

Nuclear Astrophysics with Radioactive Beams



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Part II

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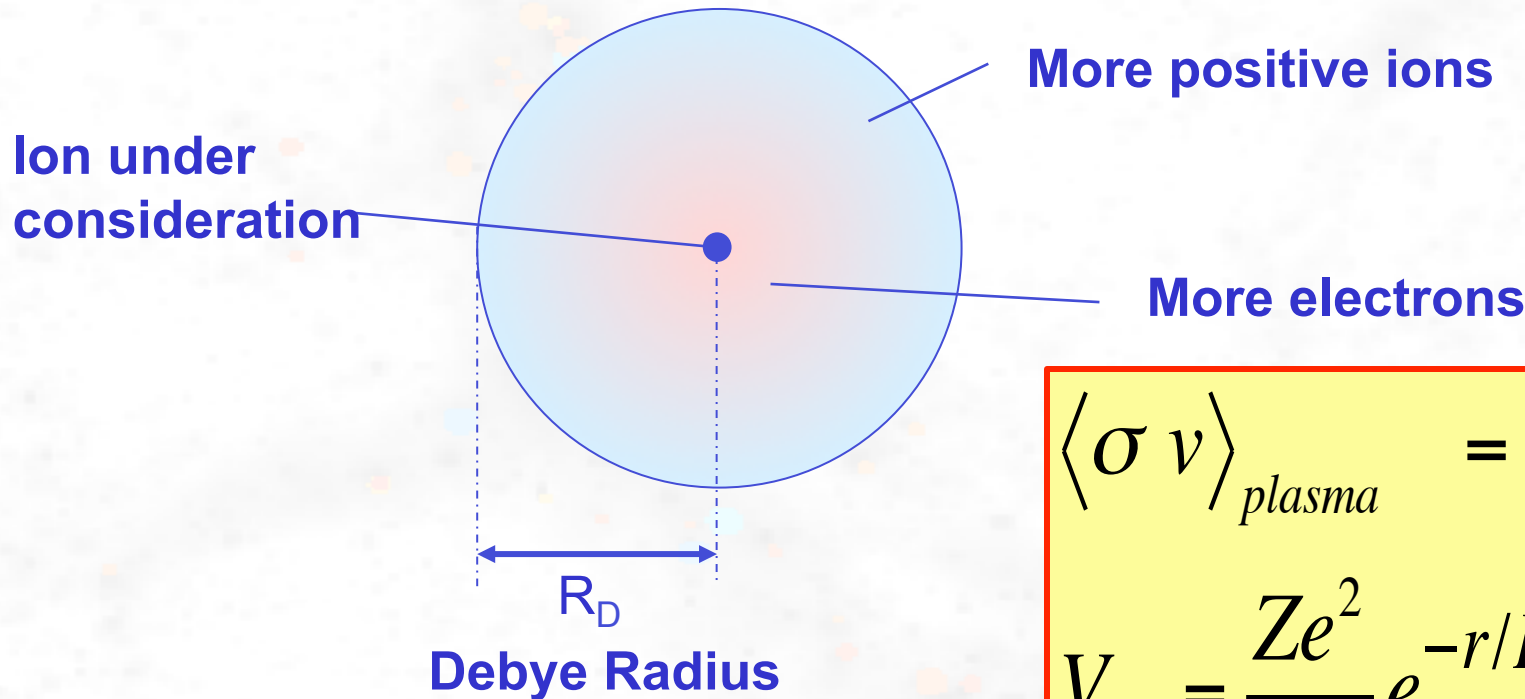
Web-page: <http://faculty.tamu-commerce.edu/cbertulani/>

Electron screening in plasmas and in experiments

Electron screening (in stars)

Debye-Hueckel screening (Salpeter 1959)

$$n R_D^3 \gg 1$$



$$\langle \sigma v \rangle_{\text{plasma}} = f(E) \langle \sigma v \rangle_{\text{bare}}$$
$$V_{\text{eff}} = \frac{Ze^2}{r} e^{-r/R_D}$$

$$f(E) = e^{U_e/kT}, \quad R_D \sim \sqrt{kT/\rho} \sim 0.218 \text{ \AA}^0 \text{ (Sun)}$$

$${}^7\text{Be}(p, \gamma){}^8\text{B} \quad (T \sim 10^7 \text{ K}): \quad f(E) \sim 1.2 \quad (20 \% \text{ effect})$$

Gamow energy \gg thermal energy (dynamical screening)

Carraro, Schaefer, Koonin, ApJ 1988

$$\alpha = v / v_{thermal}, \quad s_{12} = \text{dynamic} / \text{Debye}$$

Reaction	Gamow Energy (E_g/T)	s_{12}
$p-p$	4.6	0.76
${}^3\text{He}-{}^3\text{He}$	16.6	0.75
${}^3\text{He}-{}^4\text{He}$	17.3	0.76
$p-{}^7\text{Be}$	13.9	0.80
$p-{}^{14}\text{N}$	20.6	0.82

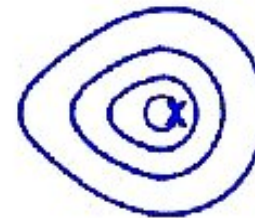
$\alpha=0$



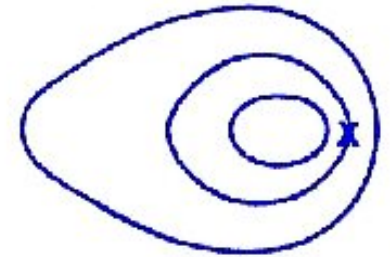
$\alpha=7$



$\alpha=20$



$\alpha=200$

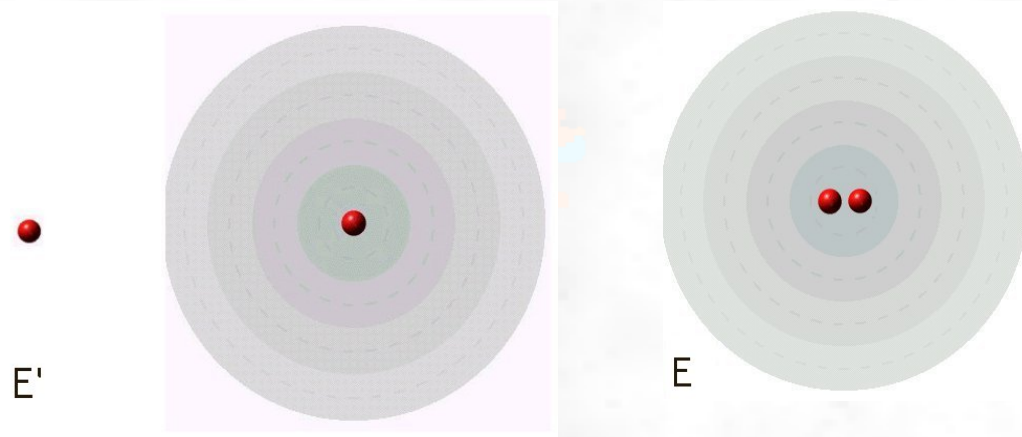


Fluctuations in the Debye sphere

$$n R_D^3 \sim 3 - 5$$

- One cannot derive the screening from thermodynamics but one has to resort to kinetic equations.
- Deviations from Debye-Hueckel can be large.

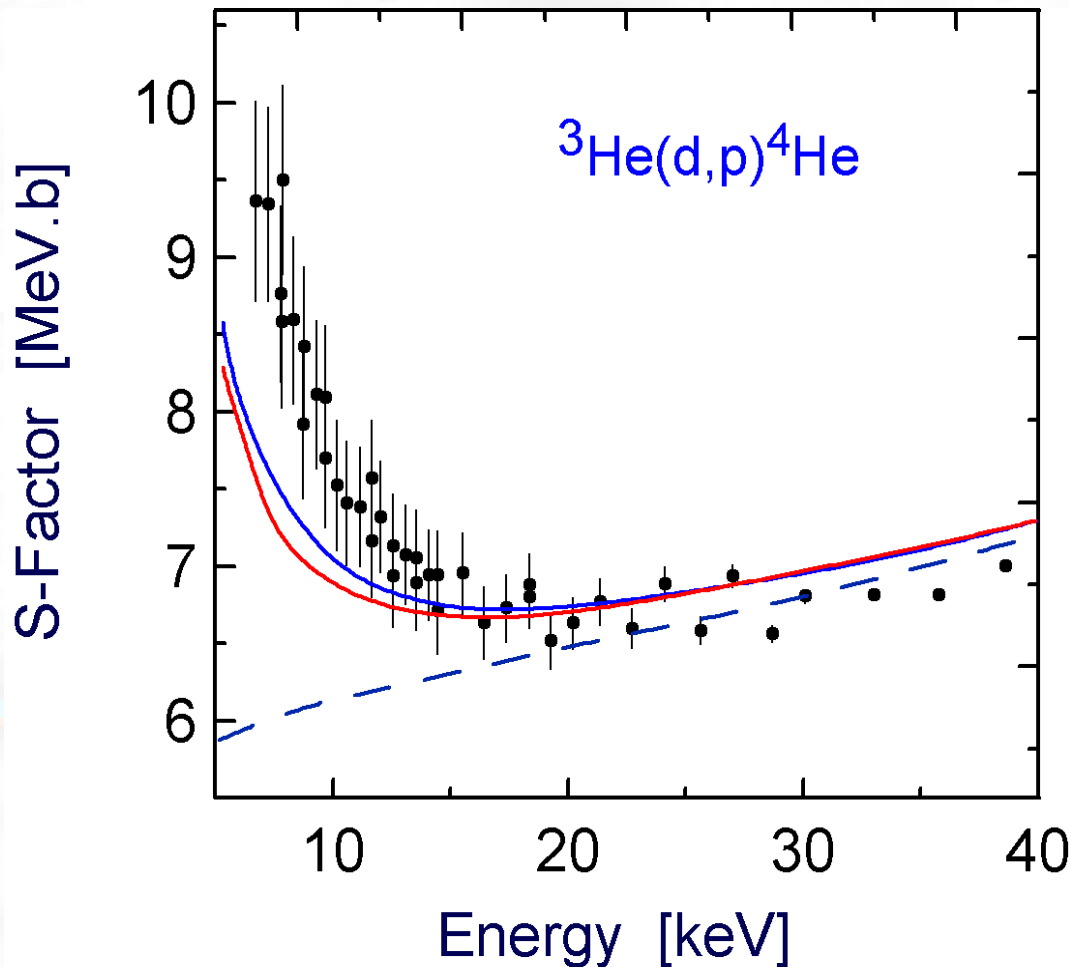
Electron screening (in the laboratory)



Adiabatic model: $\Delta E = E' - E$

$$\sigma_{lab}^{fusion} \sim \sigma_{bare}(E + \Delta E)$$

$$\sim \exp\left[\pi \eta(E) \frac{\Delta E}{E}\right] \sigma_{bare}(E)$$



--- S_{bare}
 — Dynamic
 — Adiabatic

Reaction	ΔE [eV] experiment	ΔE [eV] adiabatic limit
$d({}^3\text{He}, p){}^4\text{He}$	180 ± 30	119
${}^6\text{Li}(p, \alpha){}^3\text{He}$	470 ± 150	186
${}^6\text{Li}(d, \alpha){}^4\text{He}$	380 ± 250	186
${}^7\text{Li}(p, \alpha){}^4\text{He}$	300 ± 280	186
${}^{11}\text{B}(p, \alpha){}^2{}^4\text{He}$	620 ± 65	348

Amplification of small effects

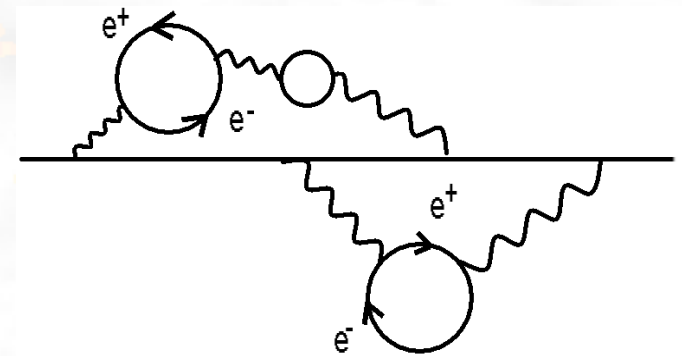
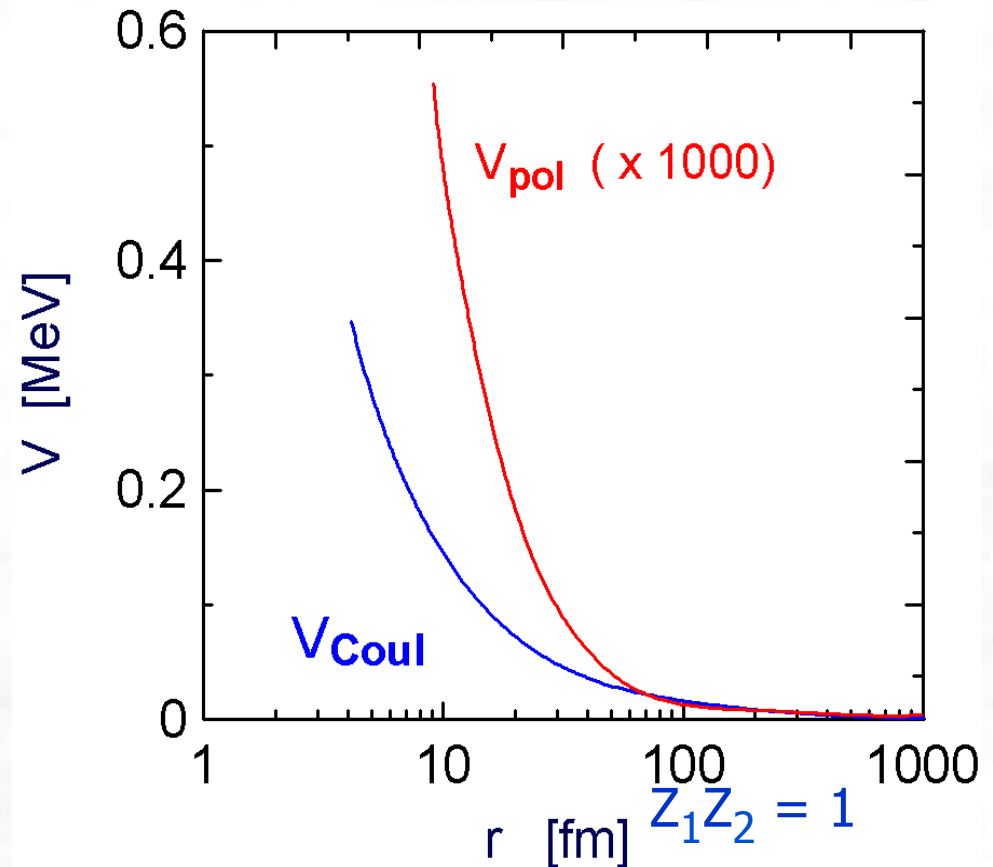
- Thermal motion, lattice vibrations, beam energy spread
- Nuclear breakup channels (in weakly-bound nuclei)
- Dynamics of tunneling

Balantekin, Bertulani, Hussein
NPA 627, 324 (1997)

<i>Corrections</i>	
Vacuum Polarization	$\sim 1\%$
Relativity	10^{-3}
Bremsstrahlung	10^{-3}
Atomic polarization	10^{-5}
Nuclear polarization	$< 10^{-10}$

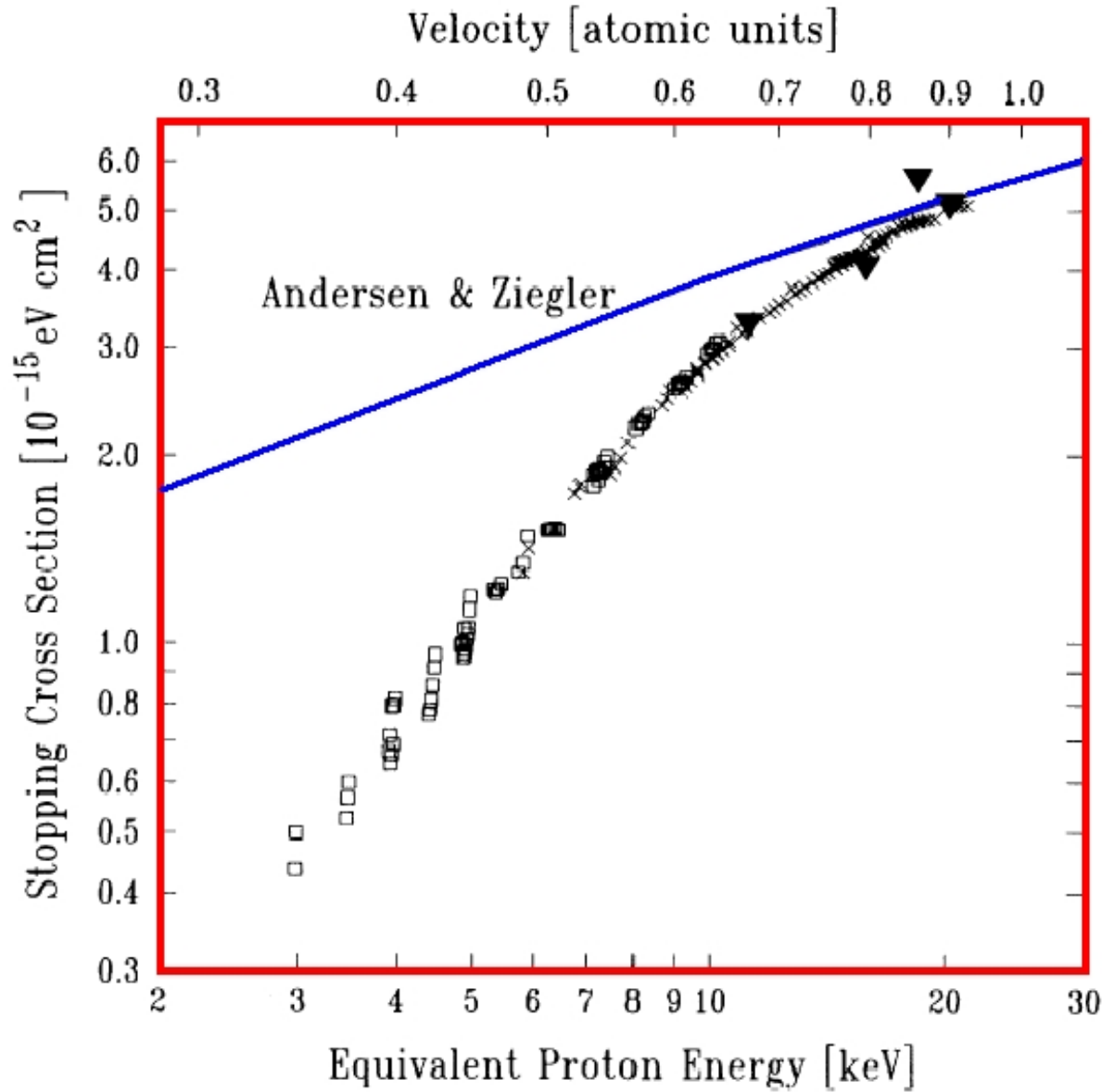
all $\leq 1\%$

Not a solution! (we need $\sim 100\%$)

