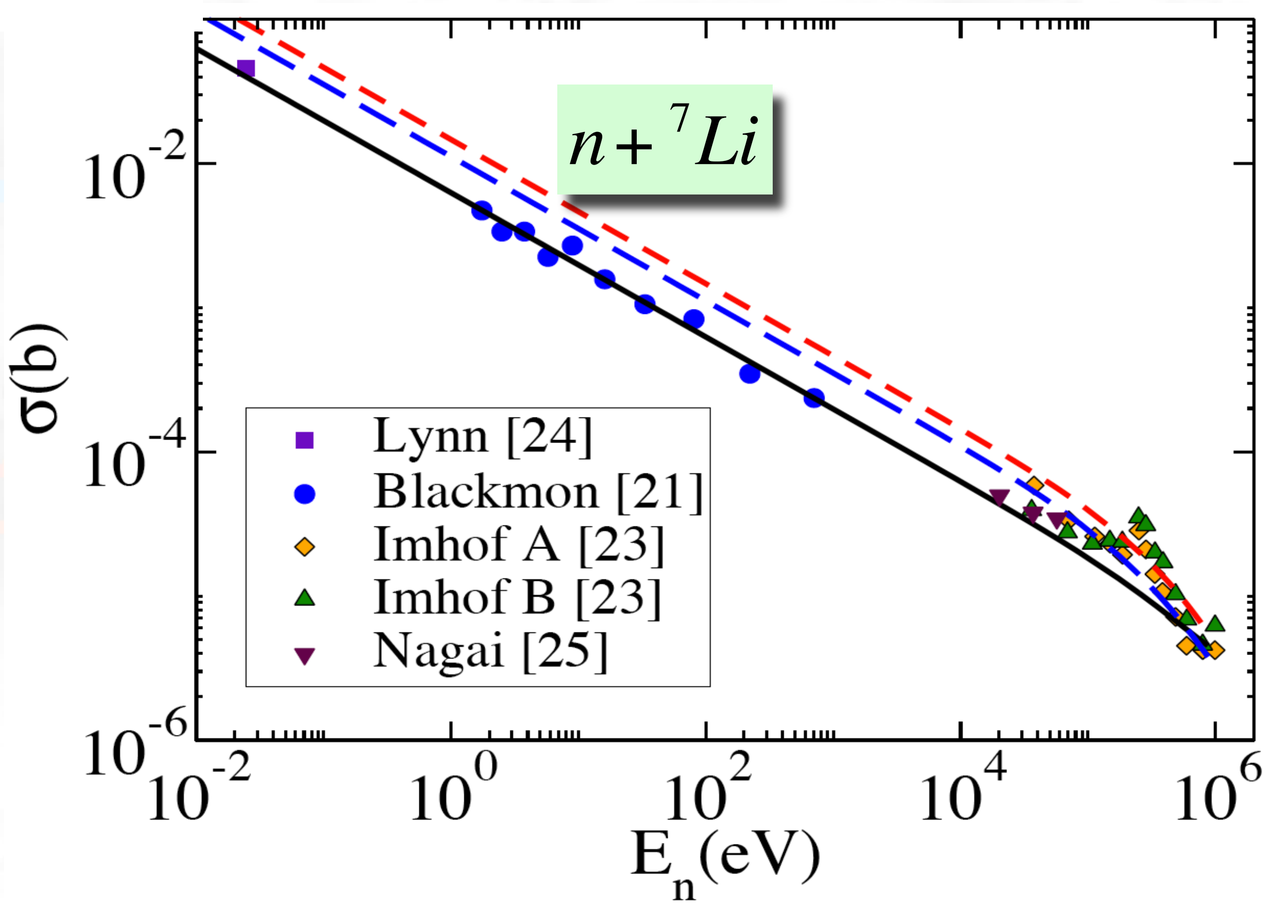
Bertulani, Hammer, van Kolck, NPA 712, 37 (2002)



Pion-less EFT - Example



Higa, Rupak, PRL 106, 222501 (2011)



Potential model

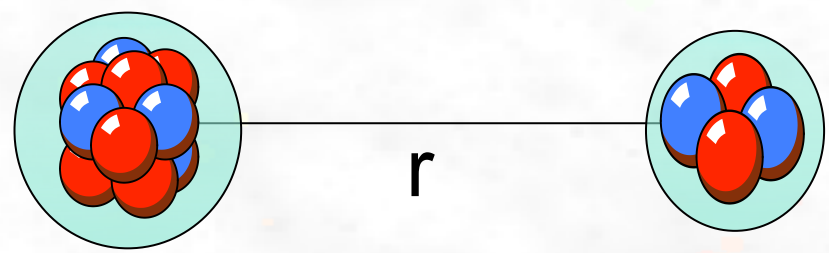
- Internal structure neglected
- Schrödinger equation:

Potential
Ex: Gauss,
Woods-Saxon

$$-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2}\psi_l(r)+\left[V(r)+\frac{\hbar^2}{2\mu}\frac{l(l+1)}{r^2}\right]\psi_l(r)=E\psi_l(r)$$

Very simple to solve numerically for $E < 0$ or $E > 0$

- initial state: scattering $E_i > 0$
  final state: bound $E_f < 0$



- Capture cross section (electric)

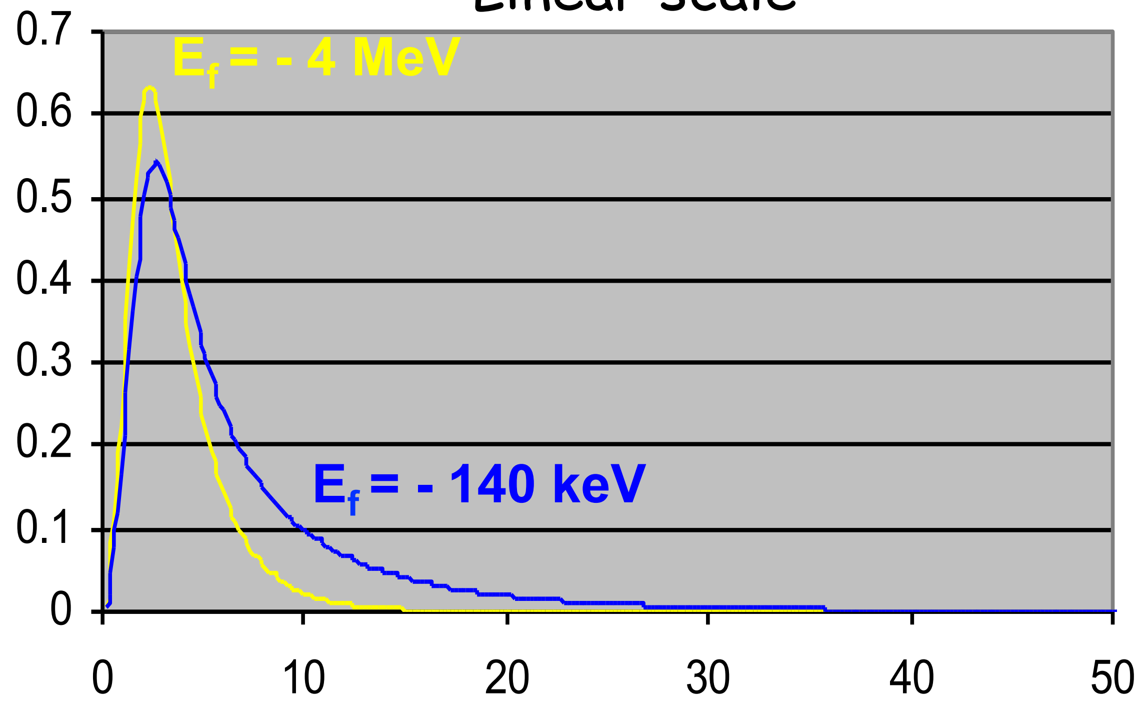
$$\sigma_{E\lambda} \sim \left|\int_0^\infty r^\lambda \psi^l(E_i,r)\psi^{l'}(E_f,r)dr\right|^2$$