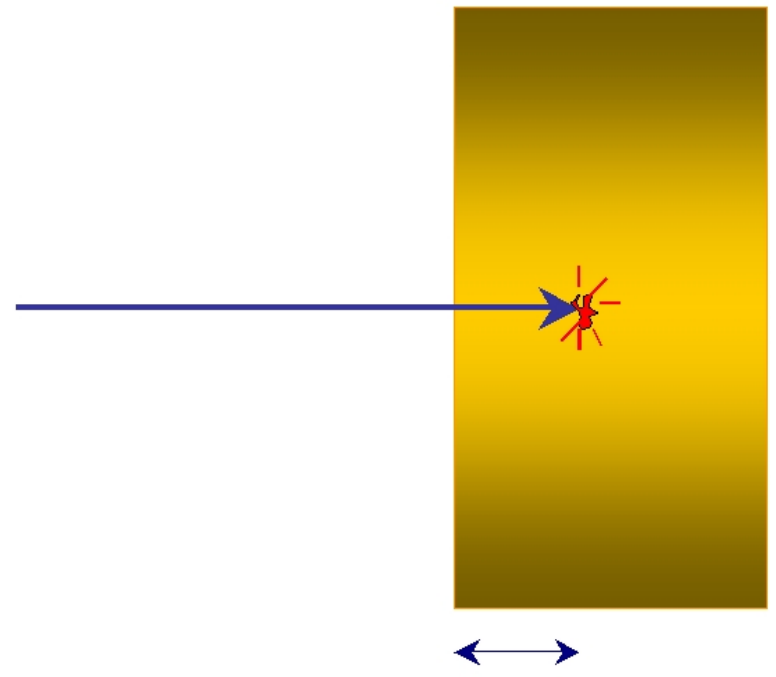


Vacuum polarization

Stopping power



Golser, Semrad, PRL 1991



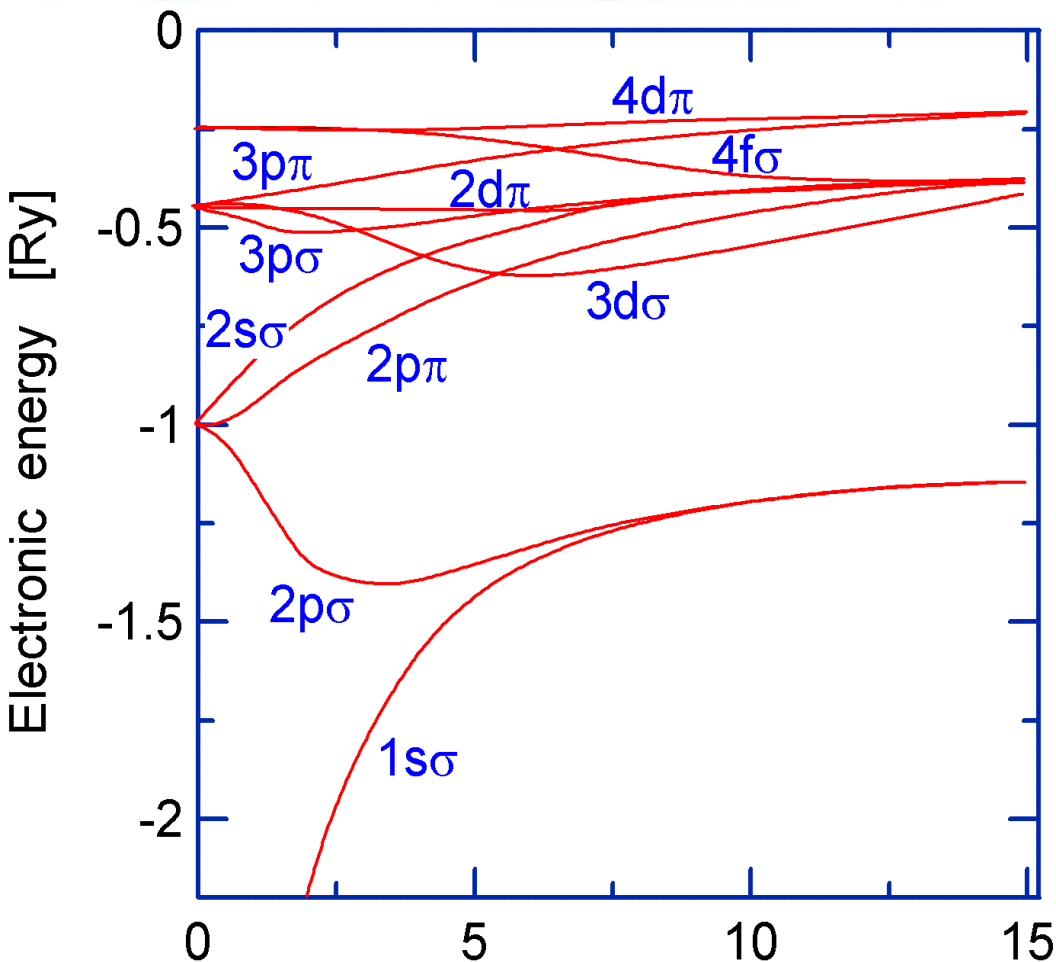
$$S = -\frac{dE}{dx}$$

$$E' = E - S_p \cdot \Delta x$$

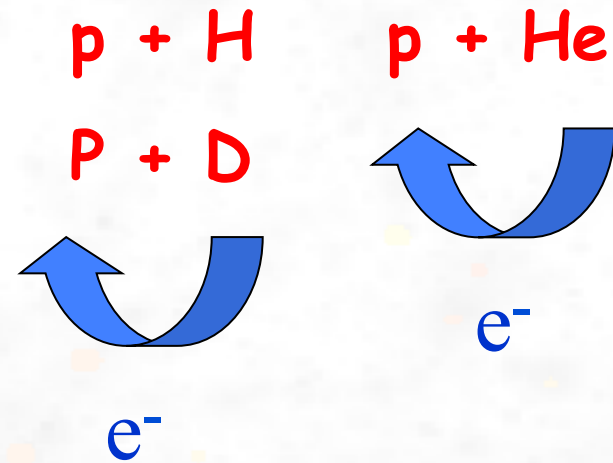
H + He

Mainly charge-exchange

Theory for the simplest systems



He^+ Internuclear separation R [a.u.] $\text{H}+\text{H}^+$



Elliptic coordinates
Molecular orbitals
Hellman-Feynmann relation
TDSE (for p + He)



Coupled-channels

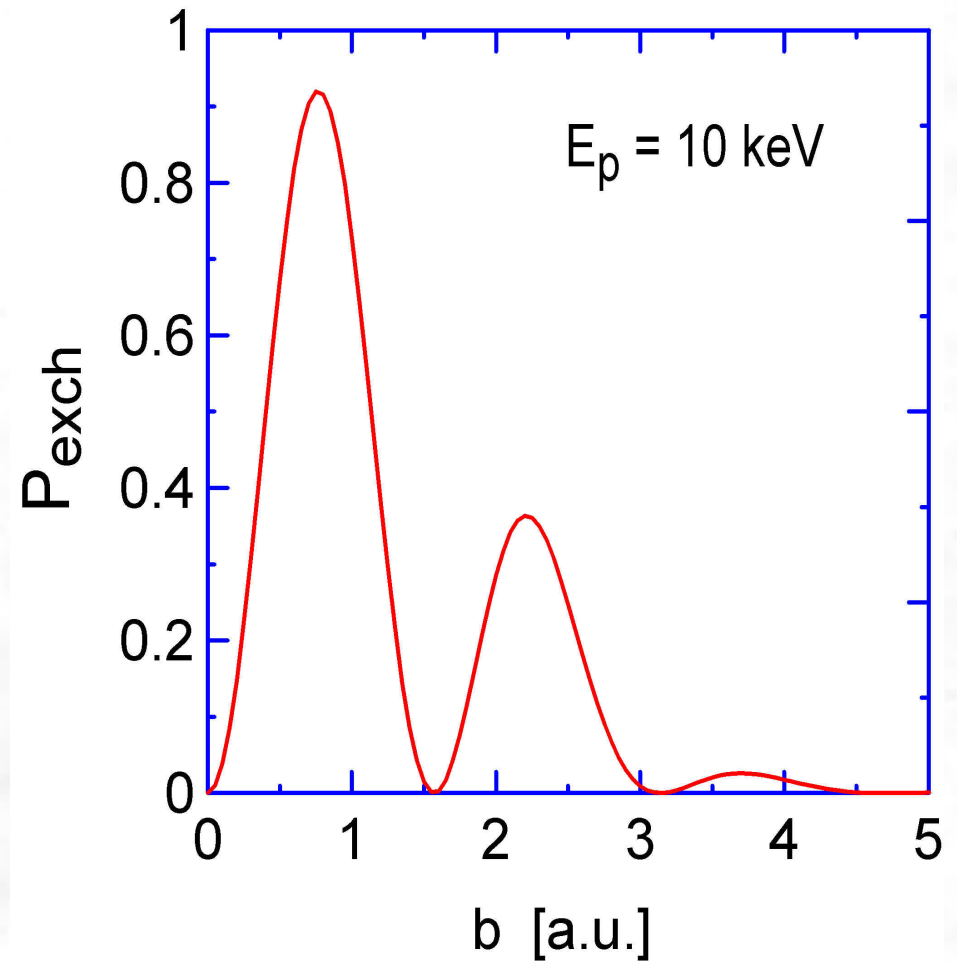
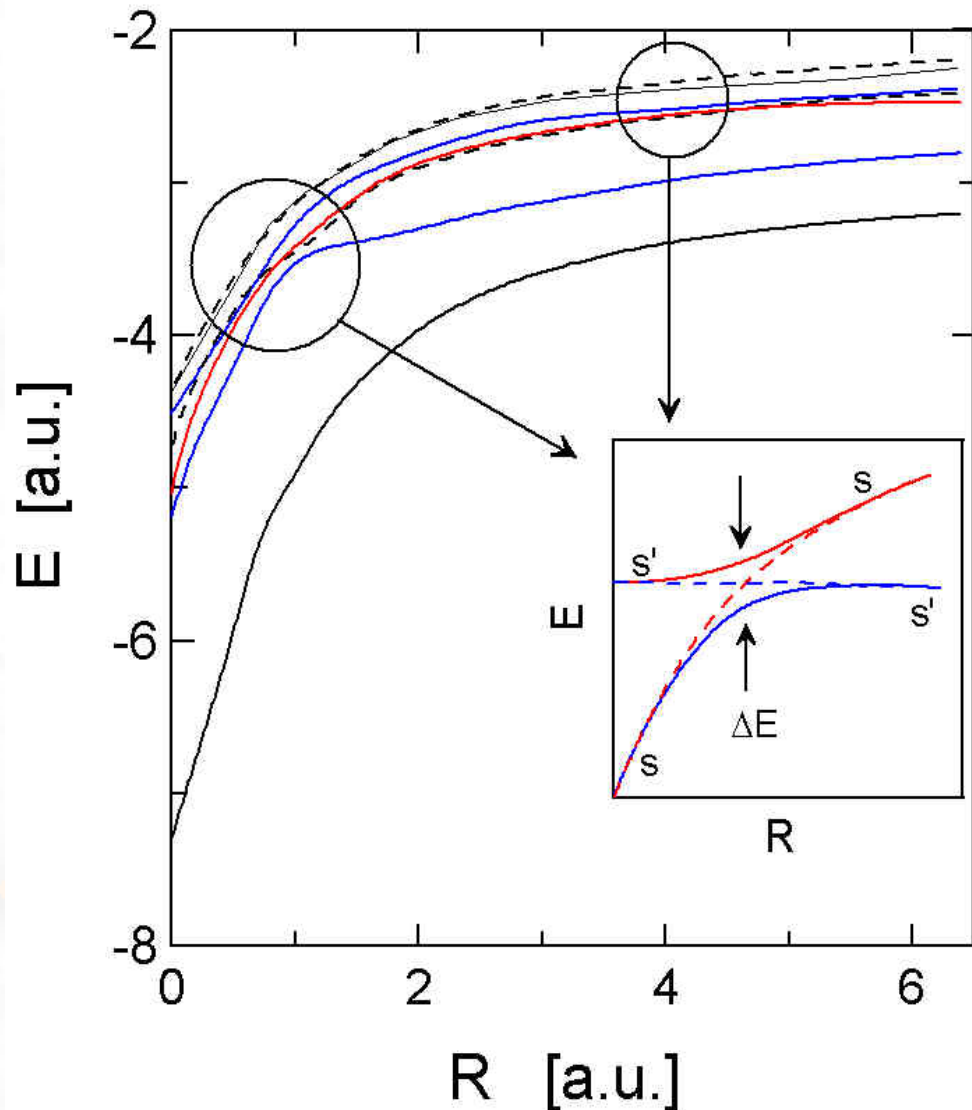
Bertulani, de Paula, PRC 62, 045802 (2000)
Bertulani, PLB 585, 35 (2004)

Damping of resonant exchange



Li^+

$He + H^+$



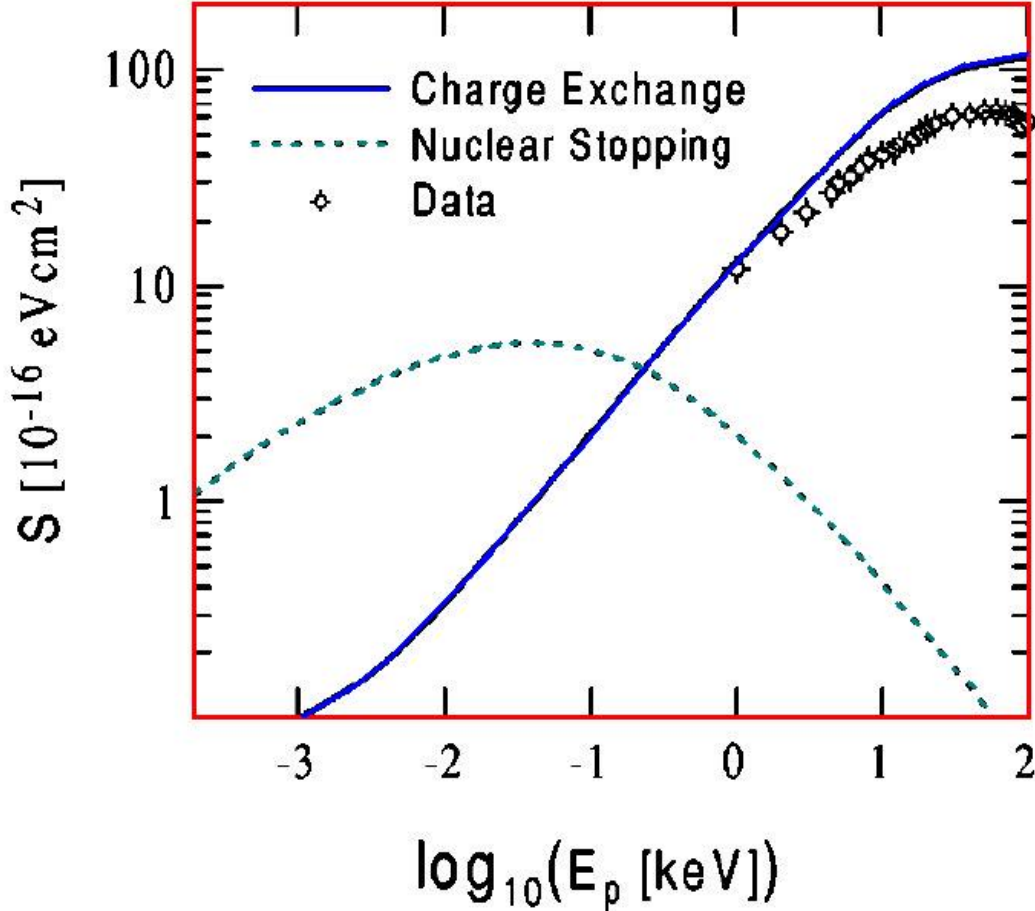
Landau-Zener

$$P = e^{-\Gamma \Delta t_{coll}} \cos^2 \left[\frac{H_{12} a}{2v} \right]$$

Stopping power at very low energies

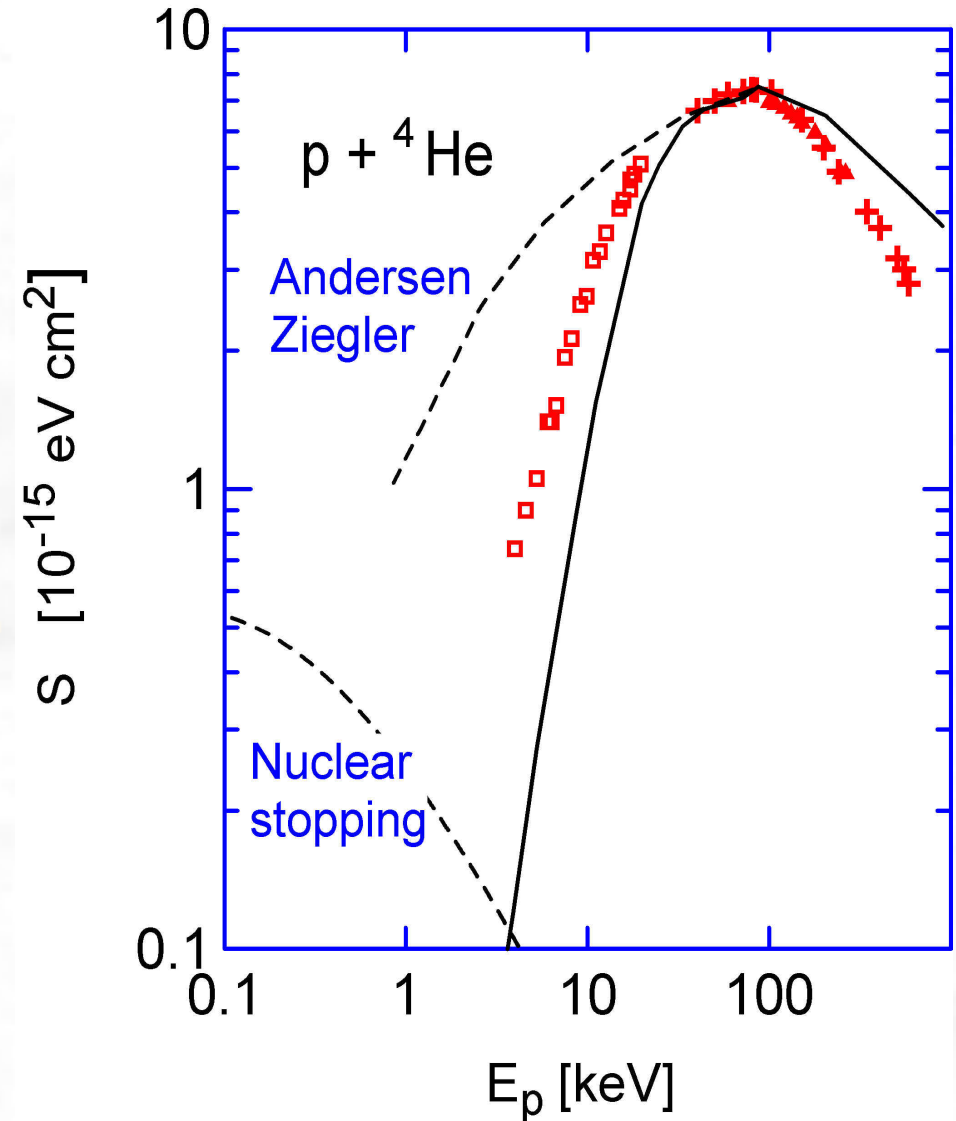
Threshold effect

$$E_P \geq \frac{\mu^2}{4M_P m_e} \Delta E \geq 8 \text{ keV}$$



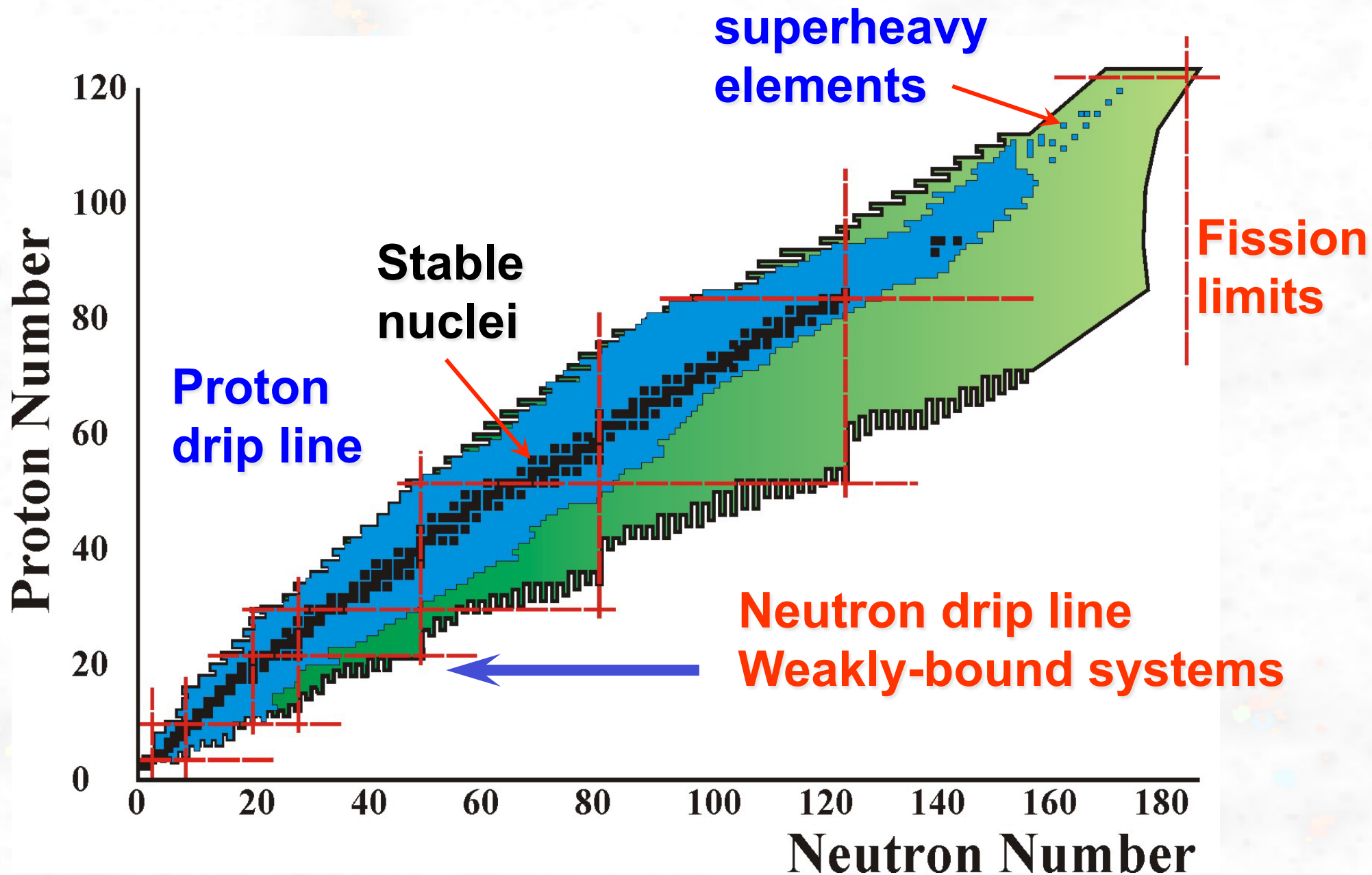
Bertulani, PLB 585, 35 (2004)