Example: $^{7}Be(p,\gamma)^{8}B$

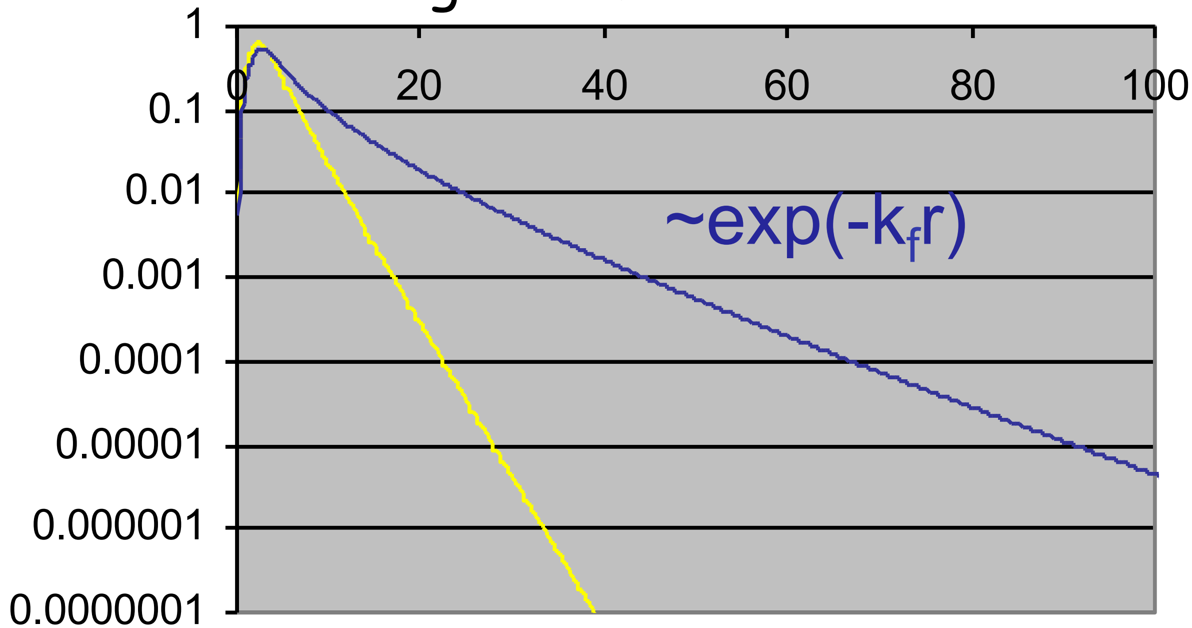
Bound state:
E_f = - 0.137 MeV

$$\psi_f(r)$$

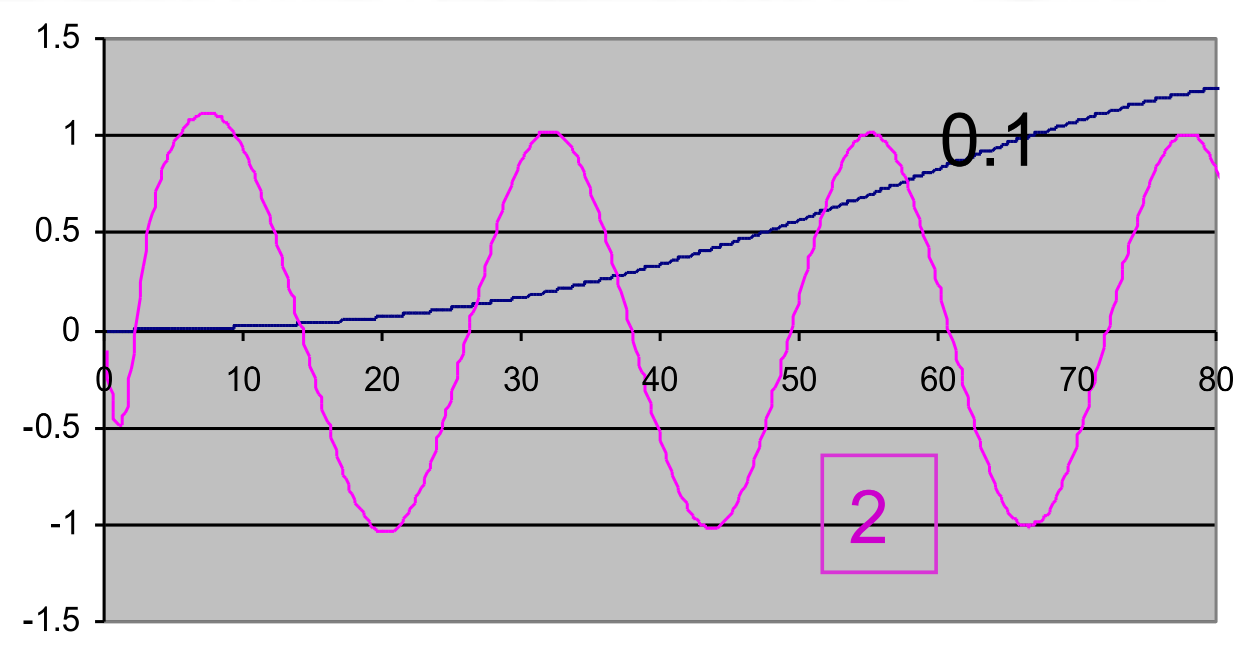
Linear scale



Logarithm scale



$^7Be(p,\gamma)^8B$ - potential model



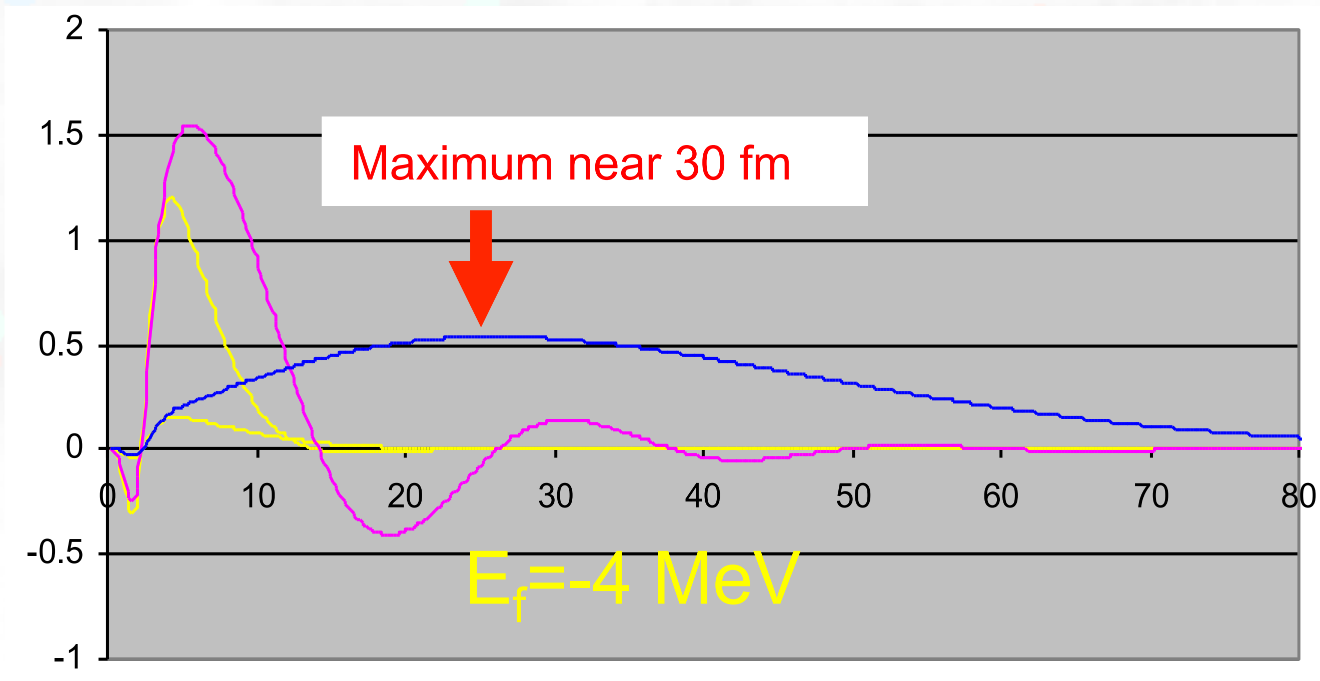
Initial state

E_i = 0.1 and 2 MeV

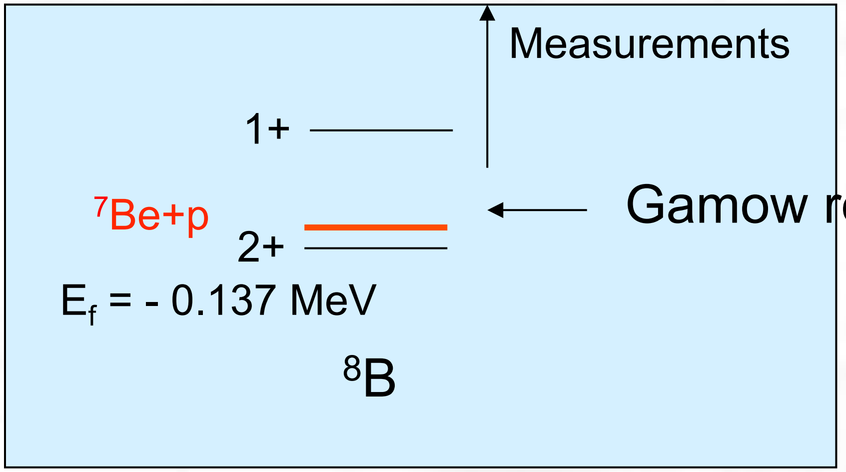
$$\psi_i(E,r)$$

Integrand

$$r\psi_i(E,r)\psi_f(r)$$

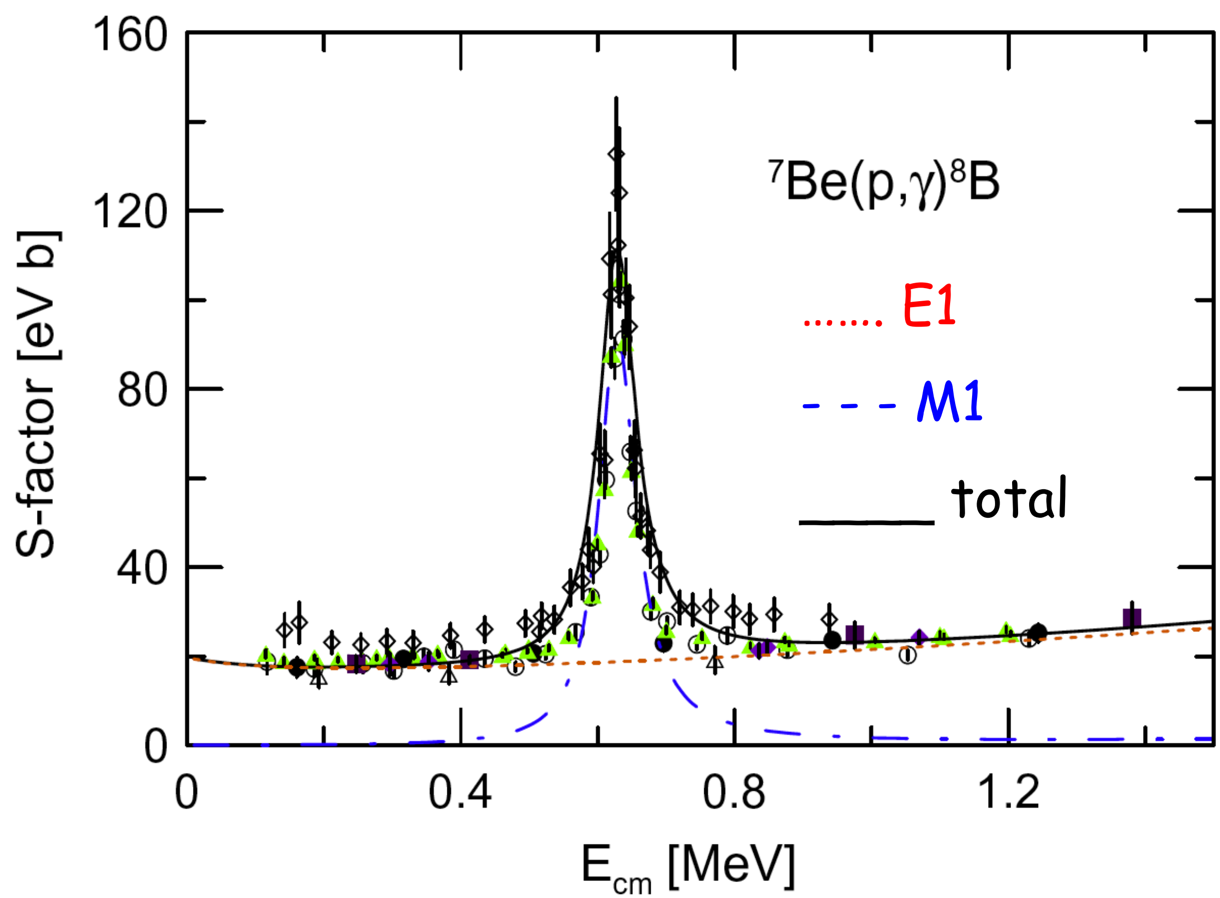


$^{7}Be(p,\gamma)^{8}B$ - potential model



Gamow region

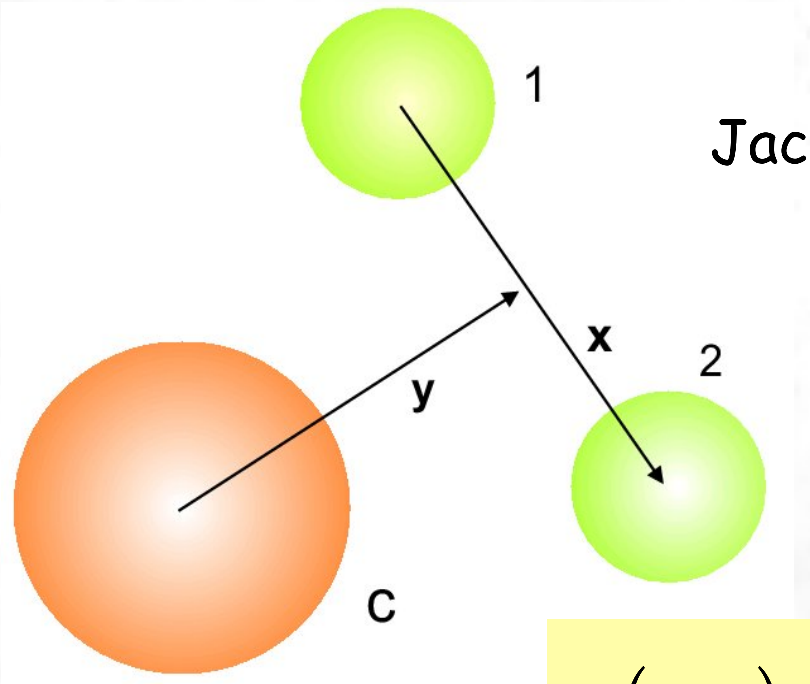
Bertulani, Z. Phys 356, 293 (1996)



M1: different sort of integrand

Explains 1+ resonance at E = 630 MeV

3-body model – example: hyperspherical harmonics



Jacobi coordinates (x, y)

Hyperspherical harmonics

$$\Psi(\mathbf{x},\mathbf{y}) = \frac{1}{\rho^{5/2}} \sum_{KLSl_xl_y} \Phi^{l_xl_y}_{KLS}(\rho)\left[\Gamma^{l_xl_y}_{KL}(\Omega_5)\otimes\chi_S\right]_{JM}$$

Hyperangle

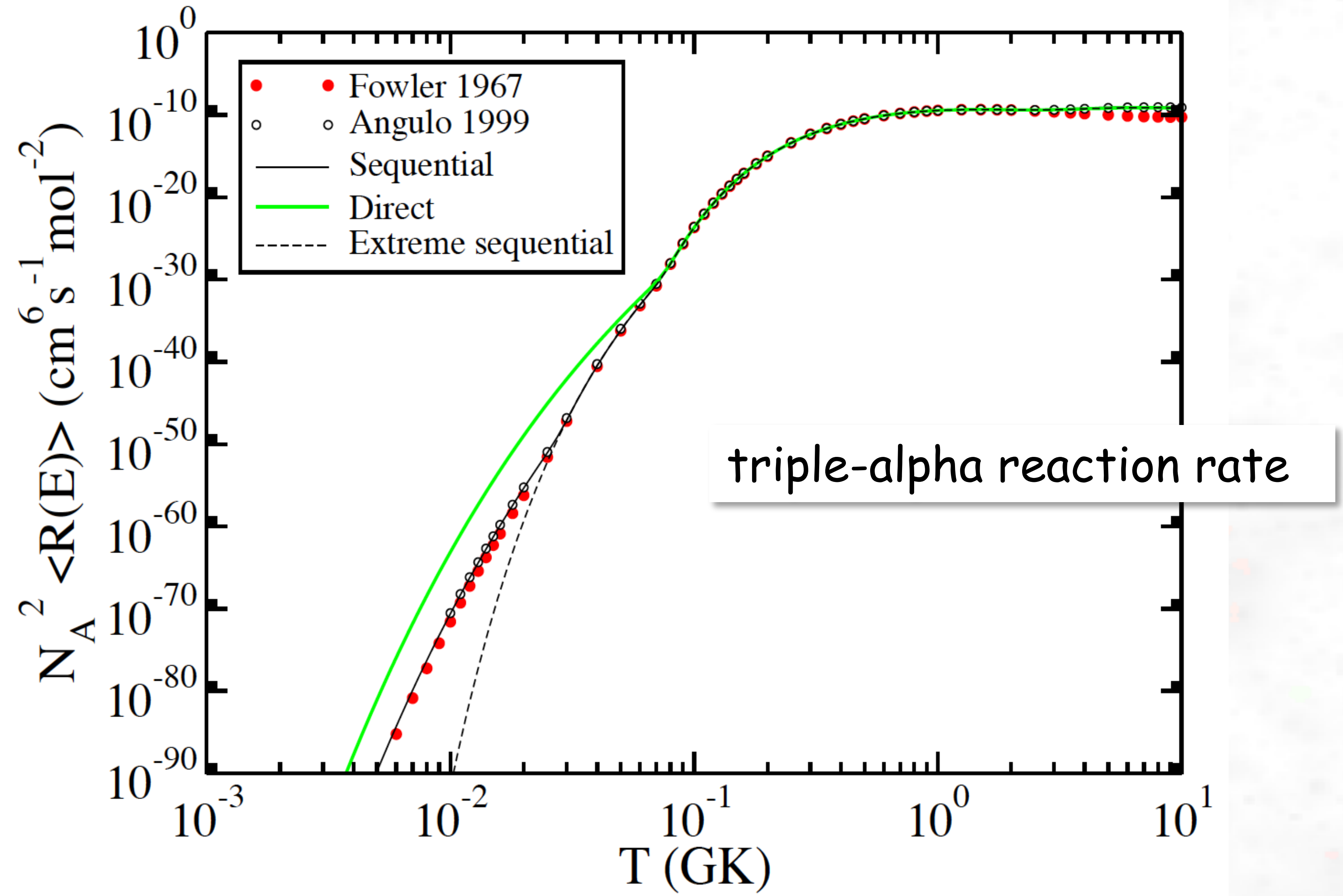
$$\Omega_5 = (\theta_x, \phi_x, \theta_y, \phi_y, \theta)$$

$$y = \rho\sin\theta, \qquad x = \rho\cos\theta$$

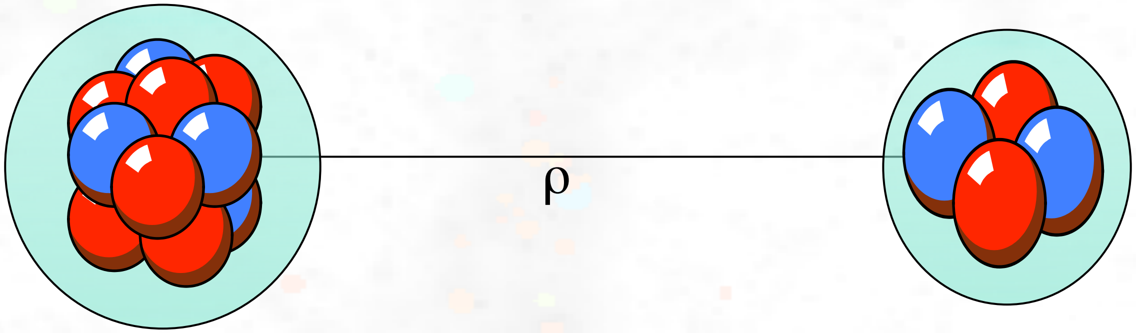
$$\sigma_{E1} \propto \left| \int dx\, dy \frac{\Phi_\alpha(\rho)}{\rho^{5/2}} y^2 x\, u_p(x) u_q(y) \right|^2$$

$$E_r = \frac{\hbar^2}{2m_N}\left(q^2 + p^2\right)$$

3-body model - example



Garrido, de Diego, Fedorov, Jensen, arXiv:1108.4811



Includes the internal structure of the nuclei: $\Phi_1\Phi_2$

$$\Psi = \mathcal{A}\ \Phi_1\Phi_2\, g(\rho)$$

Hamiltonian:

$$H = \sum_{i=1}^{A} T_i + \sum_{i<j}^{A} V_{ij} + \sum_{i<j<k}^{A} V_{ijk}$$

T_i = kinetic energy of nucleon *i*

V_{ij} = nucleon-nucleon **effective** interaction

C_{ij} = ANC

$$g(\rho) = \left\langle \chi^{(12)} \middle| \hat{A}\Phi_1\Phi_2\, \delta(\rho - \rho_{1,2}) \right\rangle$$

$$g_{bound}(\rho) \to C_{lj} \frac{W_{-\eta,l+1/2}(\rho)}{\rho}$$

$$g_{scat}(\rho \to \infty) \sim I_l(\rho) - S_l O_l(\rho)$$

Cluster models- Resonating Group Method (RGM)