He: $1s^2 \rightarrow 1s2s$: 19.8 eV

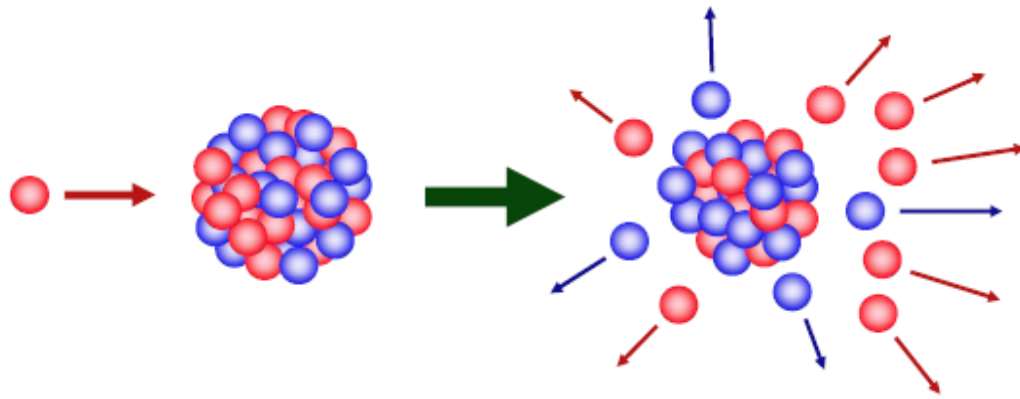


Nuclear Astrophysics with Radioactive Beams

Limits of nuclear stability

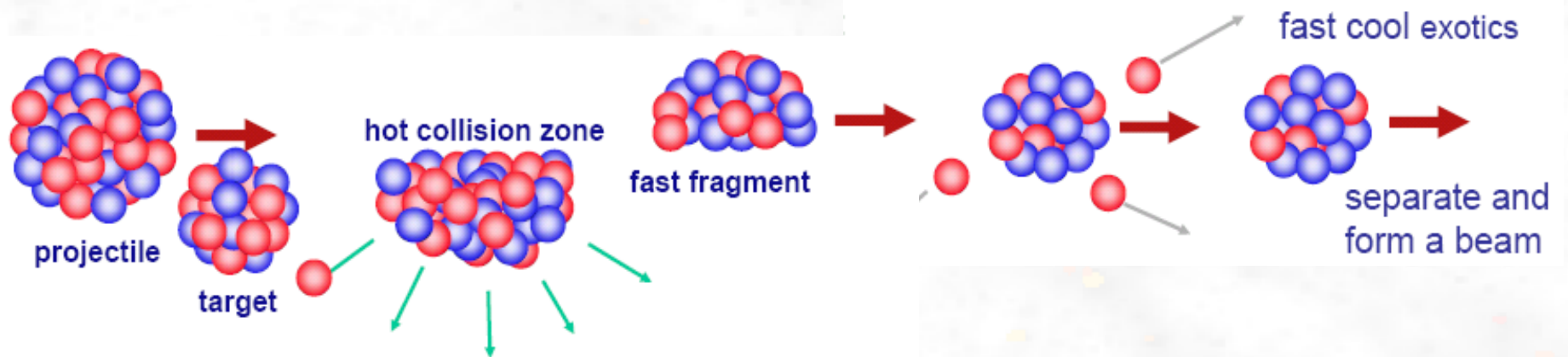


Exotic nuclei production



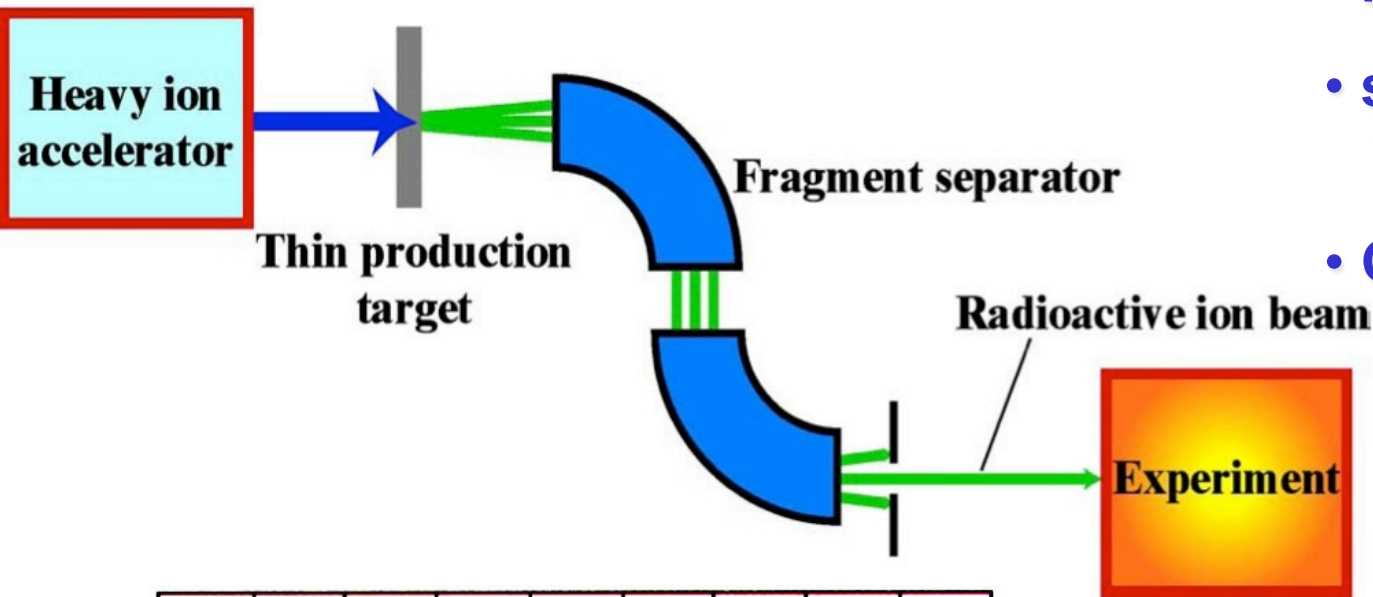
Target fragmentation: ISOL-facilities

- quality beams
- limited species, slow extraction

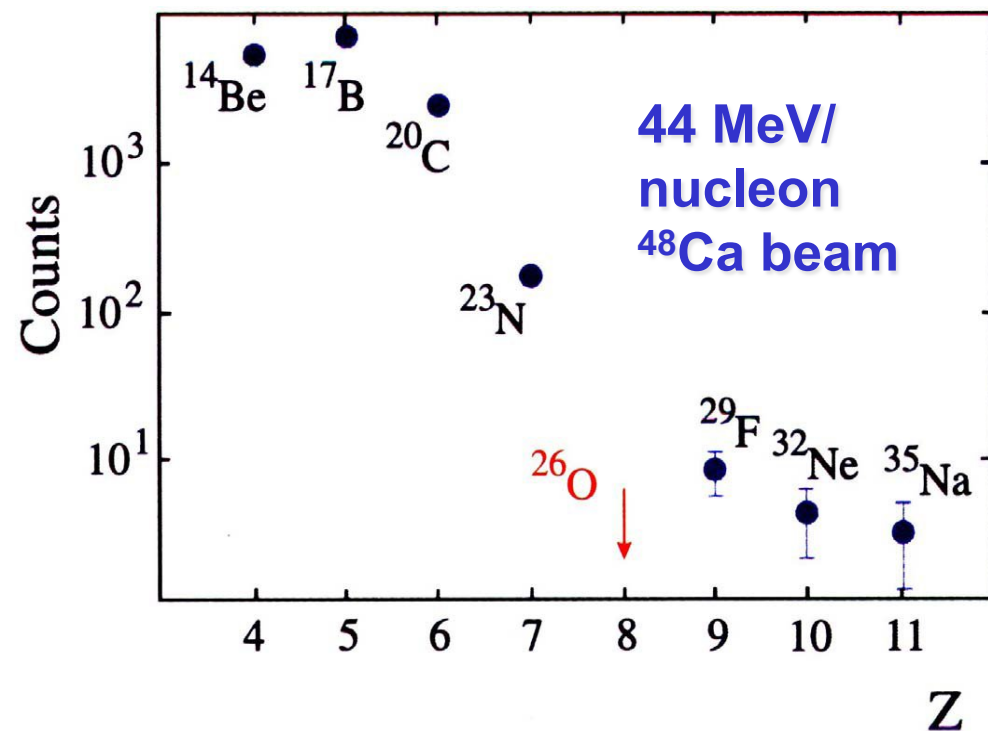


Projectile fragmentation (~ 100 MeV/nucleon)

Production of radioactive beams



- beams of modest quality
- short-lived isotopes with $T_{1/2} > 10^{-6}$ s
- GANIL, GSI, MSU, RIKEN



D. Guillemaud-Mueller et al.
PRC 41 (1990), 937

How the program of radioactive beams began

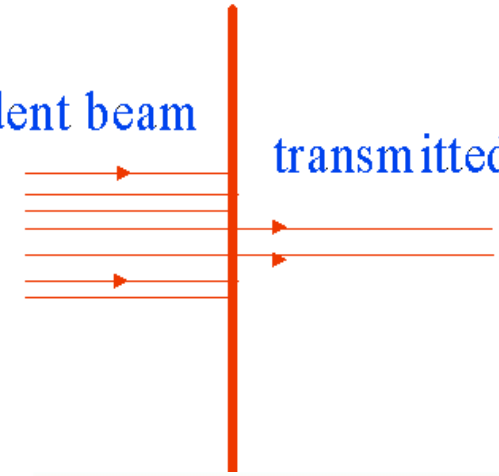
First experiments: (a) nuclear radii, (b) momentum distributions

Isao Tanihata, PRL, PLB 1985

$$\sigma_I = \pi R_I^2$$

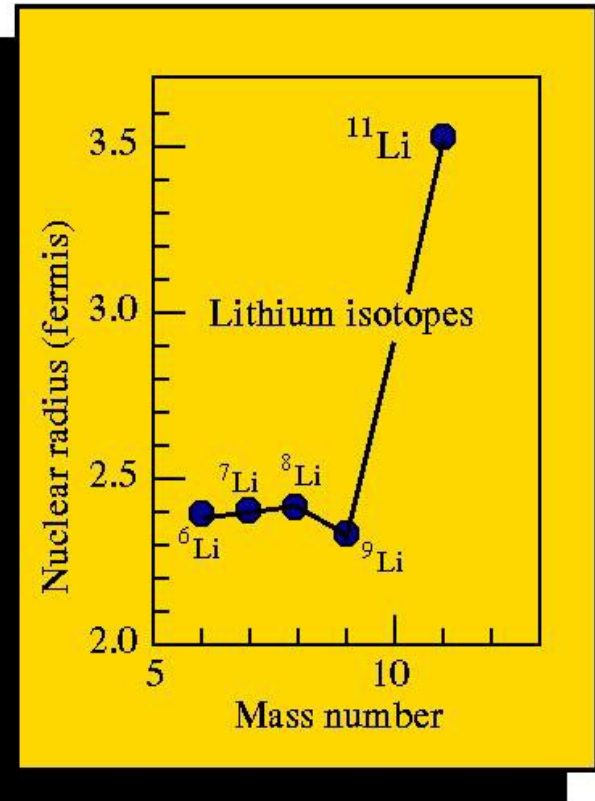
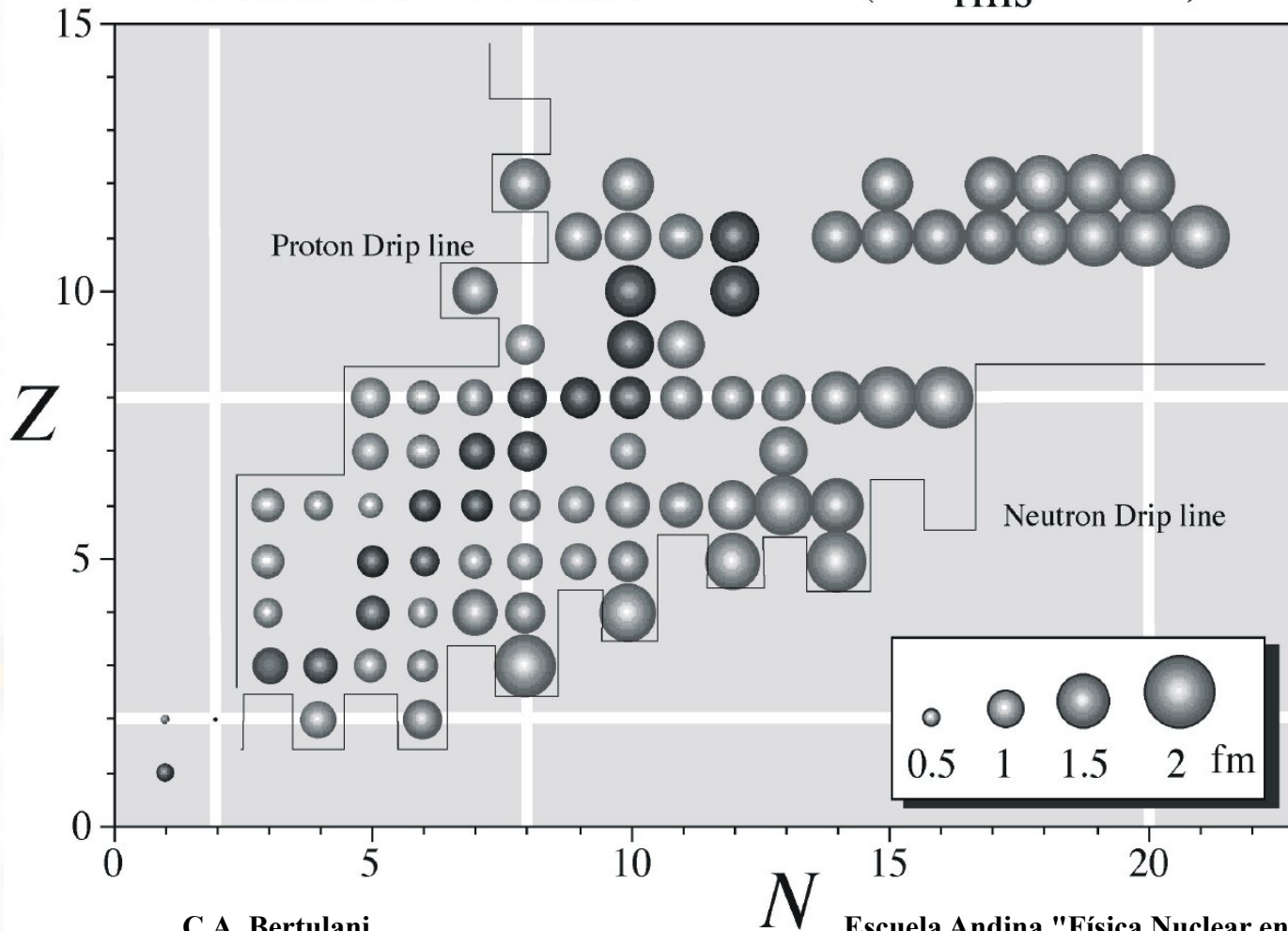
incident beam

transmitted beam



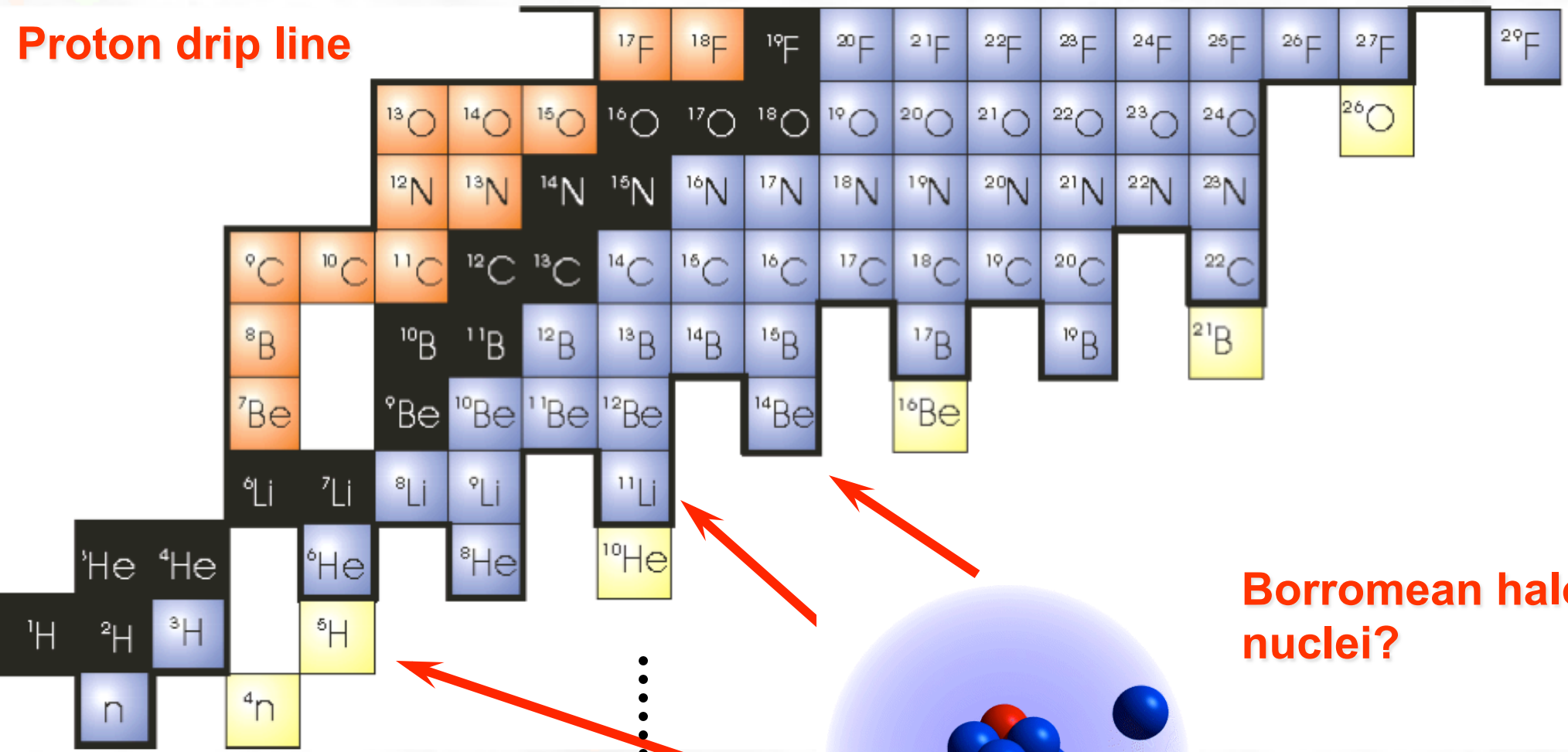
Nuclear Radii

$(R_{\text{rms}}^m - 1.47) \text{ fm}$



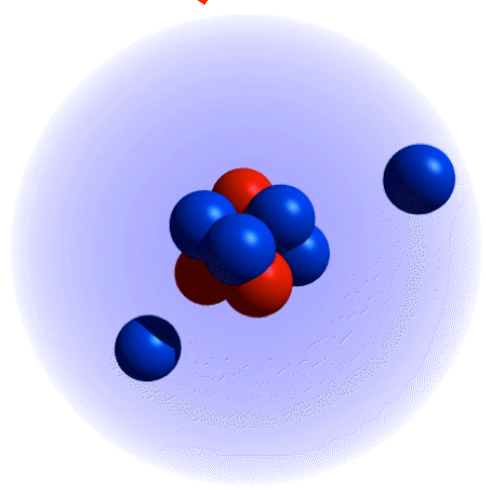
Neutron dripline in light nuclei

Proton drip line



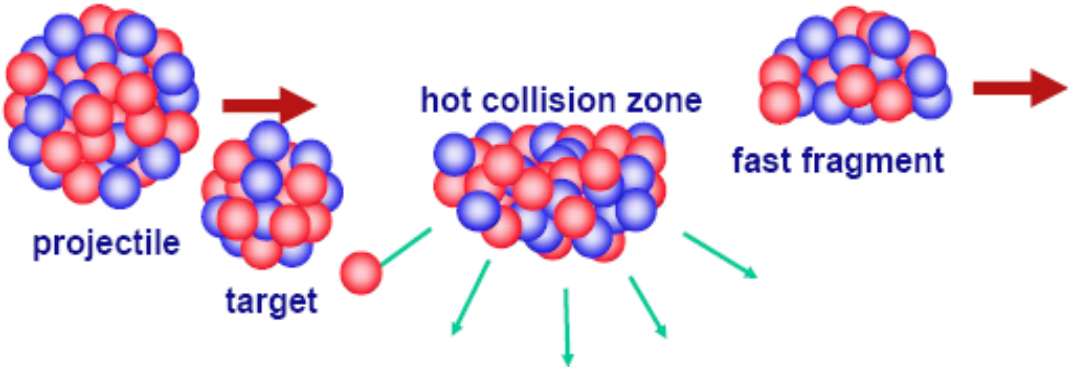
Tetra-neutron?