Variational principle:

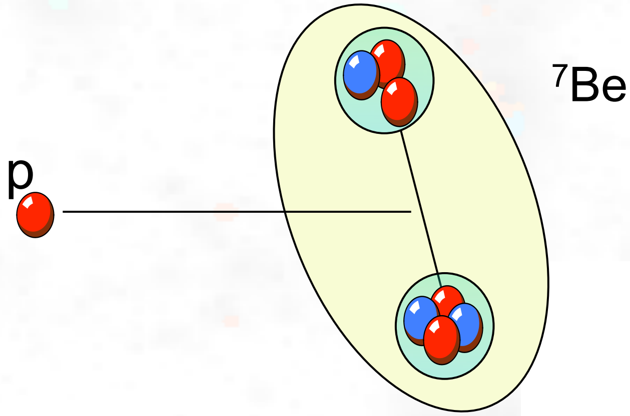
$$\delta\left[\frac{\langle\Psi_f|H|\Psi_i\rangle}{\langle\Psi_f|\Psi_i\rangle}\right]=0$$

$$\int d\rho'\left[H(\rho,\rho')-EN(\rho,\rho')\right]g(\rho')=0 \quad \text{all } \rho$$

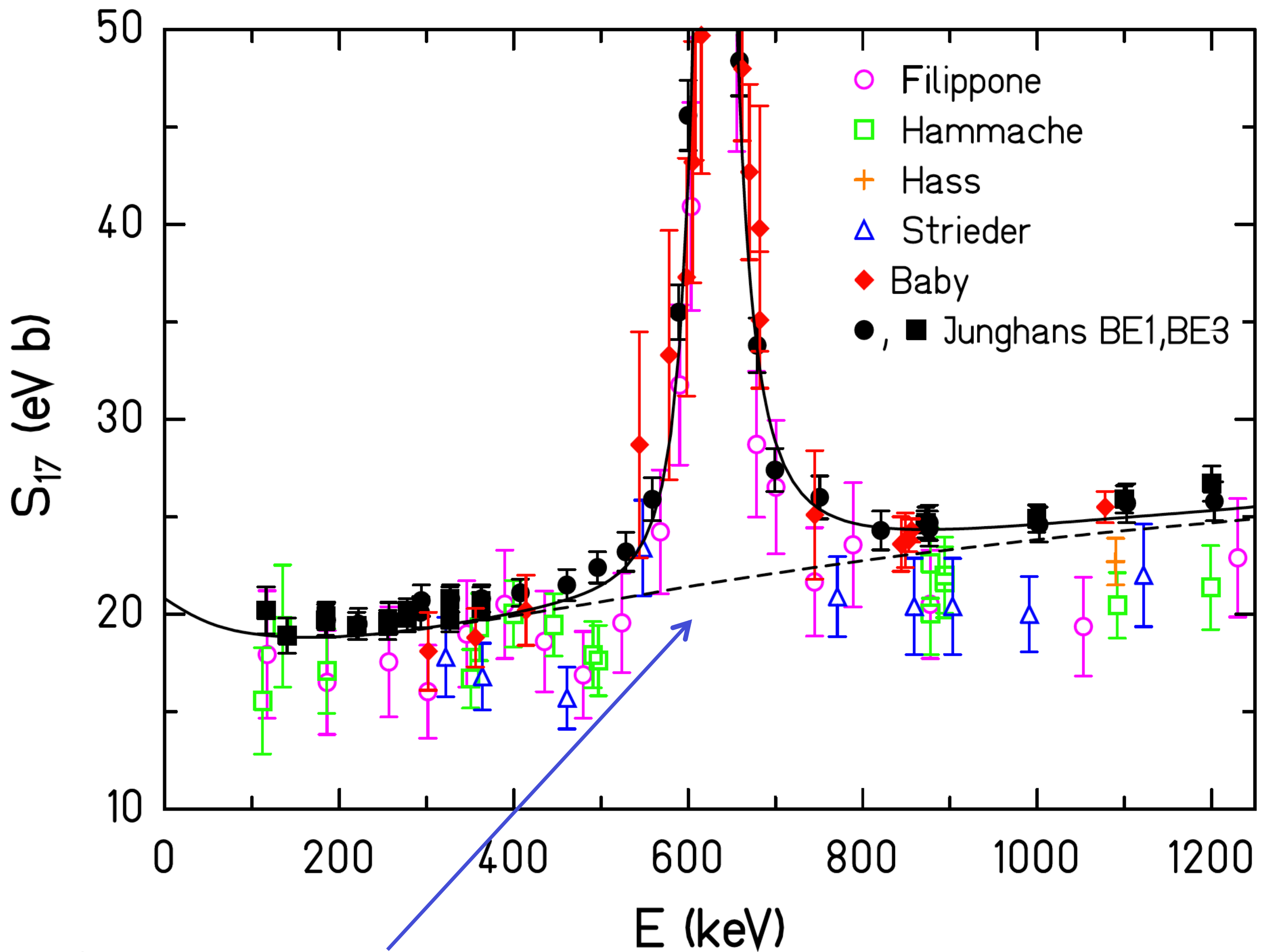
Hill-Wheeler (1955)

$$\left\{\begin{matrix} H \\ N \end{matrix}\right\}(\rho,\rho')=\left\langle \hat{\mathcal{A}}\Phi_1\Phi_2(\rho')\middle|\left\{\begin{matrix} H \\ 1 \end{matrix}\right\}\middle|\hat{\mathcal{A}}\Phi_1\Phi_2(\rho)\right\rangle$$