N=8

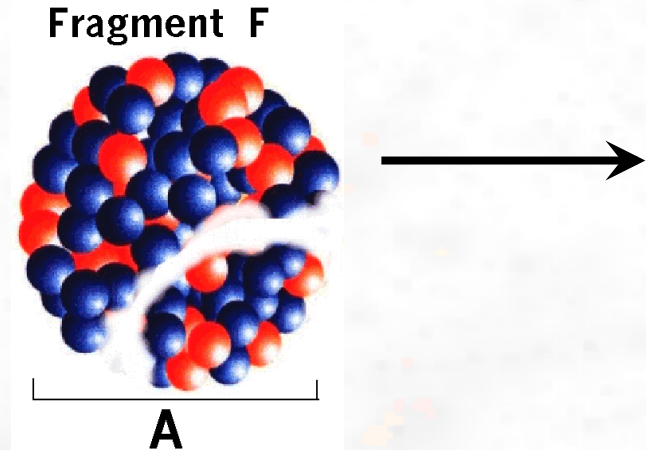


Borromean halo nuclei?

Momentum distributions of fragments



Goldhaber model,
PLB 53, 306 (1974)



$$\sigma^2 = \left\langle \left[\sum_{i=1}^F p_i \right]^2 \right\rangle = A_F \langle \mathbf{p}_i^2 \rangle + A_F(A_F - 1) \langle \mathbf{p}_i \cdot \mathbf{p}_j \rangle = 0$$

$$\left[\sum_{i=1}^F p_i \right]^2 = A \langle \mathbf{p}_i^2 \rangle + A(A - 1) \langle \mathbf{p}_i \cdot \mathbf{p}_j \rangle = 0$$

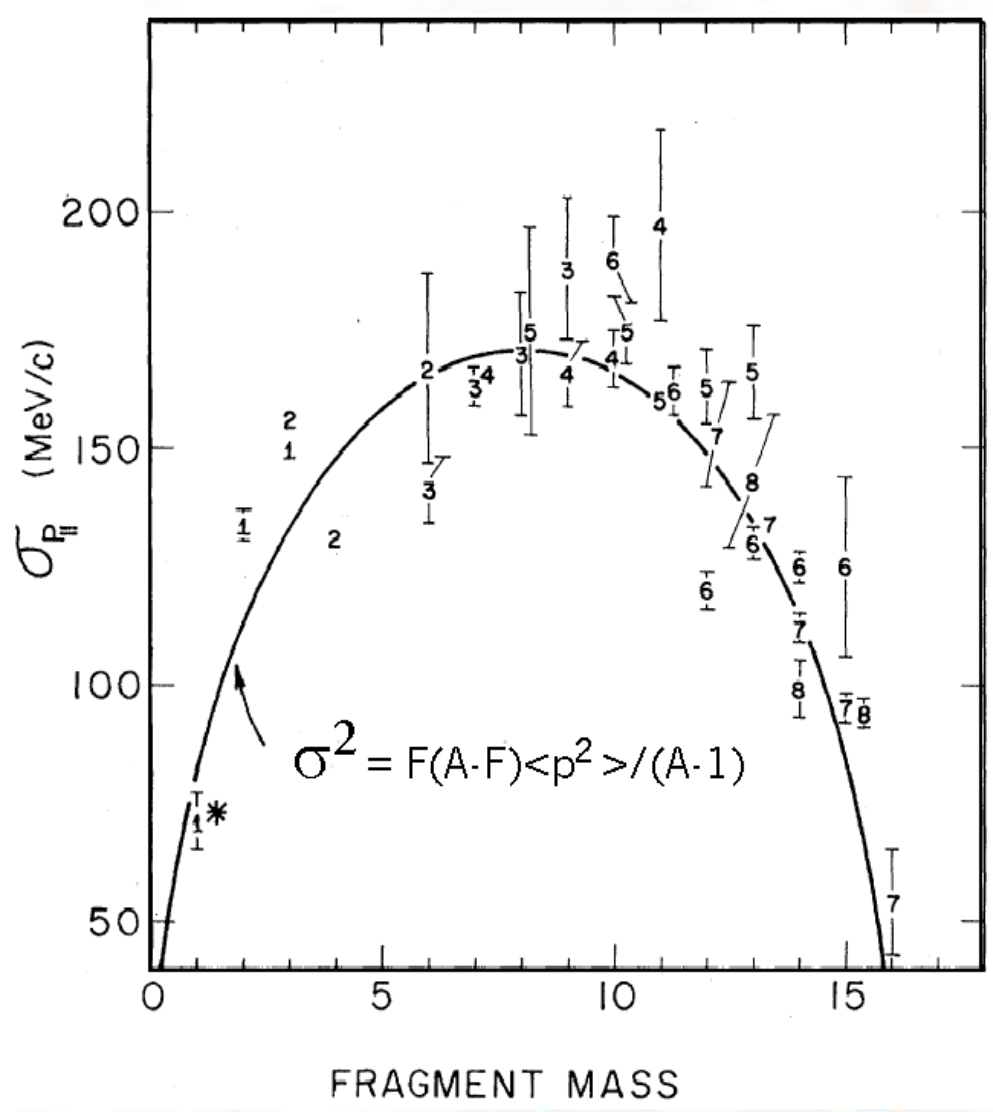
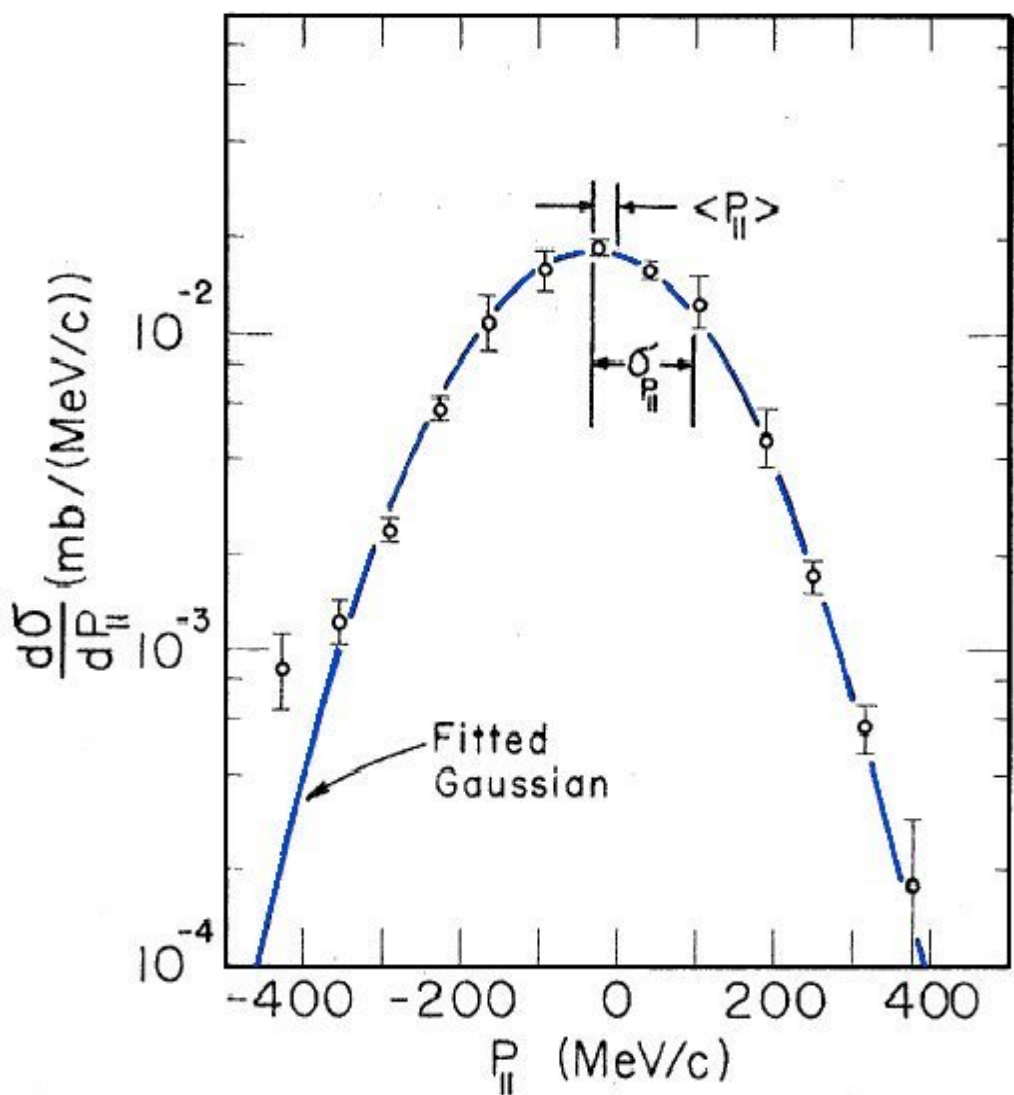


$$\sigma^2 = \frac{A_F(A - A_F)}{A - 1} \langle p_i^2 \rangle$$

nucleon fermi
momentum

$$\langle p_i^2 \rangle = \frac{3p_F^2}{5}$$

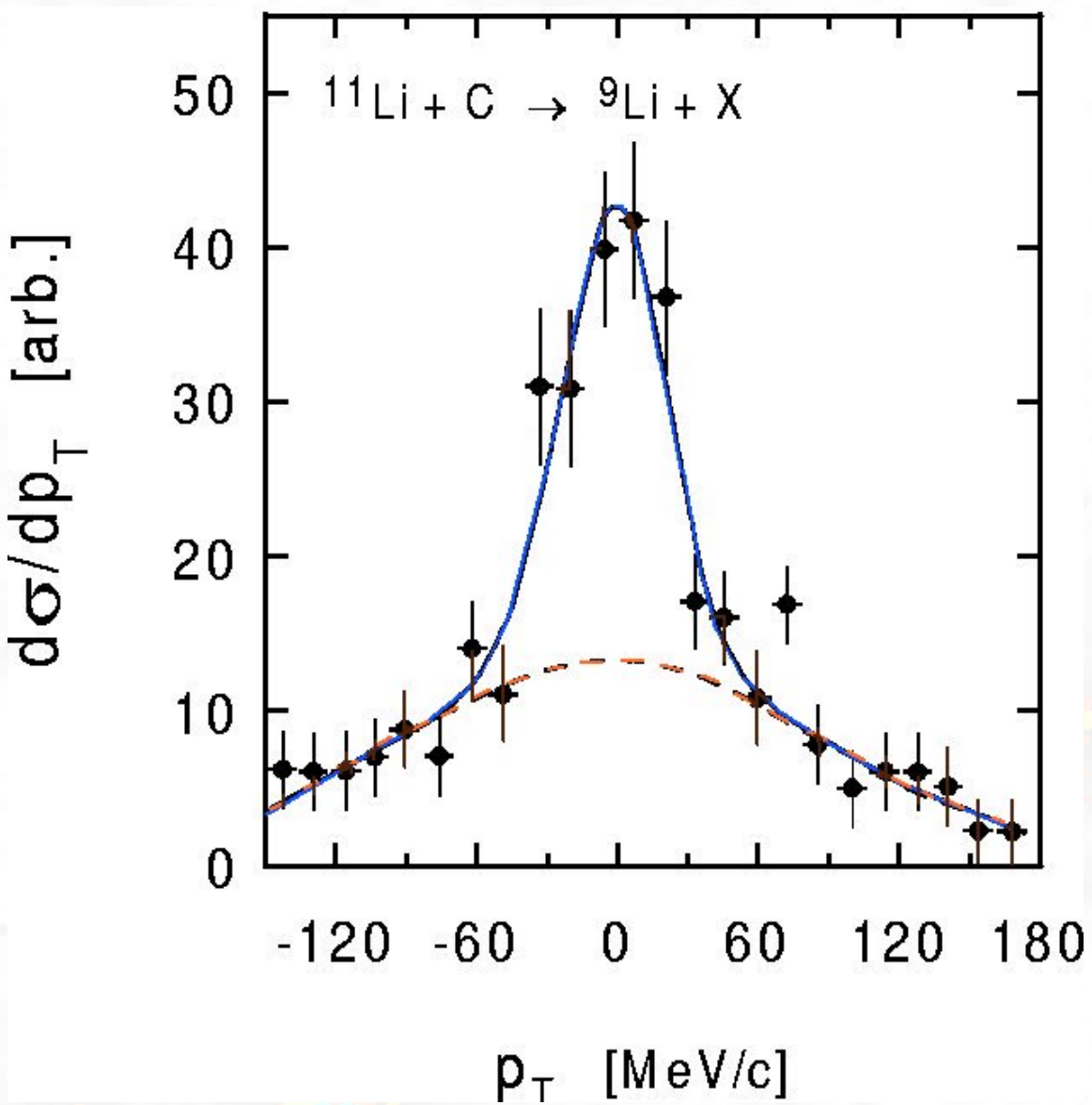
Momentum distributions of fragments (stable projectiles)



Well described by Goldhaber formula, Goldhaber, PLB 53, 306 (1974)

$$\sigma^2 = \frac{A_F(A - A_F)}{A - 1} \langle p_N^2 \rangle$$

Momentum distributions (unstable projectiles)



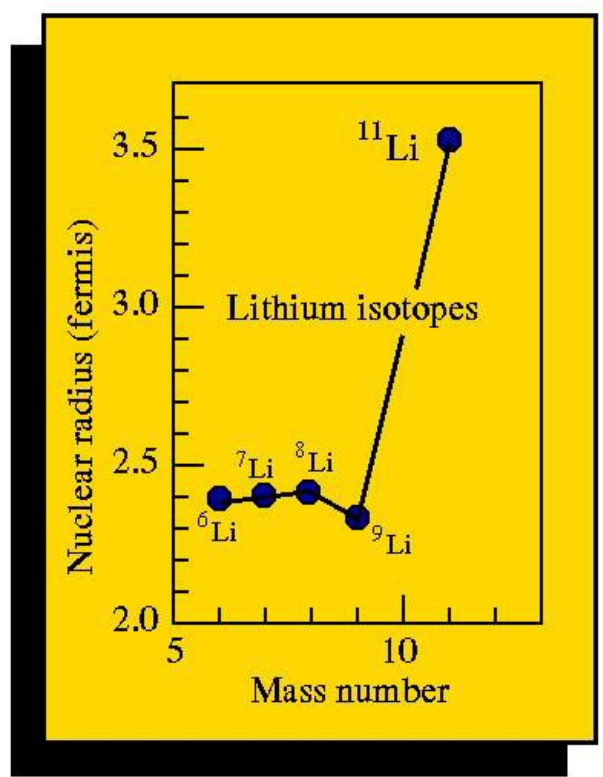
$$\sigma^2 = \frac{A_F (A - A_F)}{A - 1} \langle p_N^2 \rangle$$

Small σ^2

➔ Small $\langle p_N^2 \rangle$

Large R

Tanihata,
Bevalac, Berkeley, 1985



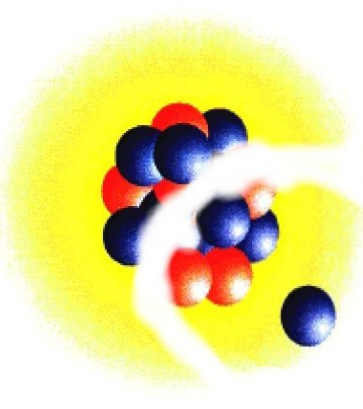
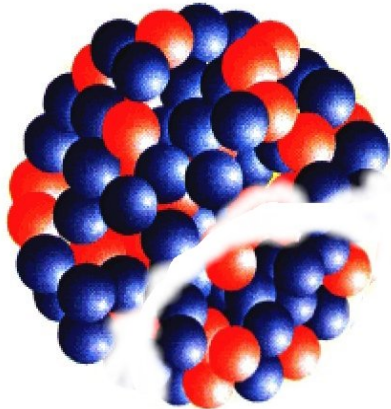
Momentum distributions (unstable projectiles)

Size due to binding, not Fermi motion **Goldhaber model with two fluids**

Valence nucleons + core nucleons

Bertulani, K. McVoy, PRC 48, 2534 (1993)

Two fluids: valence (1) + core (2) + interaction or binding (K)



$$\sigma^2 = \frac{A_F (A - A_F)}{A - 1} \left[\frac{A_1}{A} \langle p^2 \rangle_1 + \frac{A_2}{A} \langle p^2 \rangle_2 + \frac{\langle K^2 \rangle}{A_1 A_2} \right]$$



large size due to loose binding

Coulomb breakup of ^{11}Li has huge cross sections (1 - 5 barns!):

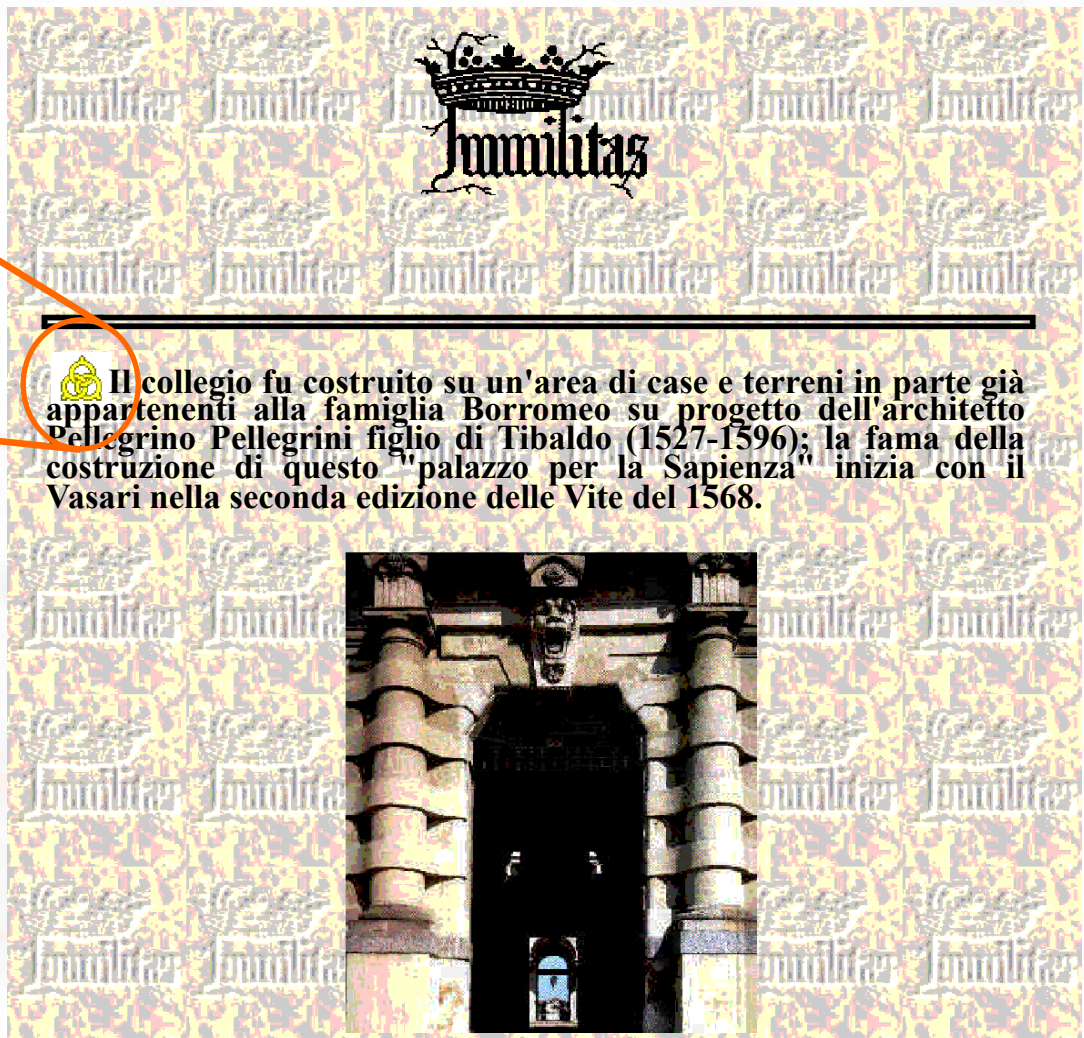
Baur, Bertulani and Rebel, Nucl. Phys. A458 (1986) 188

Bertulani and Baur, NPA 480 (1988) 615

Hansen and Jonson, Europhys. Lett. 4, 409 (1987) ← **"halo nuclei"** nomenclature

3-body calculations → **"Borromean nuclei"**

Origin of Borromean rings

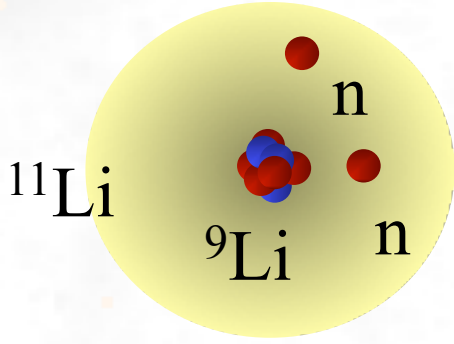
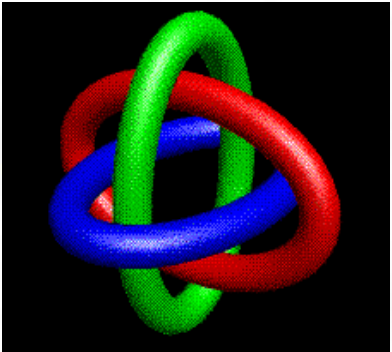


Il collegio fu costruito su un'area di case e terreni in parte già appartenenti alla famiglia Borromeo su progetto dell'architetto Pellegrino Pellegrini figlio di Tibaldo (1527-1596); la fama della costruzione di questo "palazzo per la Sapienza" inizia con il Vasari nella seconda edizione delle Vite del 1568.

Borromean nuclei are a laboratory for 3-body physics. Ex: Efimov effect

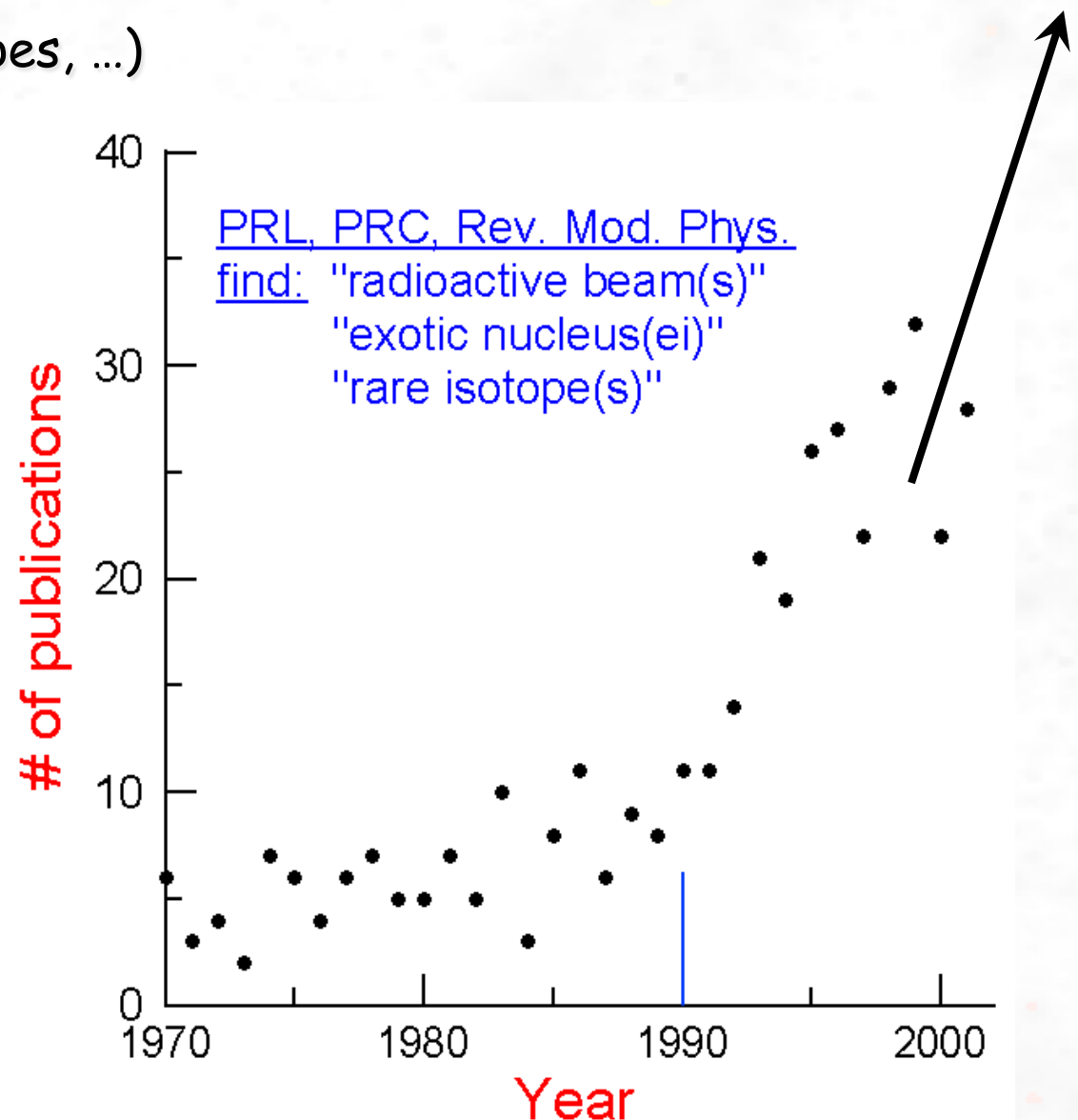
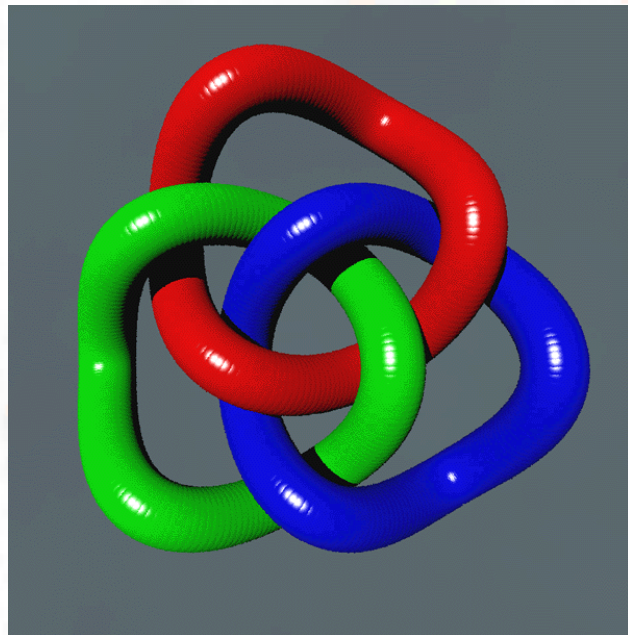
Theoretical description:

- Faddeev equations
- Hyperspherical harmonics
- etc.



Charismatic names → explosion of publications

(Halo, exotic, borromean, rare isotopes, ...)



In the 80's and 90's ^{11}Li became the center of attention

Astrophysics program of rare nuclear isotopes

Future: electron-ion scattering
skins, halos, soft multipole vibrations

Present:

(A) Trojan horse

(B) Asymptotic normalization coefficients

(C) Coulomb dissociation

(D) Charge Exchange $e^- + (Z, A) \rightarrow (Z-1, A) + \nu_e$
(p,n) (n,p) (d, ^2He) (Z, $Z\pm 1$)

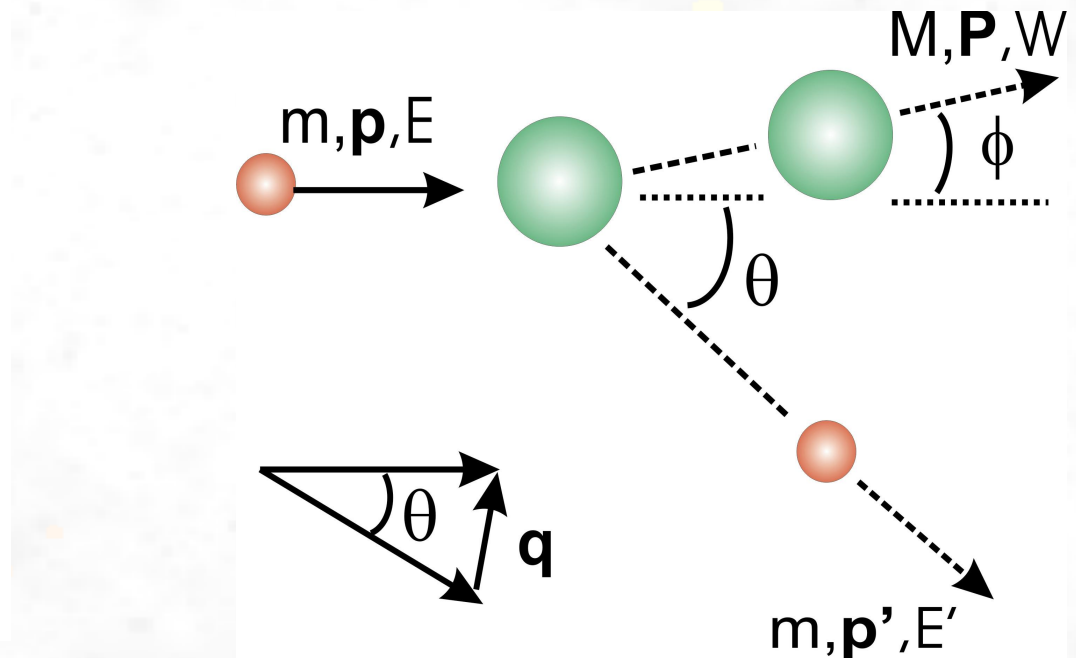
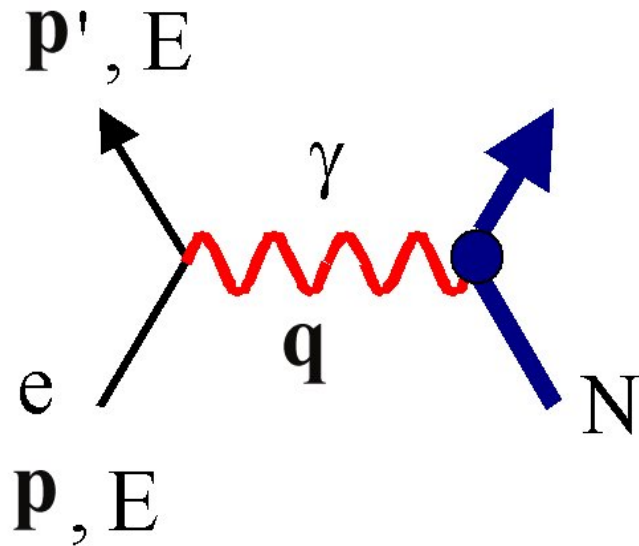
$$\left| \langle B || \sigma \tau || A \rangle \right|^2$$

(E) Knockout reactions

$$|\Psi(p)|^2; C^2S$$

The future: electron scattering

Kinematics of electron scattering



$$p' \approx \frac{p}{1 + \frac{2E}{Mc^2} \sin^2 \theta / 2}$$

electron final momentum

$$q \approx \frac{2p \sin \theta / 2}{1 + \frac{2E}{Mc^2} \sin^2 \theta / 2}$$

momentum transfer

$$E - E' \approx \frac{E^2}{Mc^2} \frac{2 \sin^2 \theta / 2}{1 + \frac{2E}{Mc^2} \sin^2 \theta / 2}$$

nucleus kinetic energy

Spinless electron

$$w = \frac{2\pi}{\hbar} \left| \langle f, \mathbf{p}' | H | i, \mathbf{p} \rangle \right|^2 \frac{dn}{dE_f}$$

golden rule

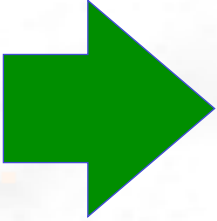
$$H = e^2 \sum_k \frac{1}{|\mathbf{r} - \mathbf{r}_k|}$$

interaction

$$\frac{dn}{dE_f} = \frac{p' d\Omega L^3}{c\hbar^3} \frac{1}{1 + \frac{2E}{Mc^2} \sin^2 \theta / 2}$$

density of states

plane wave for electrons


$$\langle f, \mathbf{p}' | H | i, \mathbf{p} \rangle = \frac{4\pi e^2}{L^3 q^2} \left\langle \Phi_f \left| \sum_{\mathbf{r}} e^{i\mathbf{q} \cdot \mathbf{r}_k} \right| \Phi_i \right\rangle$$

Φ = nuclear wavefunction



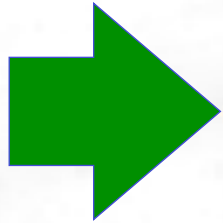
$$\frac{d\sigma}{d\Omega} = \left(\frac{e^2}{2E} \right)^2 \frac{1}{\sin^4 \theta / 2} \frac{1}{1 + \frac{2E}{Mc^2} \sin^2 \theta / 2} \left| \left\langle \Phi_f \left| \sum_{\mathbf{r}} e^{i\mathbf{q} \cdot \mathbf{r}_k} \right| \Phi_i \right\rangle \right|^2$$

Electron with spin

$$\Psi_j(\mathbf{r}, t) = u_j \exp\left[\frac{i}{\hbar}(\mathbf{p} \cdot \mathbf{r} - Et)\right] \quad \mathbf{u}_j = \text{spinor}$$

$$u_1 = -p_z / p, \quad u_2 = -(p_x + ip_y) / p, \quad u_3 = 1, \quad u_4 = 0 \quad \text{spin up}$$

$$u_1 = -(p_x - ip_y) / p, \quad u_2 = p_z / p, \quad u_3 = 0, \quad u_4 = 1 \quad \text{spin down}$$



$$\frac{d\sigma}{d\Omega}(\text{spin}) = \left| \sum_{\uparrow}^4 u_j'^* u_j \right|^2 \times \frac{d\sigma}{d\Omega}(\text{spinless})$$

$$\left| \sum_{\uparrow}^4 u_j'^* u_j \right|^2 = \frac{1}{2}(1 + \cos\theta) = \cos^2\theta / 2 \quad \text{spin average}$$

$$\frac{d\sigma}{d\Omega} = \left(\frac{e^2}{2E}\right)^2 \frac{\cos^2\theta / 2}{\sin^4\theta / 2} \frac{1}{1 + \frac{2E}{Mc^2} \sin^2\theta / 2} \left| \left\langle \Phi_f \left| \sum_{\uparrow}^Z e^{i\mathbf{q} \cdot \mathbf{r}_k} \right| \Phi_i \right\rangle \right|^2$$

PWBA

Elastic Scattering

$$\left\langle \Phi_i \left| \sum_{\mathbf{r}_k}^Z e^{i\mathbf{q}\cdot\mathbf{r}_k} \right| \Phi_i \right\rangle = \int d^3r \rho_{ch}(\mathbf{r}) e^{i\mathbf{q}\cdot\mathbf{r}} \equiv F(\mathbf{q})$$

charge form-factor

Mott cross section

$$\frac{d\sigma}{d\Omega} = \left(\frac{Ze^2}{2E} \right)^2 \frac{\cos^2 \theta / 2}{\sin^4 \theta / 2} \frac{1}{1 + \frac{2E}{Mc^2} \sin^2 \theta / 2} \equiv \sigma_M$$

$$F(q) = \frac{4\pi}{q} \int dr r \sin(qr) \rho_{ch}(r)$$

spherical nuclei

$$\frac{d\sigma}{d\Omega} = \frac{\sigma_M(\theta)}{Z^2} |F(q)|^2$$

Nuclear physics

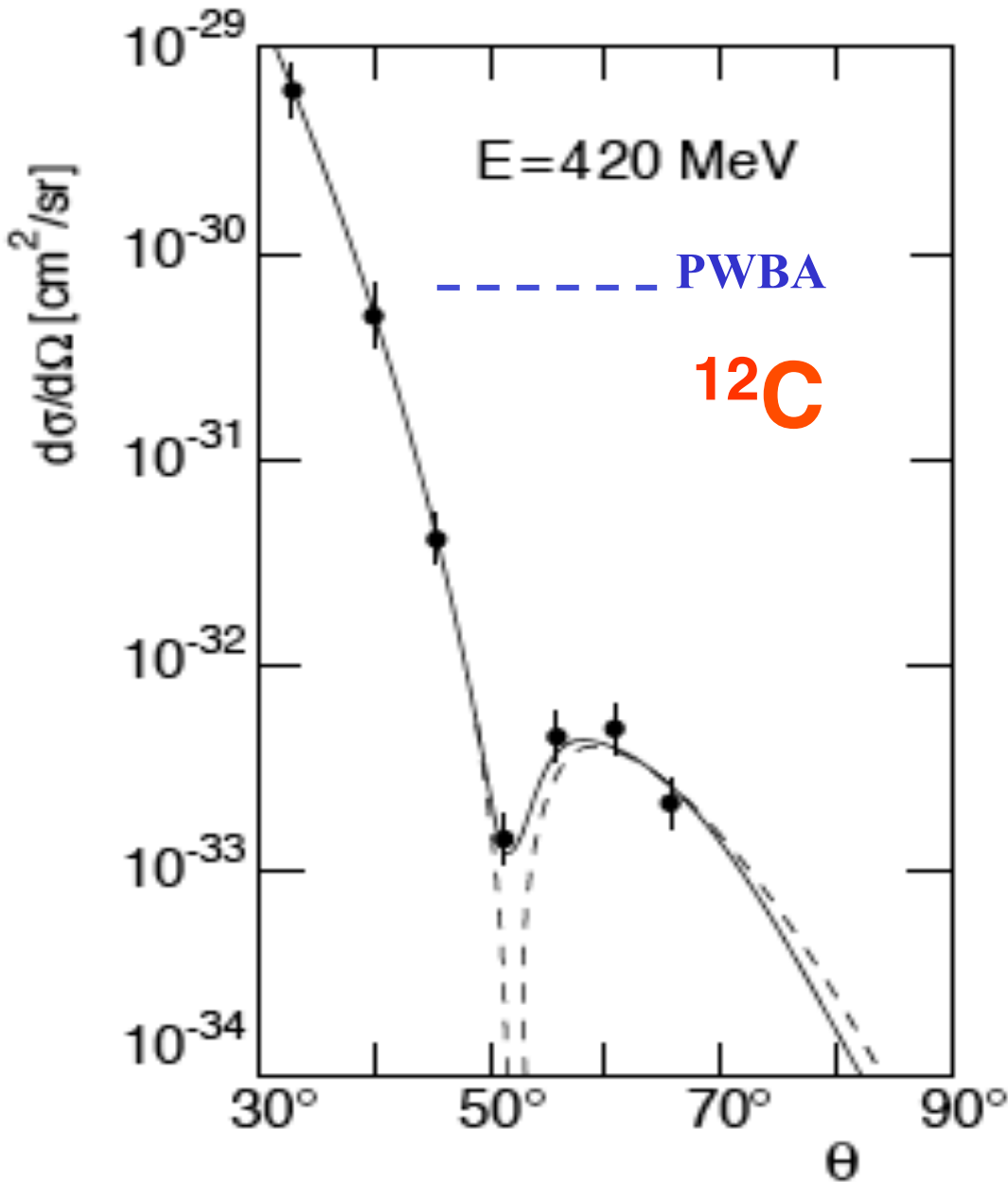
$$\rho_{ch}(r) = \int \rho_p(\mathbf{r}') f_{Ep}(\mathbf{r} - \mathbf{r}') d\mathbf{r}' + \int \rho_n(\mathbf{r}') \underline{f_{En}(\mathbf{r} - \mathbf{r}') d\mathbf{r}'}$$

f_{Ep} = charge dist. in proton

f_{En} = charge dist. in neutron

< 10% effect, large q' s

DWBA corrections



Hofstadter, 1953

- electron wavefunction attracted to the nucleus

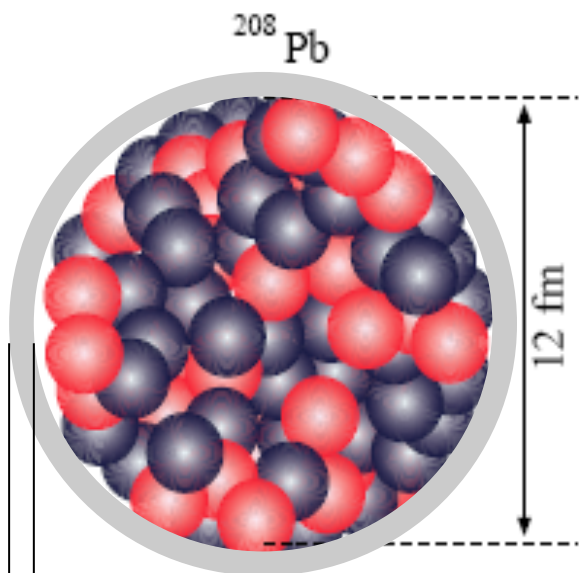
- a measured q probes a larger $q = q_{\text{eff}}$ in $F(q)$

$$\frac{d\sigma}{d\Omega} = \frac{\sigma_M(\theta)}{Z^2} |F(q_{\text{eff}})|^2$$

Still valid. But with

$$q_{\text{eff}} = q \left(1 - \frac{V(0)}{E} \right) = q \left(1 + 1.5 \frac{Ze^2}{ER} \right), \quad R \cong 1.2 A^{1/3} \text{ fm}$$