Cluster models: e.g., RGM



- Volkov (gaussians) effective interactions

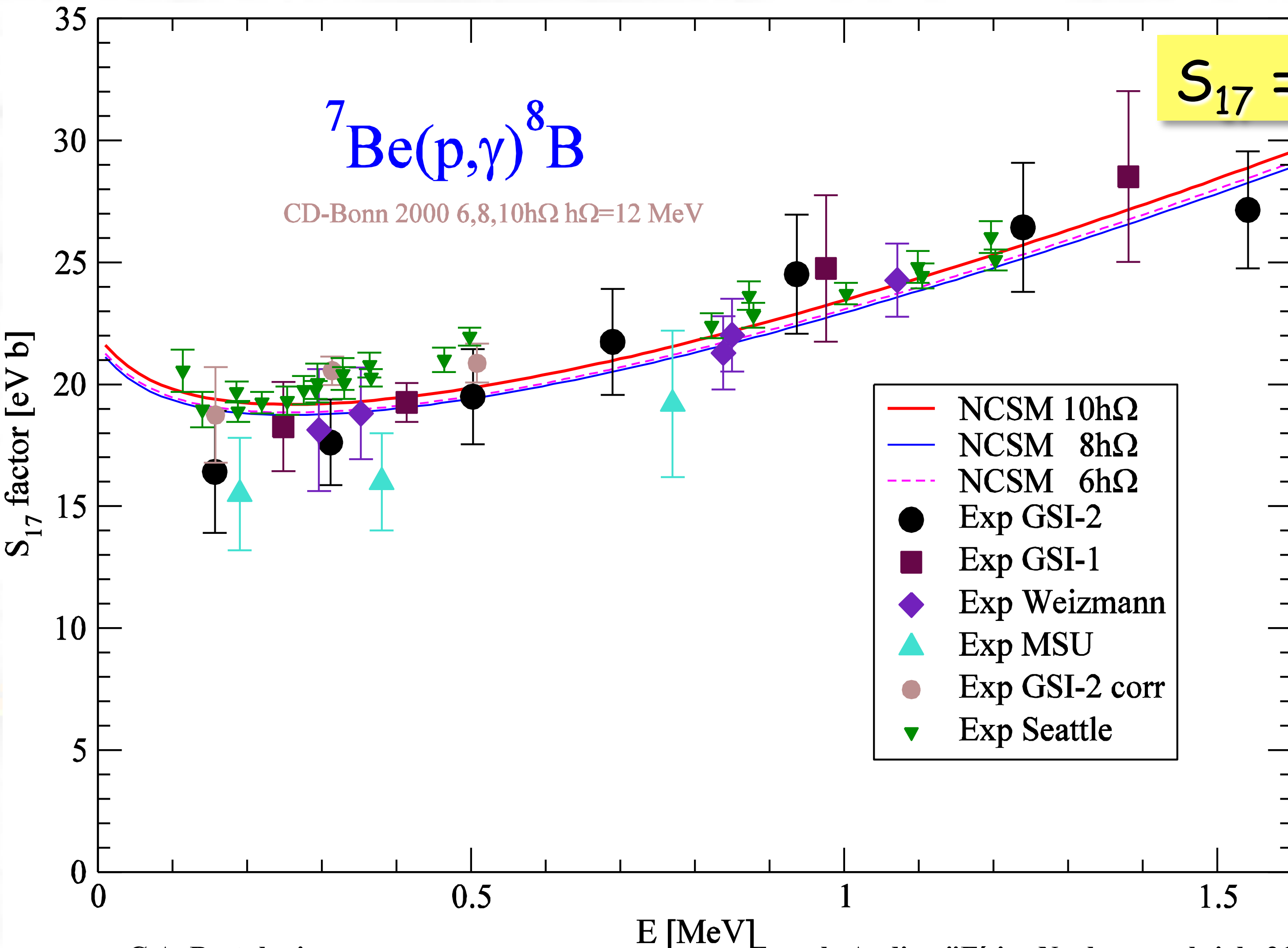


Descouvemont, Baye, 2004

Cluster models: no core shell model

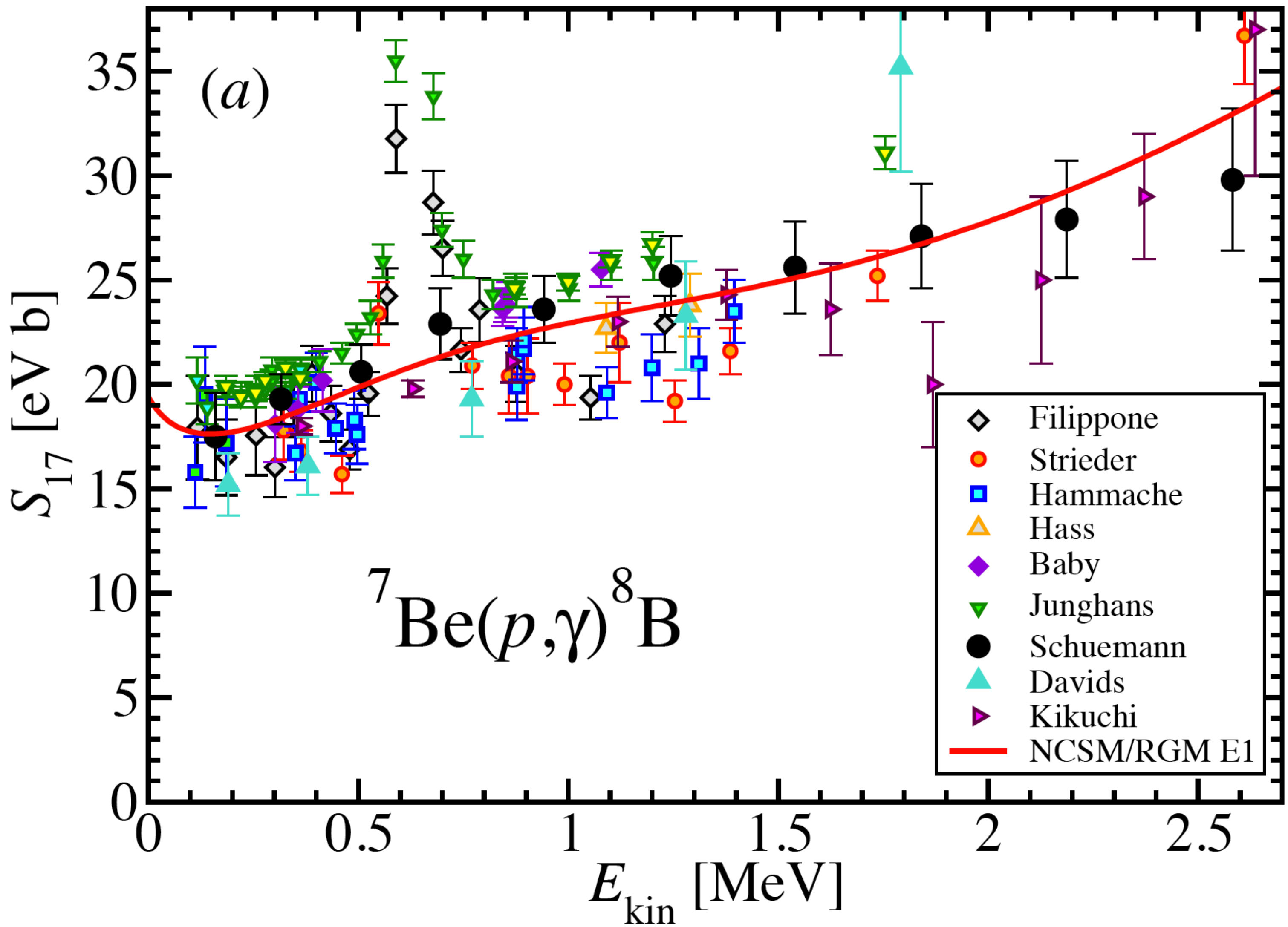
- Accurate wave functions of ^{7}Be – Ab-initio calculations
- In this example ^{7}Be+p scattering states defined in a potential model
- No Hill-Wheeler

Navratil, Bertulani, Caurier, PLB 2006
Navratil, Bertulani, Caurier, PRC 2006



S_{17} = 22 ± 1 eV.b

Cluster models: no core shell model + Hill-Wheeler



$S_{17} = 19.4 \pm 0.7$ eV.b

Quaglioni, Navratil, Roth, PLB 704 (2011) 379

Resonances: R-matrix theory

Instead of

$$-\frac{\hbar^2}{2\mu}\frac{d^2\Psi}{dr^2}+V\Psi=E\Psi$$

solve

$$-\frac{\hbar^2}{2\mu}\frac{d^2\phi_\lambda}{dr^2}+V\phi_\lambda=E_\lambda\phi_\lambda$$

with boundary conditions

$$r\left.\frac{d\phi_\lambda/dr}{\phi_\lambda}\right|_a=-b$$

a = channel radius
b = positive constant

ϕ_λ a complete basis: $\Psi=\sum_\lambda A_\lambda\phi_\lambda$

$$\left.\frac{rd\Psi/dr}{\Psi}\right|_a=\frac{1-b\mathcal{R}}{\mathcal{R}}$$

$$\mathcal{R}=\sum_\lambda\frac{\gamma_\lambda^2}{E_\lambda-E}$$

R-matrix
(here one channel)

$$\gamma_\lambda^2=\frac{\hbar^2\phi_\lambda^2(a)}{2\mu a}$$

reduced width
(does not depend on E)

One-channel R-Matrix theory

Outside the channel radius a: $\Psi \sim I + \mathcal{S}O$

I: e^{-ikr}, O: e^{ikr}, $\mathcal{S}$: S-matrix

Assume E near one of the E_λ → neglect all but channel α

Channel radius matching condition

$$\left.\frac{r\,d\Psi/dr}{\Psi}\right|_a = \frac{1-b\mathcal{R}}{\mathcal{R}}$$

$$\Gamma_\alpha = \frac{\hbar^2 k \phi_\alpha^2(a)}{m}$$

resonance width

$$\Rightarrow \quad \mathcal{S} = \left[1 + \frac{i\Gamma_\alpha}{(E_\alpha + \Delta_\alpha - E) - i\Gamma_\alpha/2}\right] e^{-2ika}$$

$$\Delta_\alpha = -\frac{b\Gamma_\alpha}{2ka}$$

resonance shift

$$\sigma = \frac{\pi}{k^2}\left|1-\mathcal{S}\right|^2 = \frac{\pi}{k^2}\left|\underbrace{e^{2ika} - 1}_{\text{non-resonant}} + \underbrace{\frac{i\Gamma_\alpha}{(E_\alpha + \Delta_\alpha - E) - i\Gamma_\alpha/2}}_{\text{Breit-Wigner}}\right|^2$$

Multi-channel R-Matrix theory

Generalize to possible many channels also using many a's

$$\mathcal{R}_{\alpha\alpha'} = \sum_{\lambda} \frac{\gamma_{\lambda\alpha}\gamma_{\lambda\alpha'}}{E_\lambda - E}$$

$$\mathcal{S}_{\alpha\alpha'} = \frac{I(a)}{O(a)}\left[\frac{1 - L^{*}\mathcal{R}}{1 - L\mathcal{R}}\right]$$

$$L = \left.\frac{r d\Psi / dr}{\Psi}\right|_a$$

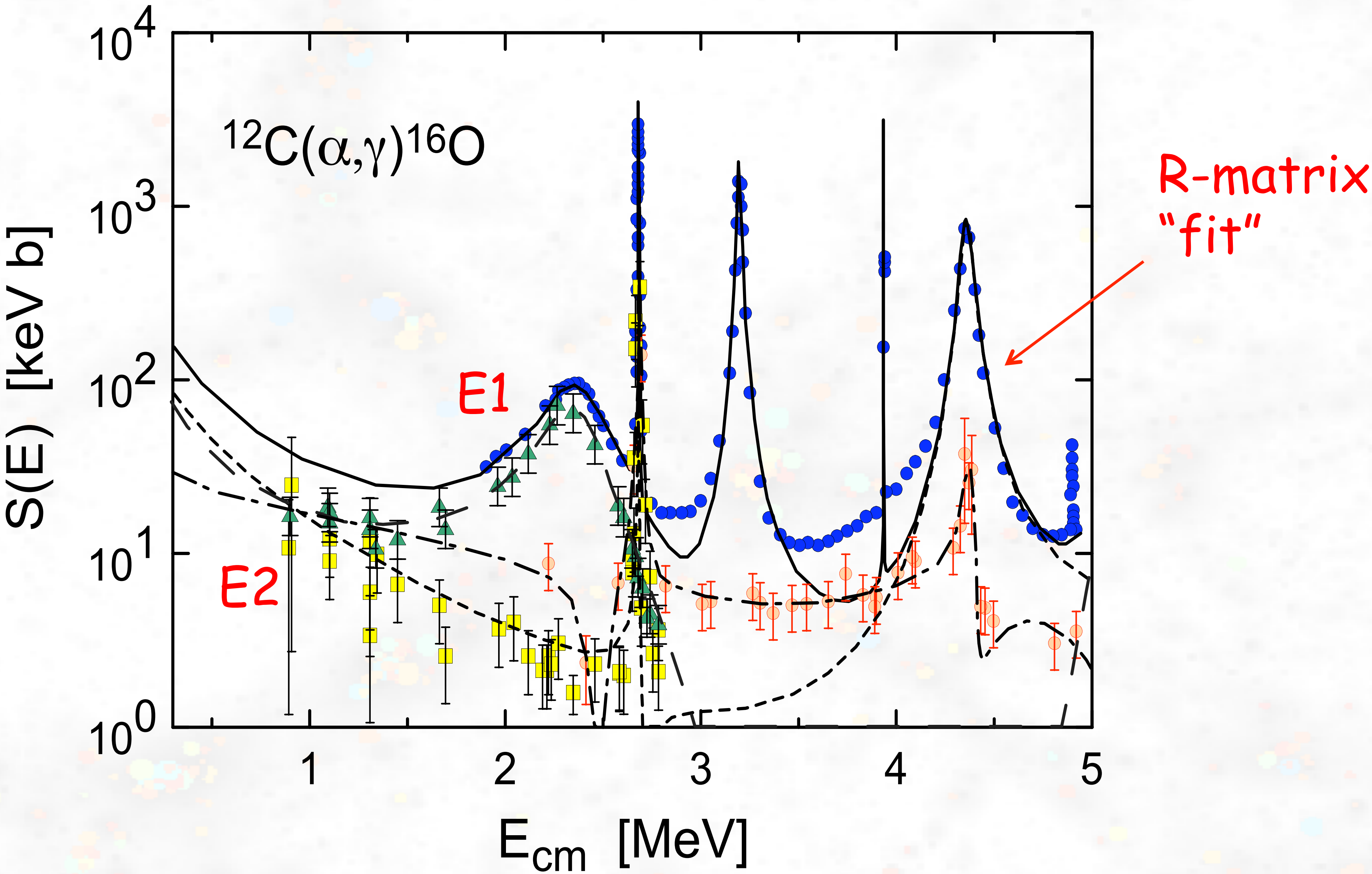
For each channel transition $\alpha \rightarrow \alpha'$