Neutron skins & neutron stars



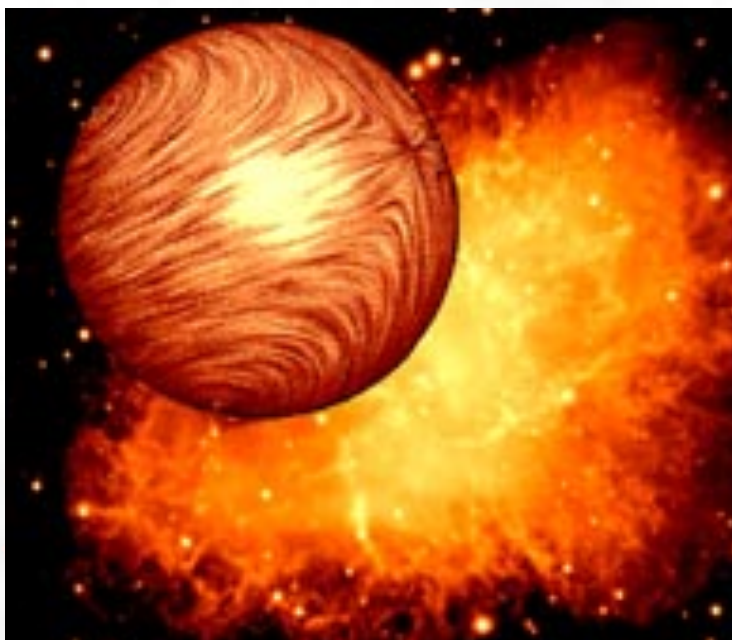
$$\frac{dP}{dr} = -\frac{G\rho(r)M(r)}{r^2} \left[1 + \frac{P(r)}{\rho(r)} \right] \left[1 + \frac{4\pi r^3 P(r)}{M(r)} \right] \left[1 - \frac{2GM(r)}{r} \right]^{-1}$$

$$\frac{dM}{dr} = 4\pi r^2 \rho(r)$$

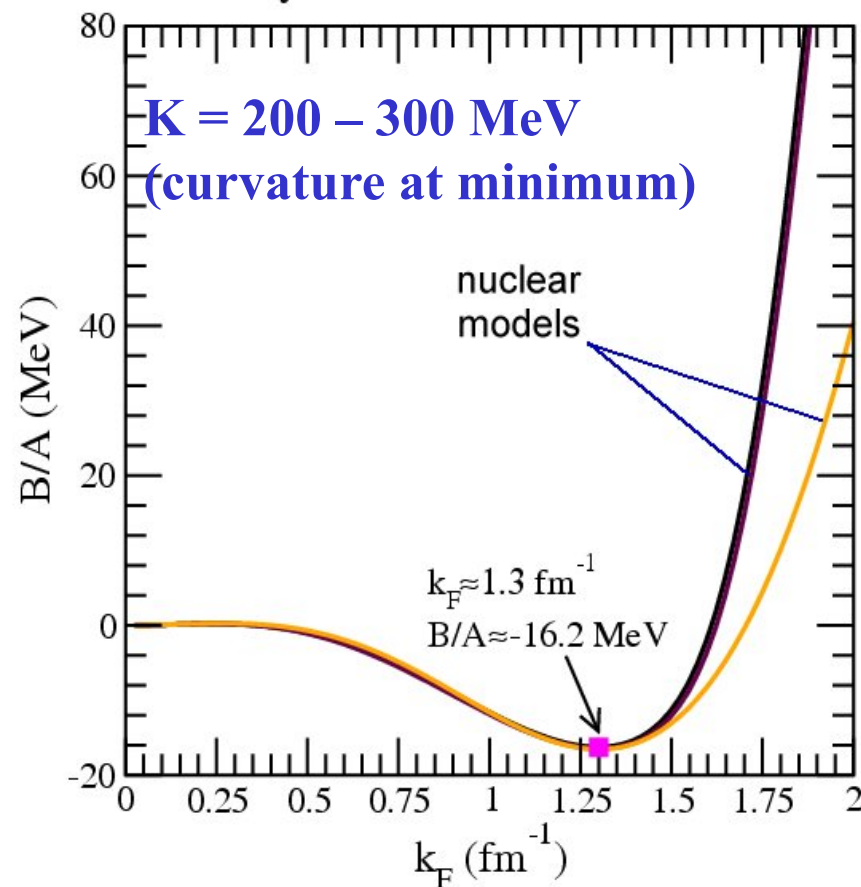
Tolman-Oppenheimer-Volkoff

- need equation of state

P x ρ



Symmetric Nuclear Matter



Symmetry energy

$$E = a_v A + a_s A^{2/3} + a_c Z^2 / A^{1/3} + S(N - Z)^2 / A + \dots$$

Bethe-Weizsaecker formula

$$\rho_0 \approx 0.16 \text{ fm}^{-3}, \quad a_v \approx -16 \text{ MeV}, \quad S \approx 30 \text{ MeV}$$

infinite nuclear matter:

$$E / A = a_v + S\delta^2, \quad \delta = (N - Z) / A \equiv b$$

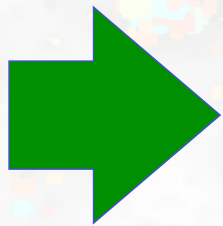
$$E(\rho; b) / A = E(\rho = 0; b) / A + b \left(\frac{\partial E / A}{\partial b} \right)_{b=0} + b^2 \frac{1}{2} \left(\frac{\partial^2 E / A}{\partial b^2} \right)_{b=0} + \dots$$

Symmetric matter 0 Symmetry energy

Pure neutron matter \approx symmetric matter + symmetry energy

Finite nuclei:

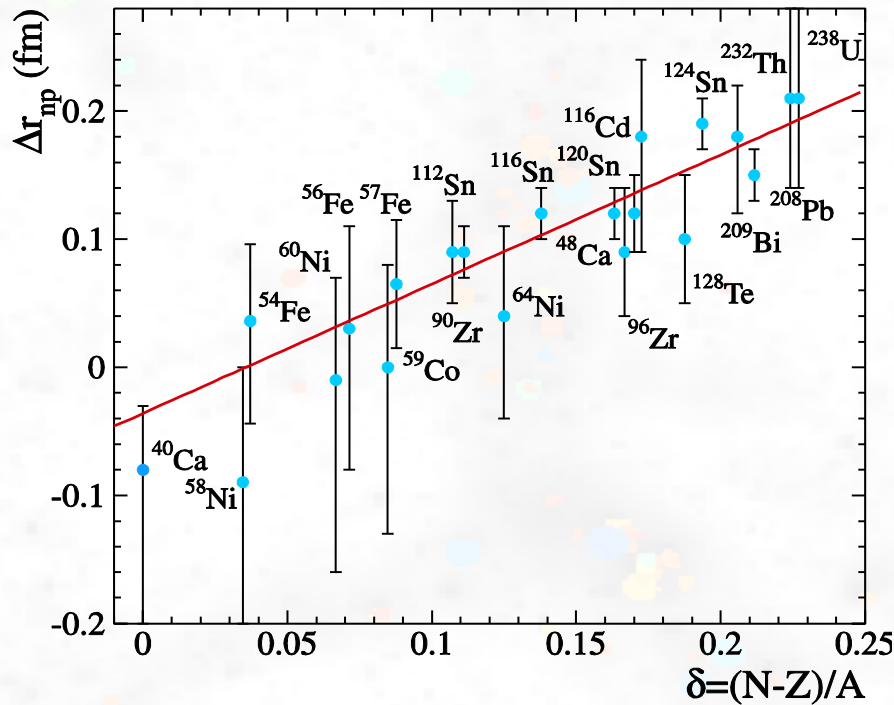
$$S \rightarrow S_V \frac{(N_V - Z_V)^2}{A} + S_S \frac{(N_S - Z_S)^2}{A^{2/3}} \quad + \text{ minimization of } E/A$$



neutron skin

$$\frac{R_n - R_p}{R} = \frac{A}{6NZ} \frac{N - Z - (a_c / 12S_V)ZA^{2/3}}{1 + (S_S / S_V)A^{1/3}}$$

Neutron skins

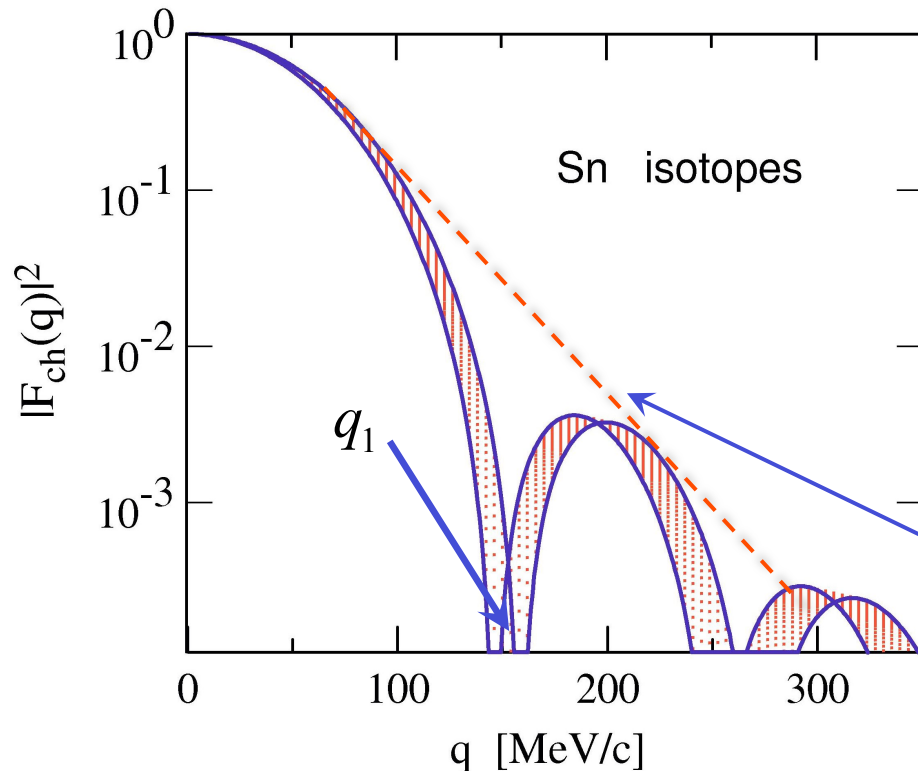


A. Trzcinska, PRL 2001

experimentally:

$$\Delta r_{np} = (-0.04 \pm 0.03) + (1.01 \pm 0.15)\delta \text{ fm}$$

$$S_V = 28 \text{ MeV}, \quad S_S = 46.6 \text{ MeV}$$



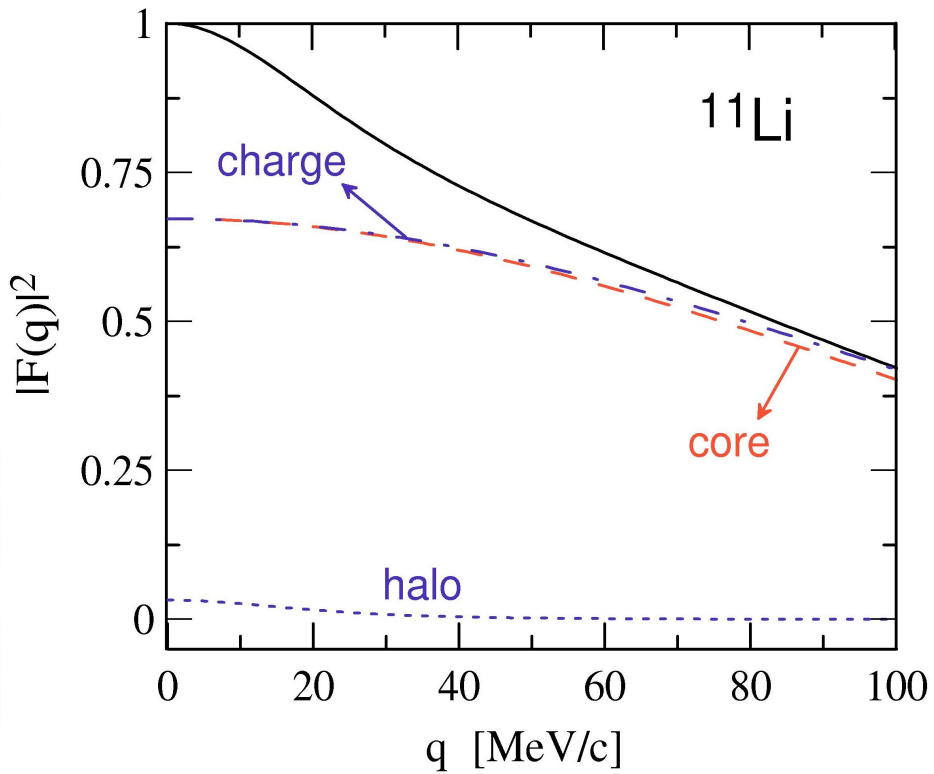
In electron-ion scattering:

Bertulani, PLB 624, 203 (2005)

$$q_1 \cong \frac{3.74}{A^{1/3}} \left[1 - 0.535 \frac{\delta}{A^{1/3}} \right] \text{ fm}^{-1}$$

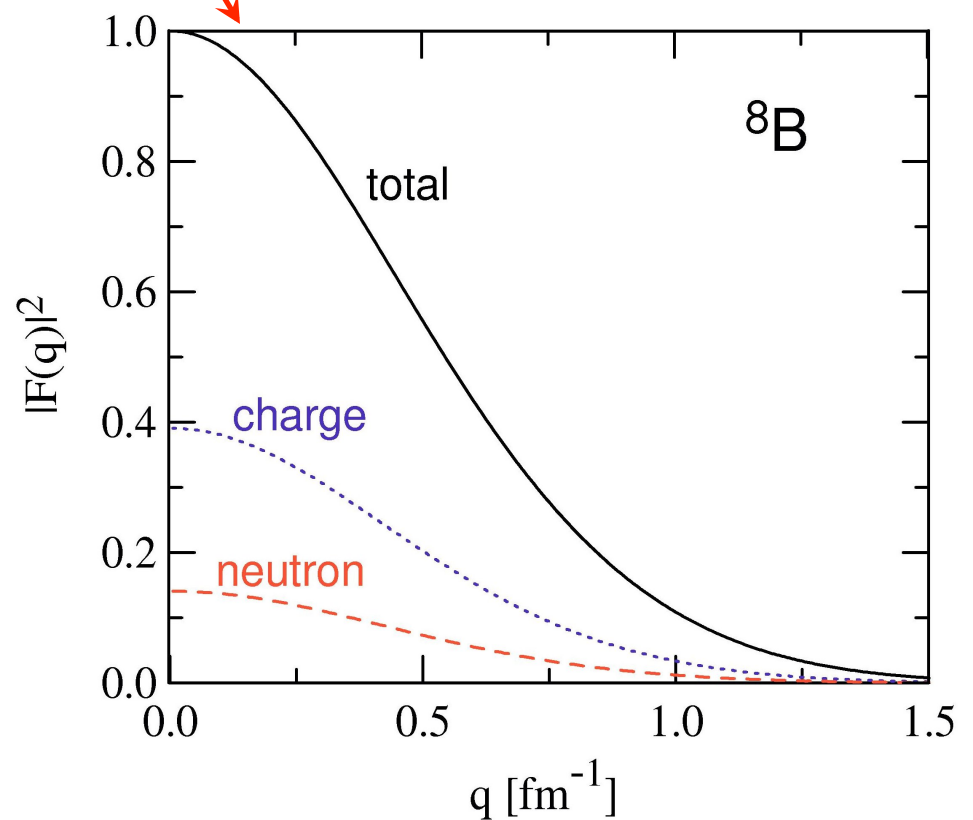
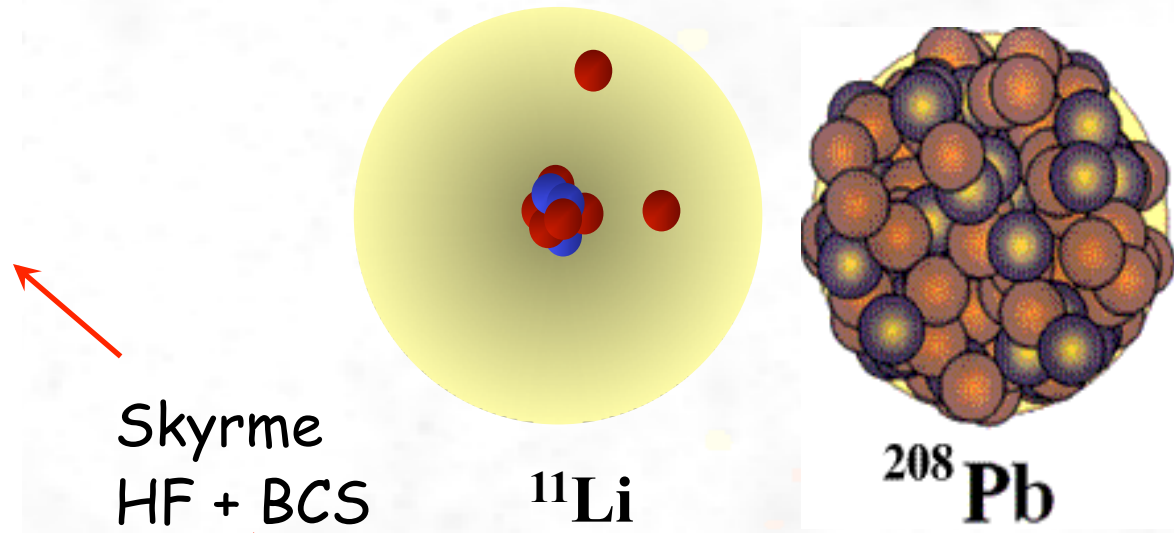
$\exp(-q^2 a^2)$

Neutron halos



nuclear halos

Bertulani, JPG 34 (2007) 315



Magnetic interaction

$$\langle i, \mathbf{p}' | H | i, \mathbf{p} \rangle = \frac{4\pi e^2}{q^2} \langle i | \left[(u_i^* u_i)(U_i^* U_i) - (u_i^* \boldsymbol{\alpha}_e u_i)(U_i^* \boldsymbol{\alpha}_N U_i) \right] | i \rangle$$

averages over initial and sum over final spins

$$\overline{\left| \sum u_i^* u_i U_i^* U_i \right|^2} = \left(\frac{Mc^2}{E} \right) 4 \cos^2 \theta / 2$$

$$\overline{\left| \sum u_i^* \boldsymbol{\alpha}_e u_i U_i^* \boldsymbol{\alpha}_N U_i \right|^2} = \left(\frac{\hbar^2 q^2 c^2}{E} \right) 2 \tan^2 \theta / 2$$

“Dirac” elastic cross section of an electron on a proton (with $\mu = eh/Mc$)

$$\frac{d\sigma^{elast}}{d\Omega_d} = \left(\frac{e^2}{2E} \right)^2 \frac{\cos^2 \theta / 2}{\sin^4 \theta / 2} \frac{1}{1 + \frac{2E}{Mc^2} \sin^2 \theta / 2} \left[1 + \frac{\hbar^2 q^2}{4Mc^2} \left(2 \tan^2 \theta / 2 \right) \right]$$

Inelastic electron scattering

$$\langle f, \mathbf{p}' | H | i, \mathbf{p} \rangle = \frac{4\pi e^2}{q^2} \left\langle f \left| \sum_{\mathbf{r}} e^{i\mathbf{q}\cdot\mathbf{r}_k} \left[(u_f^* u_i)(U_f^* U_i) - (u_f^* \boldsymbol{\alpha}_e u_i)(U_f^* \boldsymbol{\alpha}_N U_i) \right] \right| i \right\rangle$$

- expand $\exp(i\mathbf{q}\cdot\mathbf{r})$ into multipoles
- averages over initial and sum over final spins

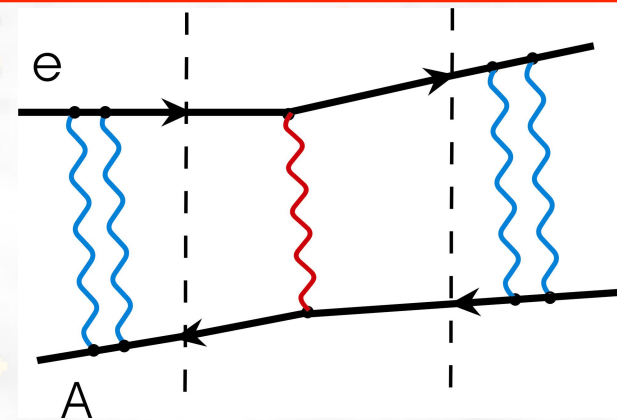
$$\left(\frac{d\sigma}{d\Omega} \right)_{inel} = \frac{\sigma_M(\theta)}{Z^2} \left[\sum_{\lambda} |F_{C\lambda}(q_{eff})|^2 + \left(\frac{1}{2} + \tan^2 \theta / 2 \right) \sum_{\lambda} |F_{E\lambda}(q_{eff})|^2 \right]$$

$$\left(\frac{d\sigma}{d\Omega} \right)_{inel} \cong \left(\frac{d\sigma}{d\Omega} \right)_{DWBA}$$

$$F_{C\lambda}(q) \propto \int dr r^2 j_{\lambda}(qr) \delta\rho_{if}(r)$$

$$F_{E\lambda}(q) \propto \int dr r^2 \left[J_{\lambda, \lambda+1}^{if}(r) j_{\lambda+1}(qr) + (\lambda, \lambda-1) \right]$$

$$J_{\lambda, \lambda+1}^{if}(r) = \langle f | \mathbf{J}_{if} \cdot \mathbf{Y}_{\lambda\lambda+1} | i \rangle$$



Electron-ion collider mode

$$E_x \ll E, \quad \theta \ll 1 \quad \text{Siegert's theorem}$$

$$qR \ll 1$$

$$F_{C\lambda}(q) \cong \frac{E_x / \hbar}{q} \sqrt{\frac{\lambda + 1}{\lambda}} F_{E\lambda}(q)$$

$$\frac{d\sigma}{d\Omega dE_\gamma} = \sum_\lambda \frac{dN^{(E\lambda)}(E, E_\gamma, \theta)}{d\Omega dE_\gamma} \sigma_\gamma^{(E\lambda)}(E_\gamma)$$

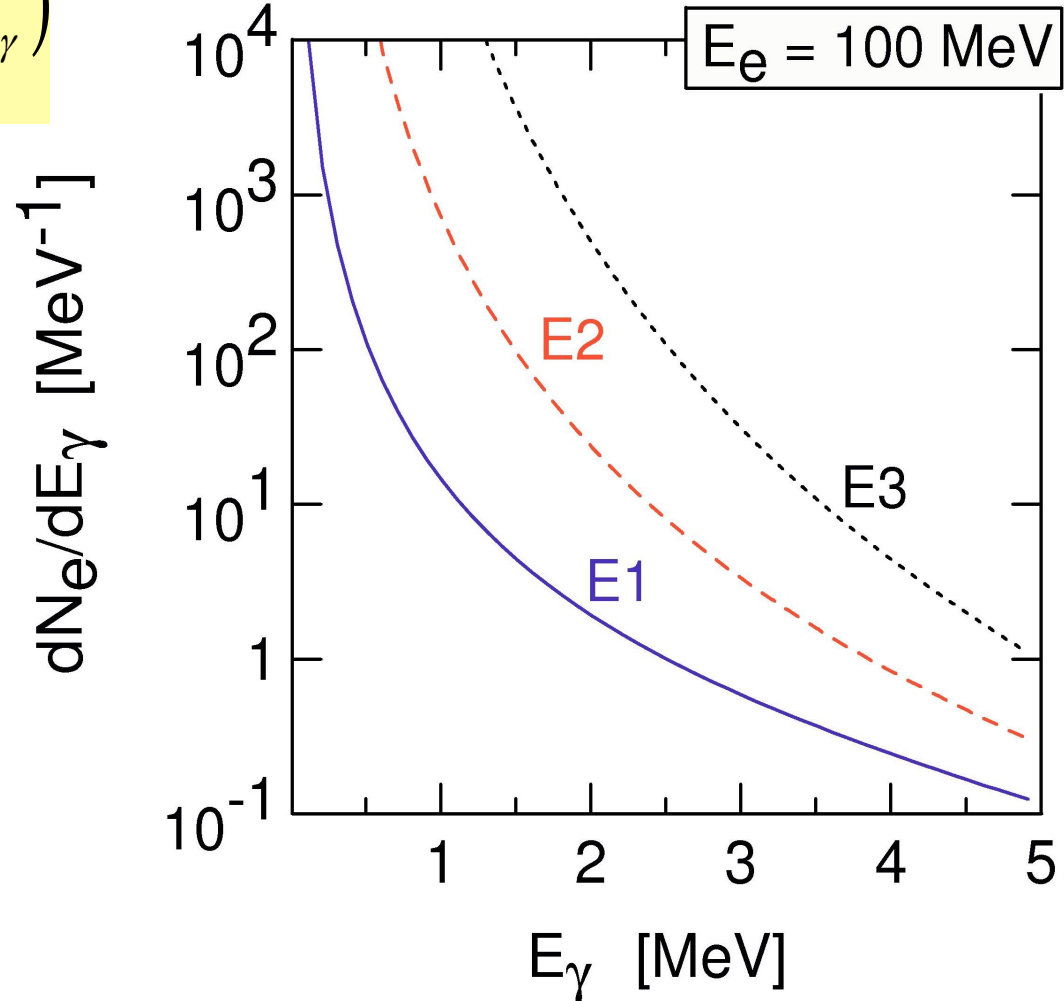
virtual photon spectrum

$$\frac{dN^{(E\lambda)}(E, E_\gamma)}{dE_\gamma} = \int_{E_\gamma/E}^{\theta_m} \frac{dN^{(E\lambda)}(E, E_\gamma, \theta)}{d\Omega dE_\gamma}$$

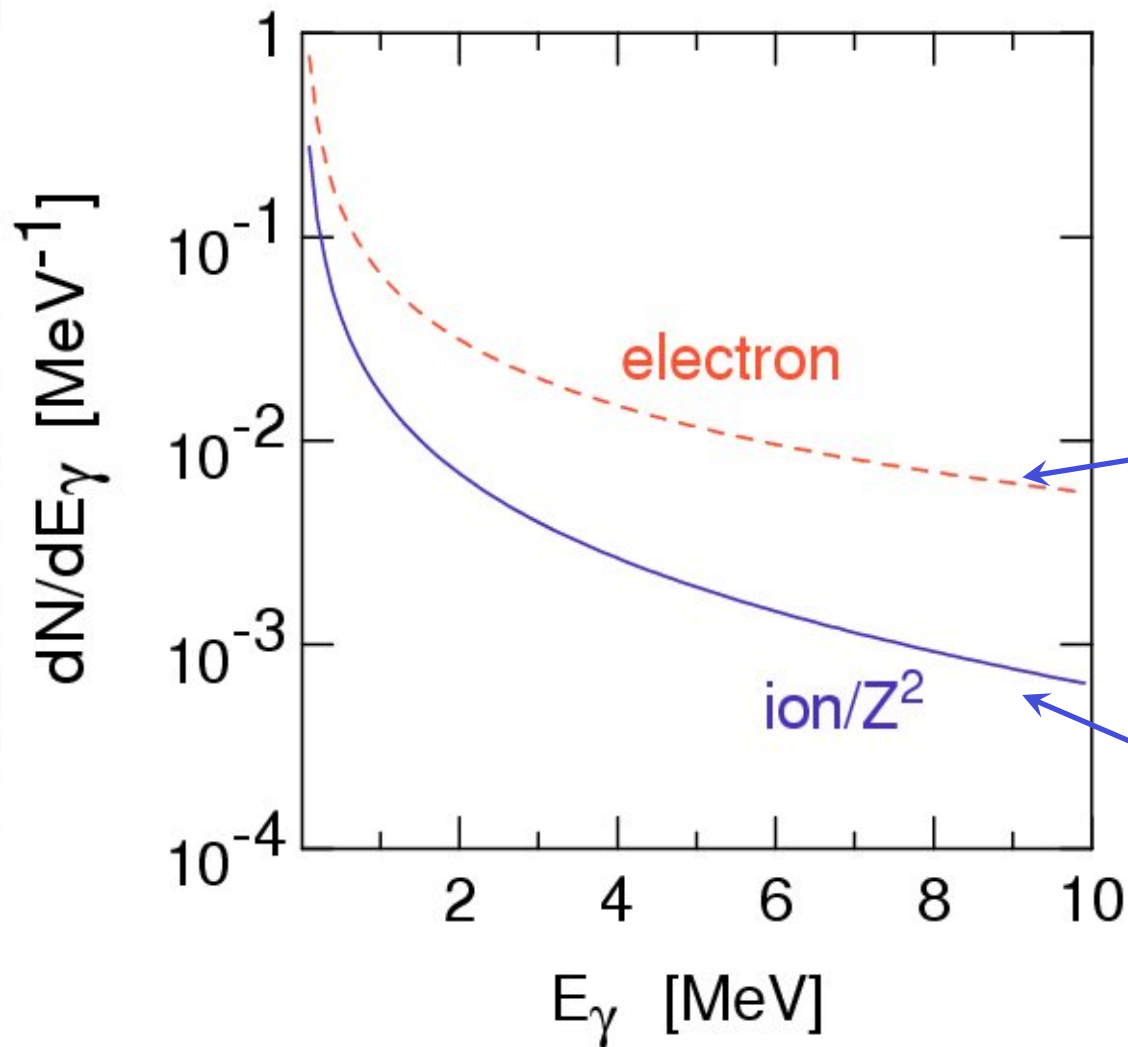
response function

$$\sigma_\gamma^{(E\lambda)}(E_\gamma) \propto \frac{dB(E\lambda)}{dE_x}$$

$$\frac{dB(E\lambda)}{dE_x} \propto \int dr r^2 r^\lambda \delta\rho_{if}(r)$$



Electron-ion collider mode



comparison with Coulomb excitation

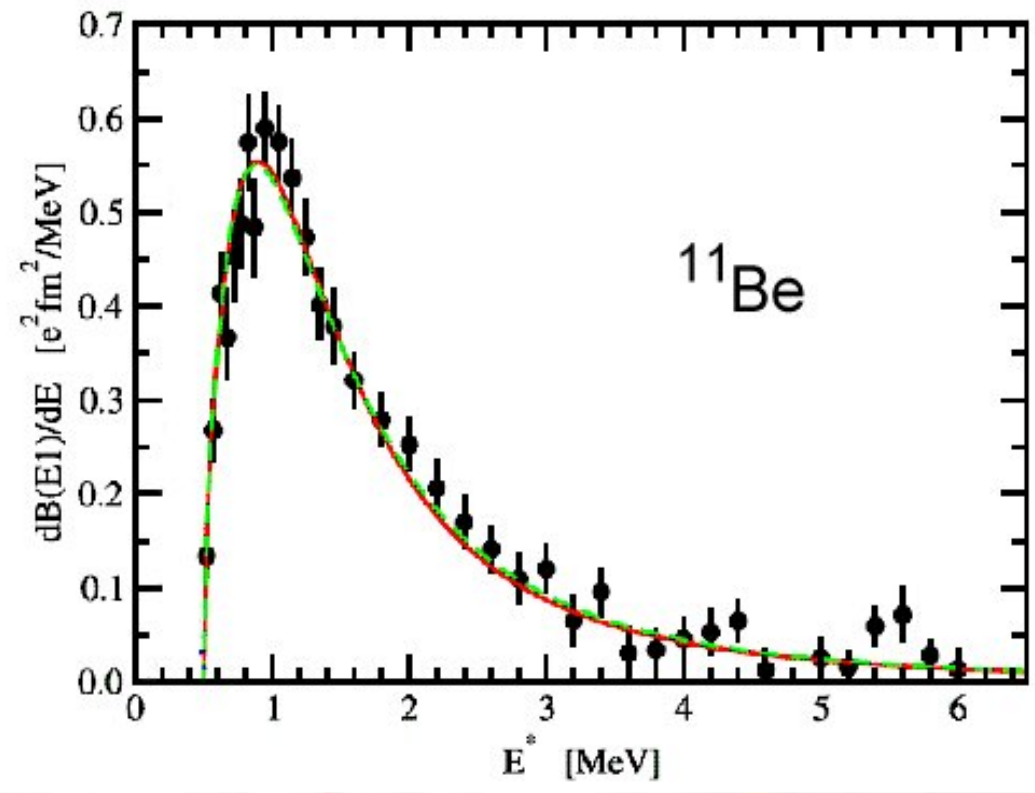
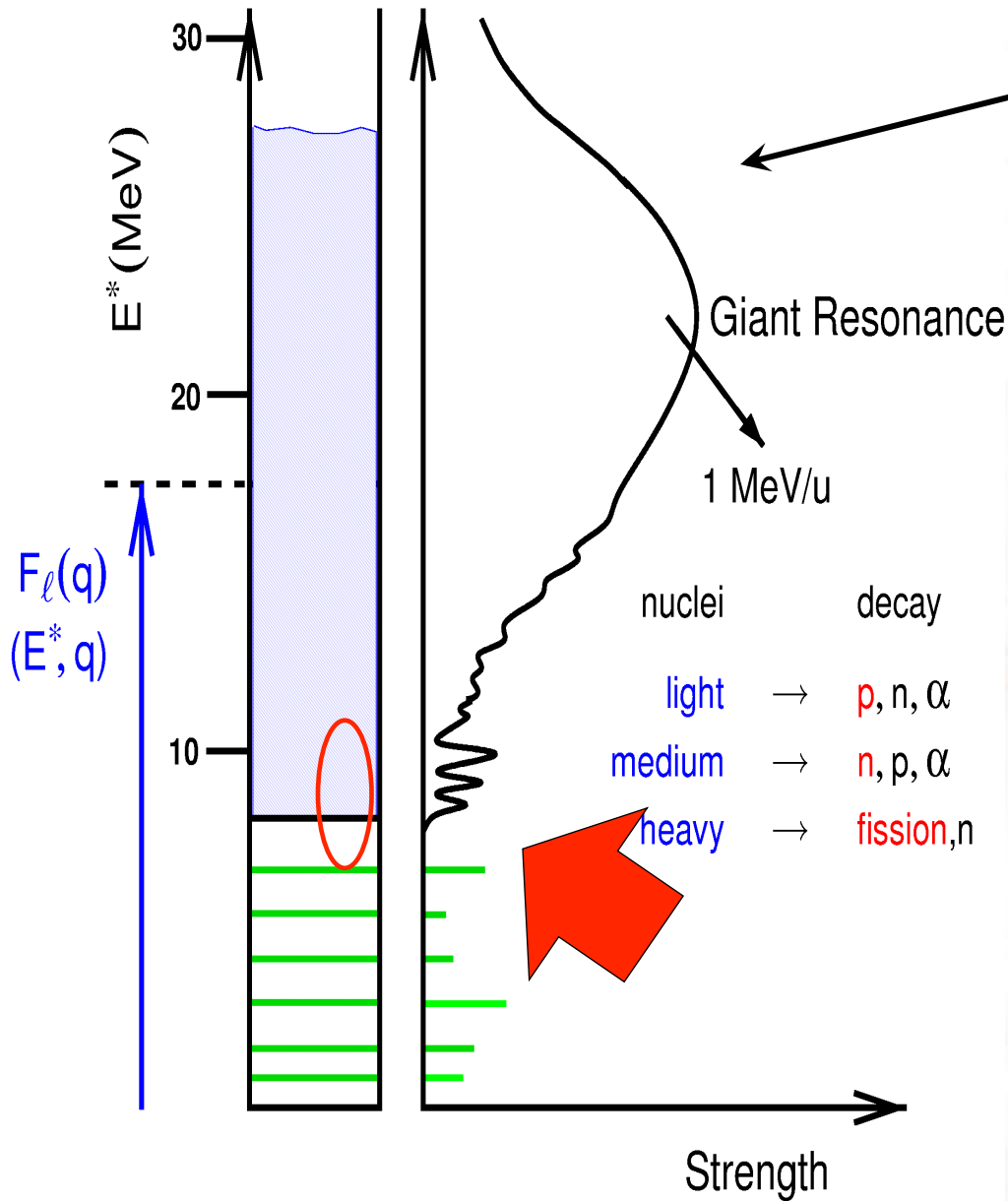
$E = 1 \text{ GeV}$

$E = 1 \text{ GeV/nucleon}$

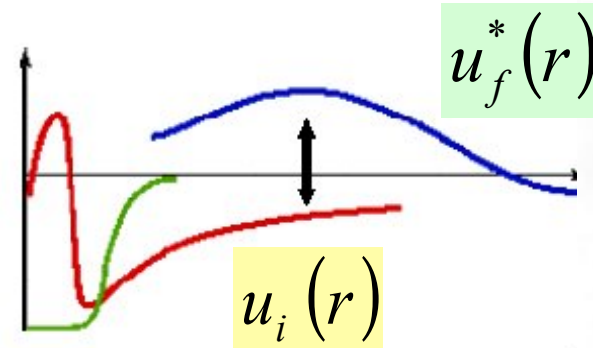
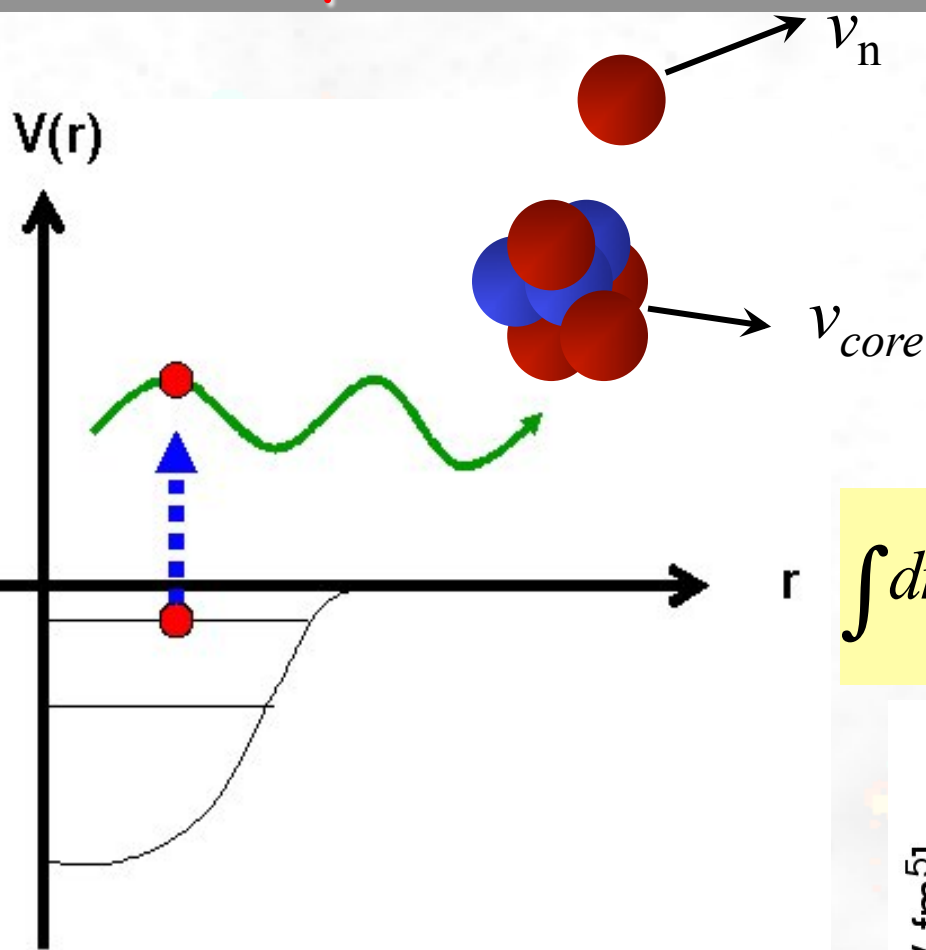
Bertulani, PLB 624, 203 (2005)

Electron-ion scattering: Response in exotic nuclei

Ex: EM response in exotic nuclei
 Collective response or
 Direct breakup?