Finally:

$$\sigma_{\alpha\alpha'} = \frac{\pi}{k_\alpha^2}\left|\delta_{\alpha\alpha'} - \mathcal{S}_{\alpha\alpha'}\right|^2 = \frac{\pi}{k_\alpha^2} T_{\alpha\alpha'}$$

transition (transmission) probability (matrix)

R-Matrix theory - example



Compound nucleus theory

Heisenberg relation: $\Delta E \Delta t \sim \hbar$ → for a state with width Γ

→ decay time: $\Delta t \sim \dfrac{\hbar}{\Gamma_\alpha}$

If many decay channels → decay probability = $\dfrac{\Gamma_\alpha}{\sum_\alpha \Gamma_\alpha} = \dfrac{\Gamma_\alpha}{\Gamma}$

Bohr hypothesis: formation independent of decay

$$\sigma_{\alpha\alpha'} = \sigma_{CN}(\alpha)\frac{\Gamma_{\alpha'}}{\Gamma}$$

a + b
or
c + d
or ...

α → formation → X → α' → decay

Compound nucleus theory - Ewing-Weisskopf

detailed balance: $g_\alpha k_\alpha^2 \sigma_{\alpha\alpha'} = g_{\alpha'} k_{\alpha'}^2 \sigma_{\alpha'\alpha}$

spin counting

for CN:

$$g_\alpha k_\alpha^2 \sigma_{CN}(\alpha)\Gamma_{\alpha'} = g_{\alpha'} k_{\alpha'}^2 \sigma_{CN}(\alpha')\Gamma_\alpha$$

$$\frac{\Gamma_{\alpha'}}{g_{\alpha'} k_{\alpha'}^2 \sigma_{CN}(\alpha')} = \frac{\Gamma_\alpha}{g_\alpha k_\alpha^2 \sigma_{CN}(\alpha)} \quad \Rightarrow \quad \Gamma_\alpha = g_\alpha k_\alpha^2 \sigma_{CN}(\alpha)$$

introducing density of levels ρ of final states:

$$\sigma_{\alpha\alpha'} = \sigma_{CN}(\alpha) \frac{\left(2J_{\alpha'}+1\right)\mu_{\alpha'} E_{\alpha'} \sigma_{CN}(\alpha')\rho(E_{\alpha'})}{\sum_\alpha \int \left(2J_\alpha+1\right)\mu_\alpha E_\alpha \sigma_{CN}(\alpha)\rho(E_\alpha)\, dE_\alpha}$$

Compound nucleus theory - Hauser-Feshbach

Herman Feshbach: "In a rainy weekend in Boston I've written a paper where I included the angular momentum in the Ewing-Weisskopf theory..."

CN ang. mom.

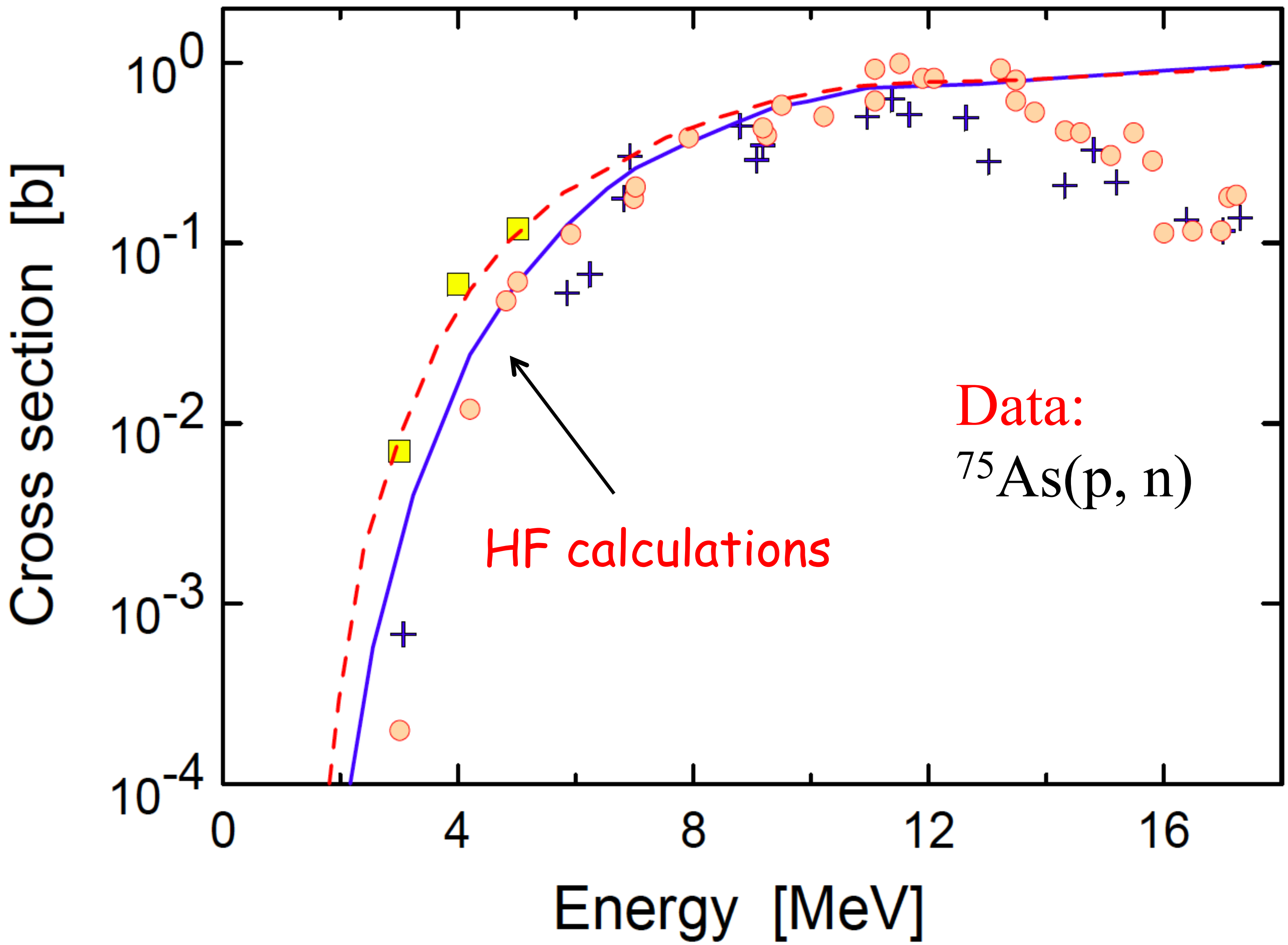
$$\sigma_{\alpha\alpha'} = \frac{\pi}{k^2} \sum_J \frac{2J+1}{(2i_c+1)(2I_c+1)} \frac{\sum_{s,l} T_{l,s}(c) \sum_{s',l'} T_{l',s'}(c')}{\sum_c \sum_{s,l} T_{l,s}(c)}$$

projectile spin

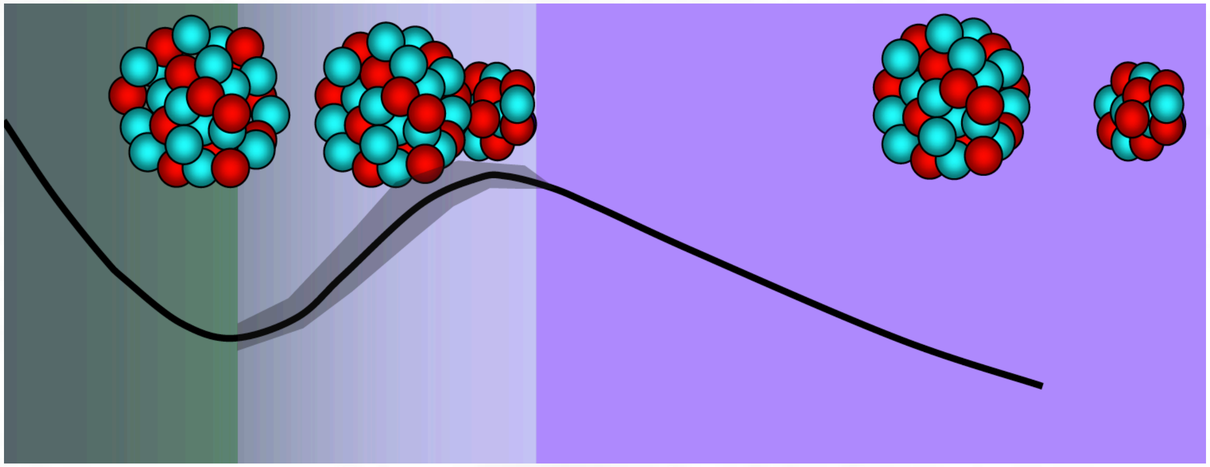
target spin

transmission probability

Hauser-Feshbach Theory - Example



Fusion of heavy nuclei



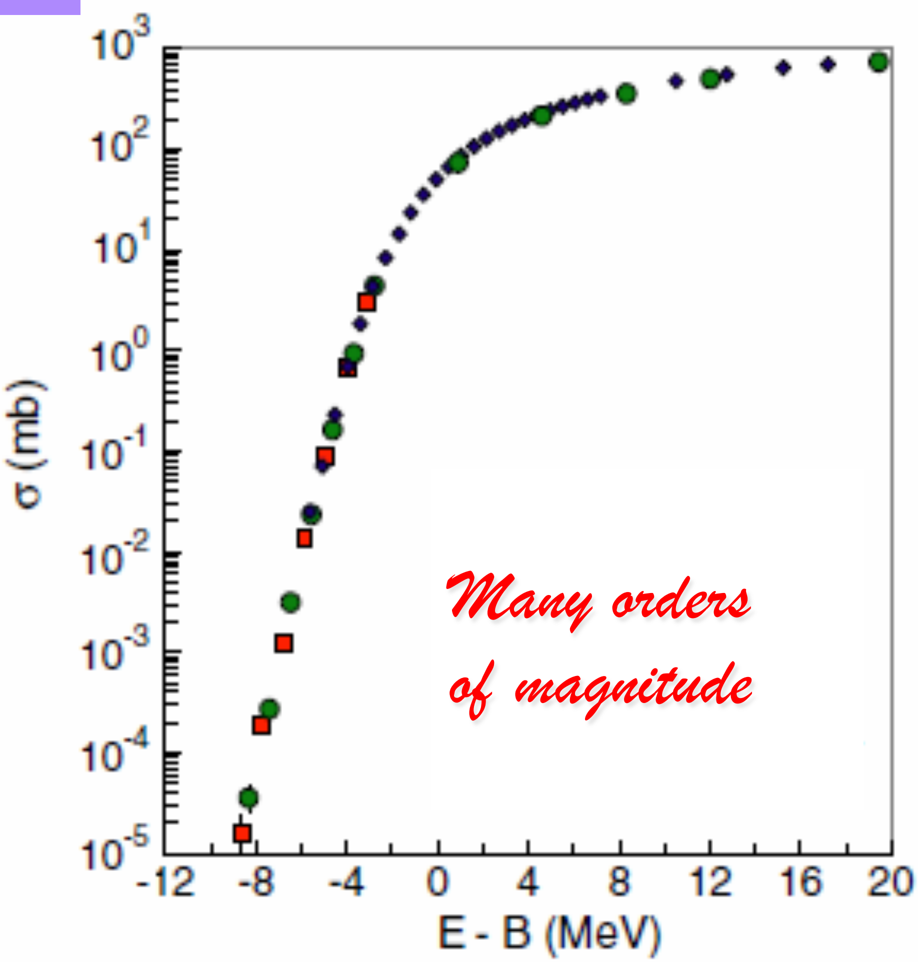
Fusion cross section

$$\sigma(\mathrm{E}) = \frac{\pi\hbar^2}{2\mu\mathrm{E}} \sum_l (2l+1) T_l(E)$$

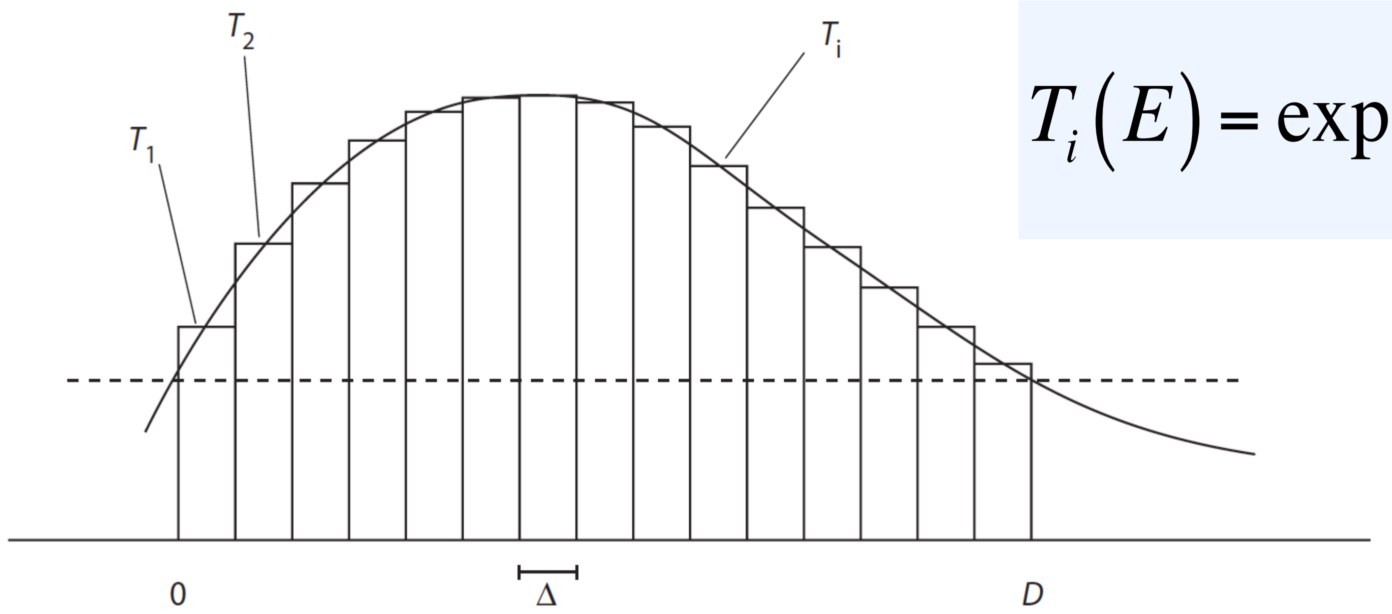
Fusion or transmission probability

or

"Penetrability factor"



Fusion of heavy nuclei – Barrier penetration model



$$T_i(E) = \exp\left[-\frac{2}{\hbar}\Delta\sqrt{2\mu(U-E)}\right]$$

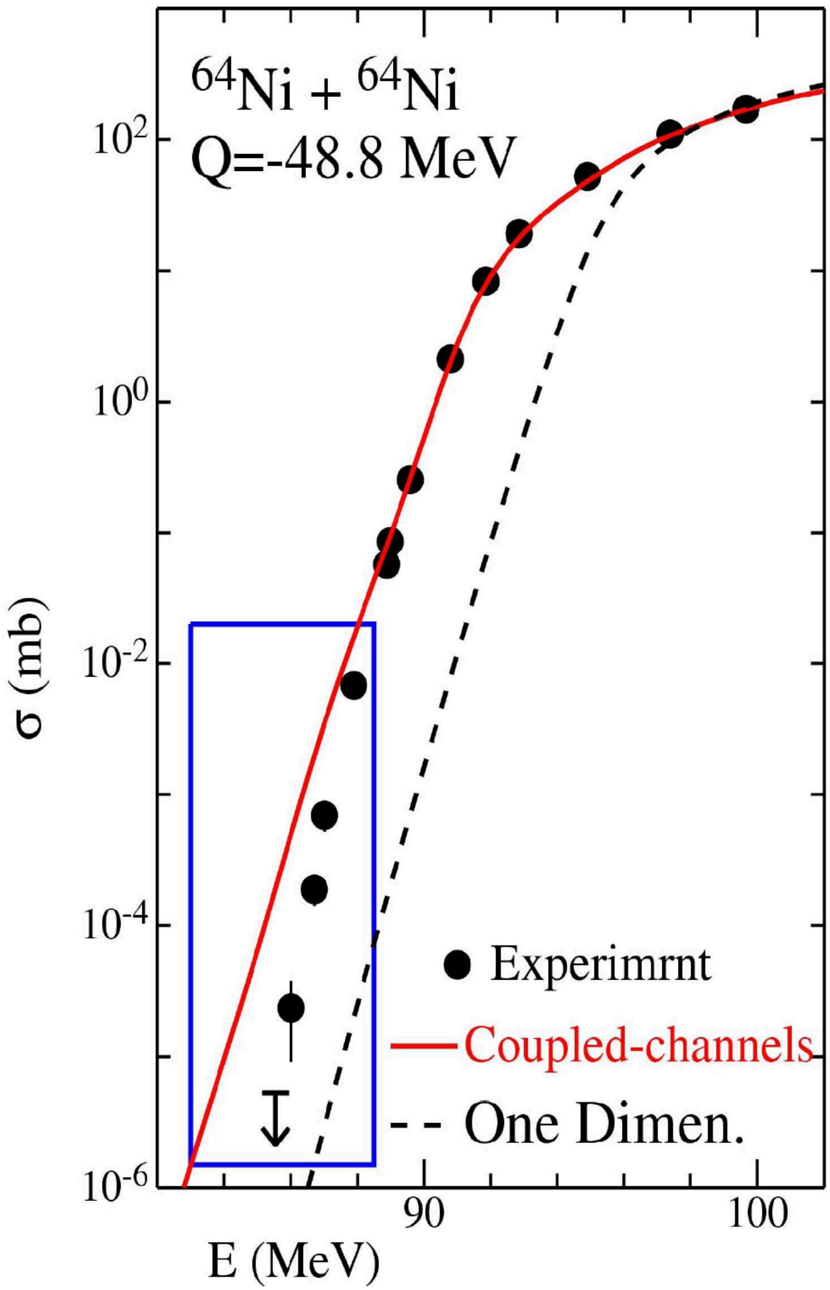
$$T(E) = \prod_i T_i(E)$$

(e.g., WKB approximation)

$$T_l(E) = \exp\left[-\frac{2}{\hbar}\int_{R_1}^{R_2} dr\sqrt{2\mu\left[U_l(r)-E\right]}\right]$$

Barrier Penetration Model (BPM)

Fusion of heavy nuclei – example

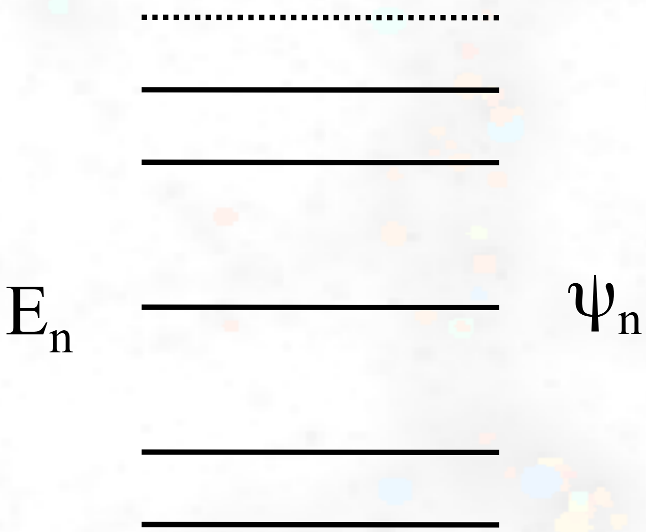


C.L. Jiang et al, PRL 93, 012701 (2004)

Often BPM does not work

Coupled-channels and/or microscopic models often necessary.

Coupled-channels equations (t.d. version)



$H = H_0 + U$ solution

$$\psi = \sum_n a_n(t)\psi_n e^{-iE_n t/\hbar}$$

H_0 spectrum: $H_0\psi_n = E_n\psi_n$

$$H\psi = i\hbar\frac{\partial\psi}{\partial t}$$

$$\frac{da_k}{dt} = -\frac{i}{\hbar}\sum_n a_n(t)U_{kn}(t)e^{i(E_k-E_n)t/\hbar}$$

$$U_{kn}(t) = \int \psi_k^* U(t)\,\psi_n\,d^3r$$

1st order: $a_n \sim \delta_{n0}$

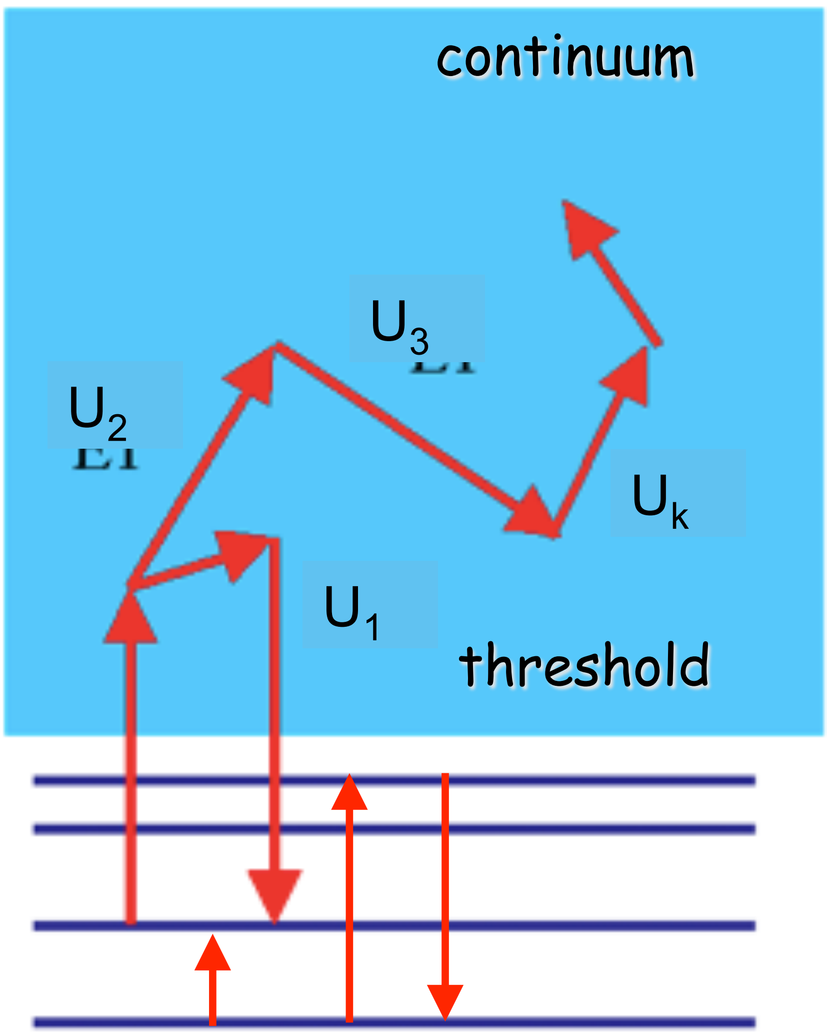
$$a_k = -\frac{i}{\hbar}\int dt\,U_{k0}(t)\,e^{i(E_k-E_0)t/\hbar}$$

Continuum discretized coupled-channels (CDCC)

$$|\varphi_0\rangle = |E_0, J_0 M_0\rangle e^{-iE_0 t/\hbar}$$

$$|\varphi_{jJM}\rangle = e^{-iE_j t/\hbar} \int \Gamma_j(E) |E, JM\rangle dE$$

$$\int \Gamma_i(E) \Gamma_j(E) dE = \delta_{ij}$$



Bertulani, Canto, NPA 539, 163 (1992)

Continuum discretized coupled-channels

(Semiclassical)

$$\frac{da_k}{dt} = -\frac{i}{\hbar} \sum_j a_j(t) U_{kj}(t) e^{i(E_k - E_j)t/\hbar}$$

$$U_{kj}(t) = \int \psi_k^* U(t)\, \psi_j\, d^3 r$$

Quantum version in coordinate space straightforward

Schroedinger equation on a space-time lattice

$$i\hbar\frac{\partial\psi}{\partial t}=\hat{H}\psi$$

$$\psi(t+\Delta t)=\exp\left[-\frac{i}{\hbar}\hat{H}\Delta t\right]\psi(t)$$

$$H=\frac{\hat{p}^2}{2\mu}+V_N(x)+U(x,t)$$

discrete space lattice: x_j, $j = 1, 2, \ldots, N$

$$\hat{S}\psi_j(t)=\sum_k U_k\psi_k(t)$$

$$\psi_j(t+\Delta t)=\frac{\left[\frac{1}{i\tau}+\Delta^{(2)}-\frac{\Delta t}{2\hbar\tau}V_{Nj}+\frac{\Delta t}{\hbar\tau}\hat{S}\right]}{\left[\frac{1}{i\tau}-\Delta^{(2)}+\frac{\Delta t}{2\hbar\tau}V_{Nj}\right]}\psi_j(t)$$

$$\tau=\frac{\hbar\Delta t}{4\mu(\Delta x)^2}$$

Good to order $(\Delta t)^3$
preserves unitarity

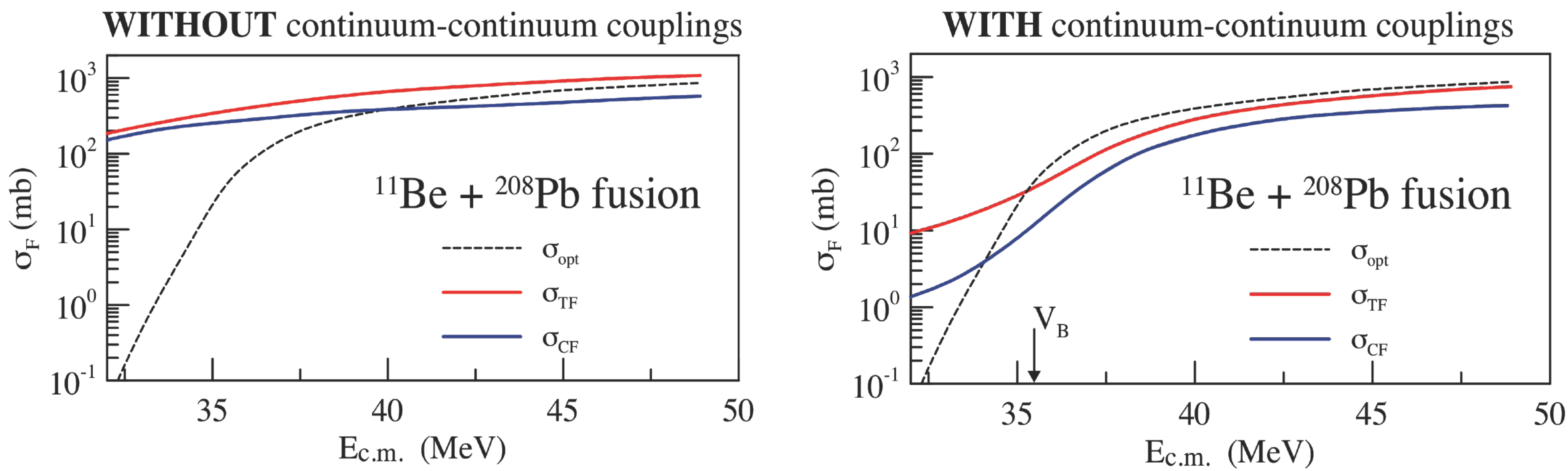
$$\Delta^{(2)}=\left(\psi_{j-1}-2\psi_j+\psi_{j+1}\right)/2(\Delta x)^2$$

Bertsch, Bertulani
NPA 556, 136 (1993)
PRC 49, 2839 (1994)

→ One-particle in three-dimensions straightforward

→ Extension to many-body: e.g., TDHF theory

CDCC fusion calculations: example



Canto, Fusion 2011

Continuum-continuum couplings hinders fusion …. but what is the mechanism?

Coupled channels one of the least controllable calculations: couplings can add as +-+--++-+ or as +++-+++ or ----+----, depending on the system

→ Suppression or enhancements are difficult to understand.

End of part I