Direct breakup



$$r^2 \delta\rho_{if}(r) \propto u_f^*(r)u_i(r)$$

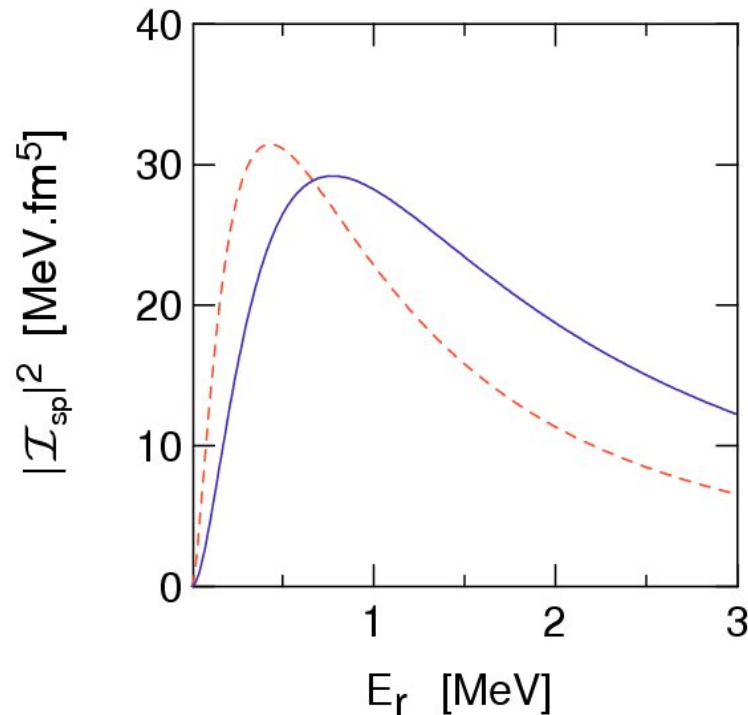
$$\int dr r^{2+1} \delta\rho_{if}(r) \propto \frac{E_r}{(S_n + E_r)^2} (1 + FSI)$$

$$\frac{dB(E\lambda)}{dE_r} \propto \frac{E_r^{\lambda+1/2}}{(S_n + E_r)^{2\lambda+2}} (1 + FSI)^2$$

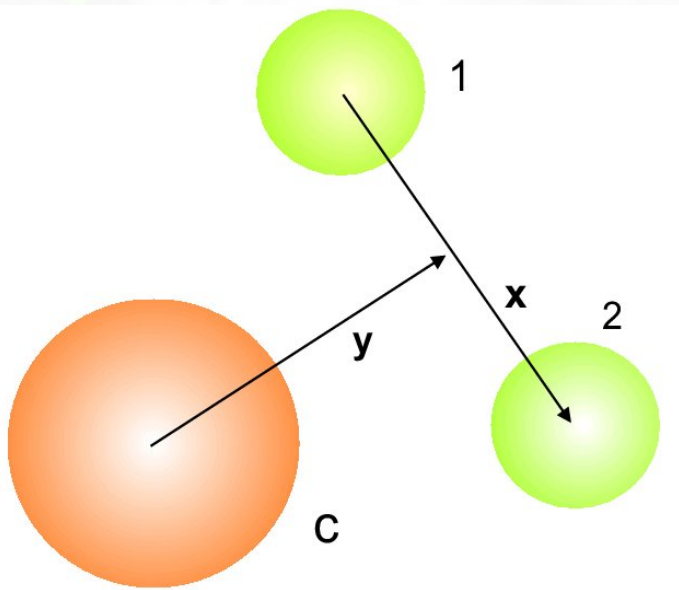
Bertulani, Sustich, PRC 46 (1992) 2340

$$E_r^{(E\lambda)peak} \cong \frac{\lambda + 1/2}{\lambda + 3/2} S_n$$

$$E_r^{(E1)peak} \cong \frac{3}{5} S_n$$



Direct breakup in the 3-body model



$$\Psi(\mathbf{x}, \mathbf{y}) = \frac{1}{\rho^{5/2}} \sum_{KLS l_x l_y} \Phi_{KLS}^{l_x l_y}(\rho) \left[\Gamma_{KL}^{l_x l_y}(\Omega_5) \otimes \chi_S \right]_{JM}$$

$$\Omega_5 = (\theta_x, \phi_x, \theta_y, \phi_y, \theta)$$

$$y = \rho \sin \theta, \quad x = \rho \cos \theta$$

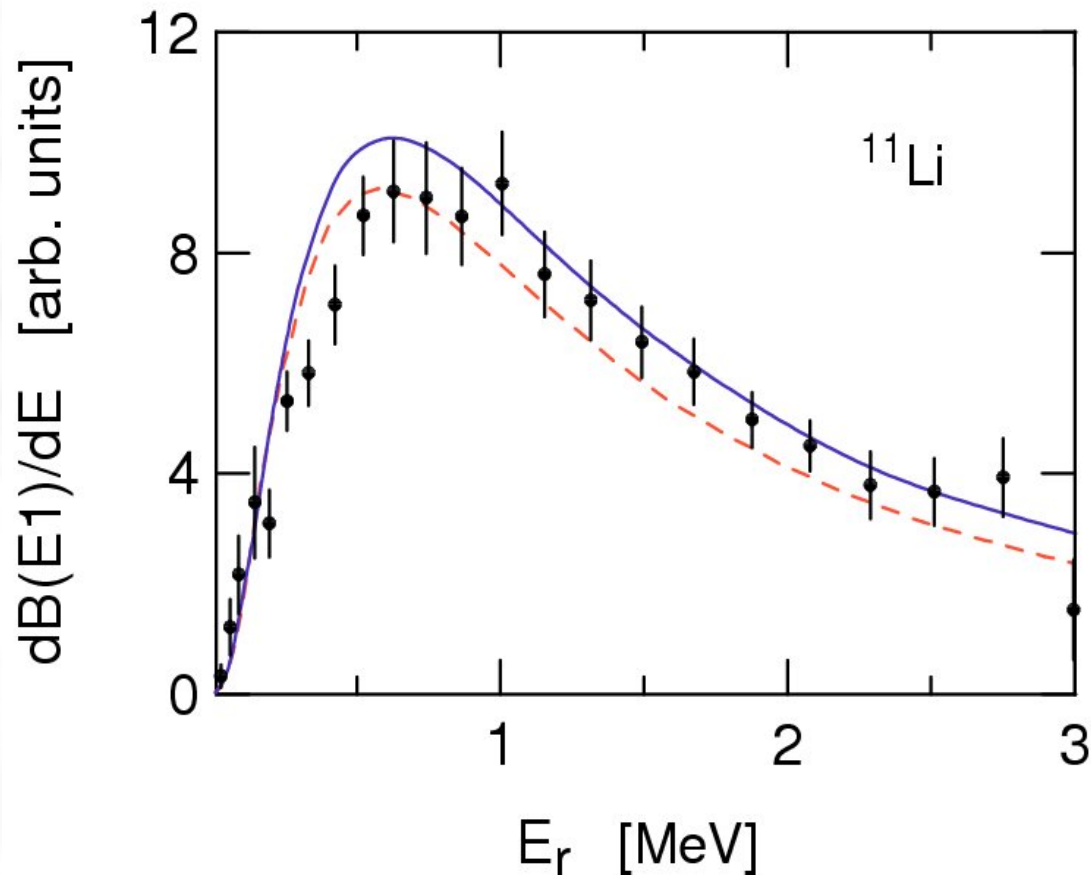
$$\delta\rho_{fi}^{E1} \propto \int dx dy \frac{\Phi_\alpha(\rho)}{\rho^{5/2}} y^2 x u_p(x) u_q(y)$$

$$E_r = \frac{\hbar^2}{2m_N} (q^2 + p^2)$$

$$\frac{dB(E1)}{dE_r} \propto \frac{E_r^3}{(S_{2n}^{eff} + E_r)^{11/2}} (1 + FSI)^2$$

$$S_{2n}^{eff} \cong 1.8 S_{2n}$$

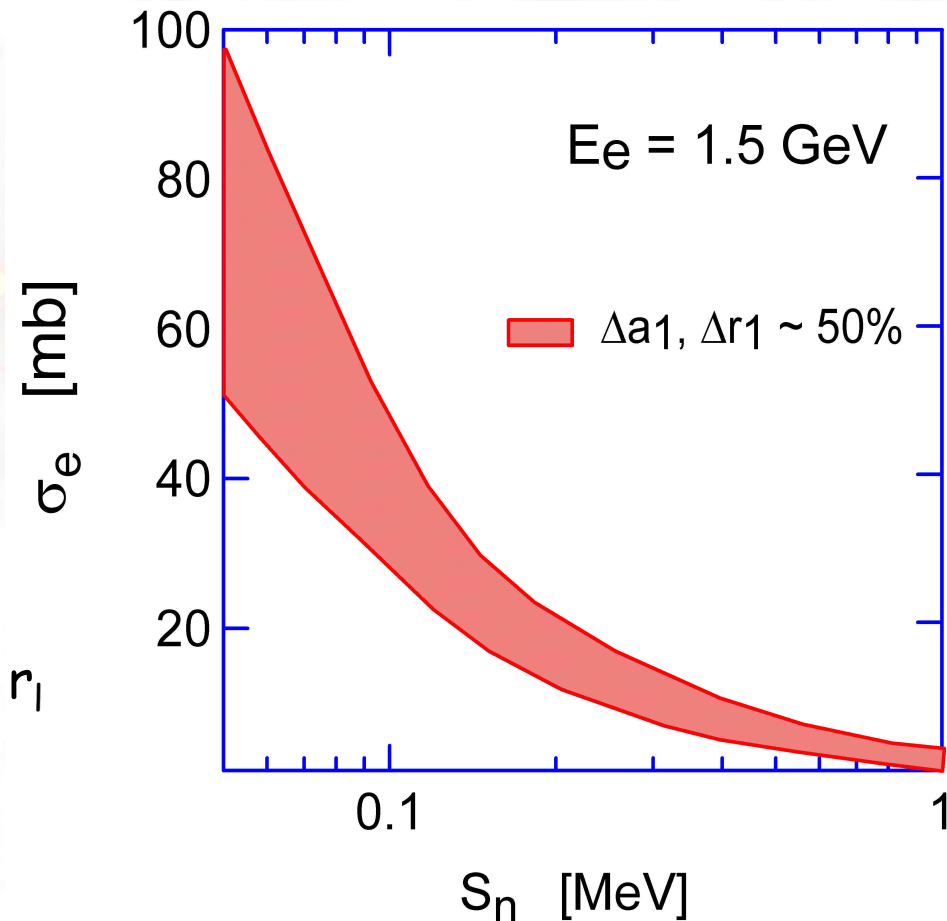
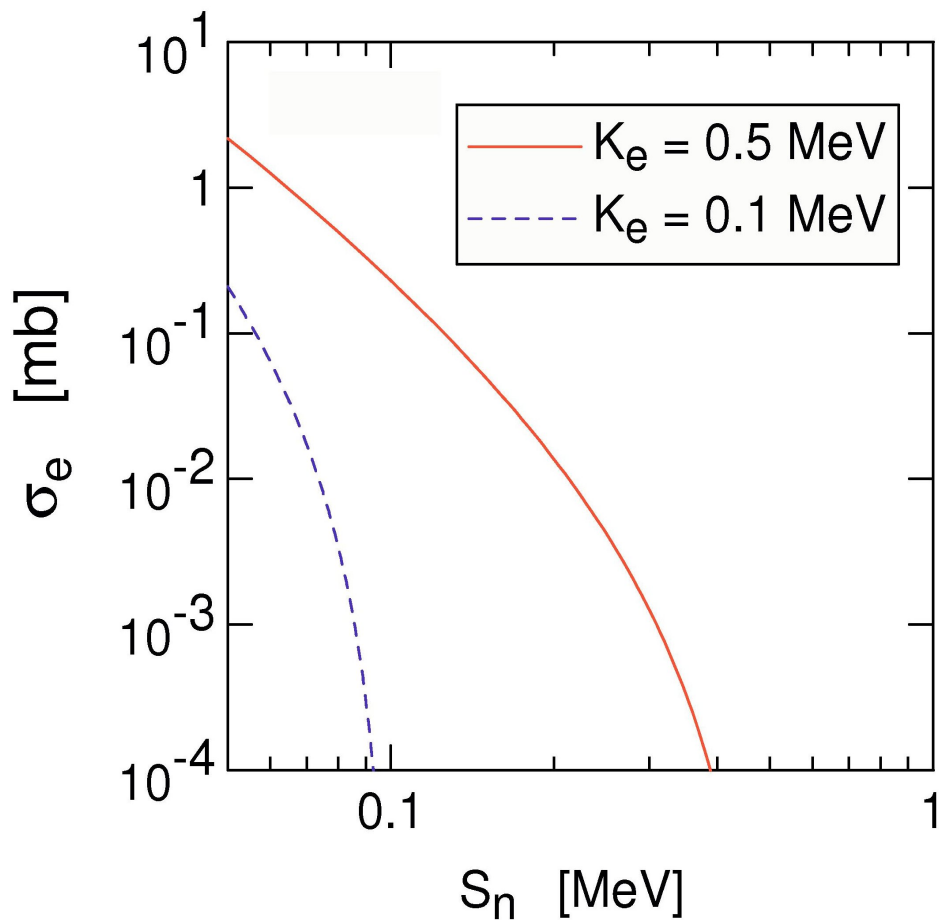
Bertulani, PRC 75, 024606 (2007)



Dependence on bind. energy and FSI

$$\frac{d\sigma}{dE_e d\Omega} \sim |f_l(q)|^2$$

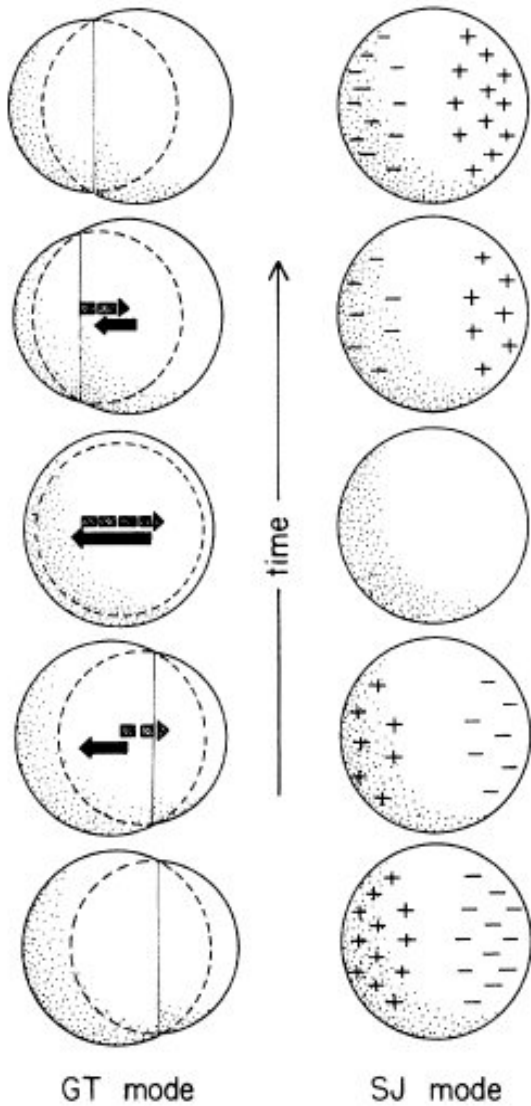
$$f_l \equiv f_l(q, S_n, a_l, r_l)$$



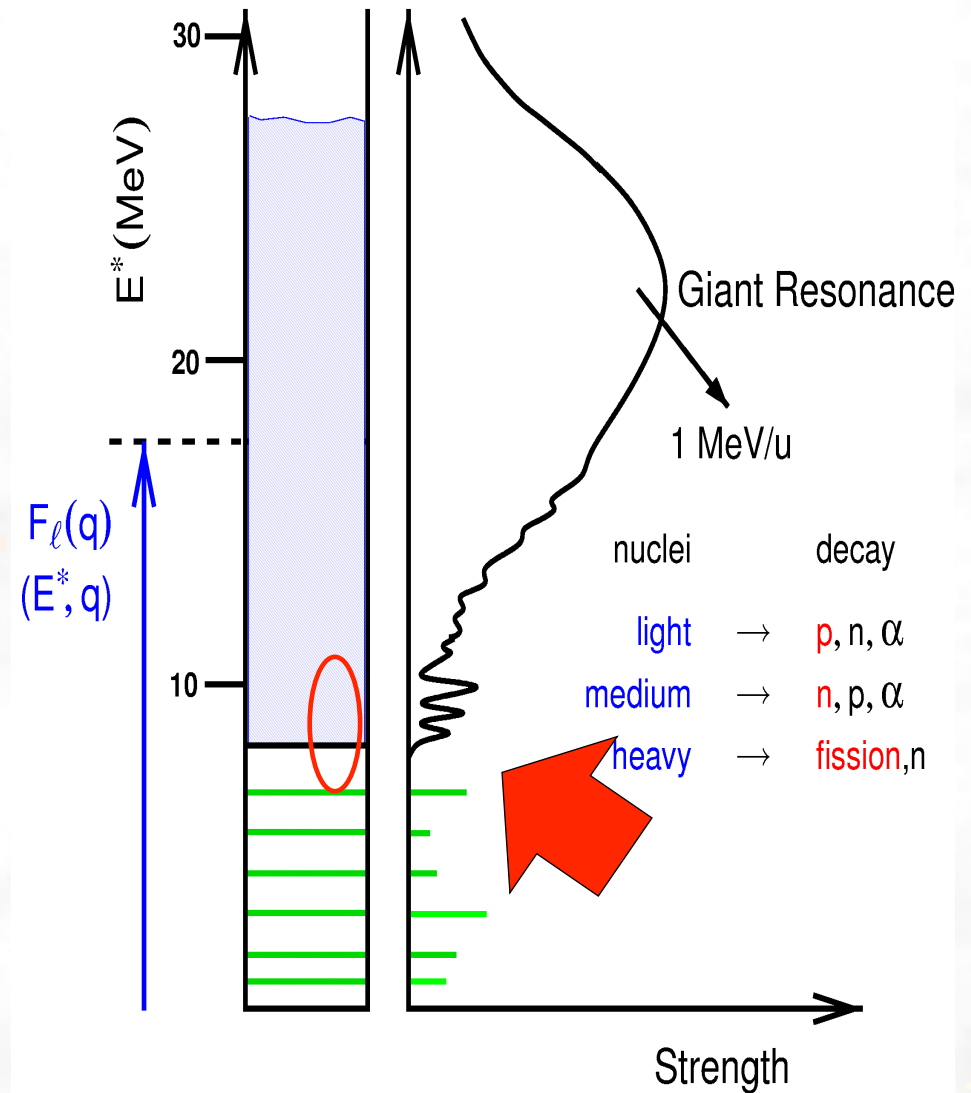
Halo nuclei: very strong dependence on effective range expansion parameters, a_l, r_l

Bertulani, PLB 624, 203 (2005)

Hydrodynamical model for collective vibrations



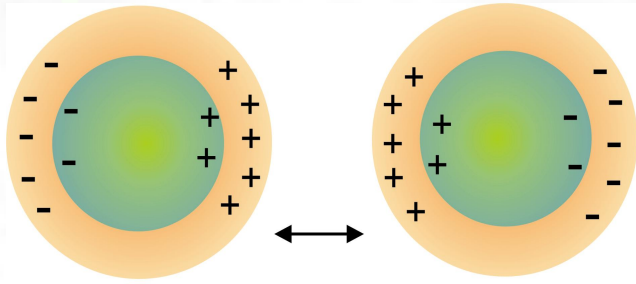
$$T = \frac{1}{2} m^* \int \rho_p \left(\mathbf{v}_{SJ}^{(p)} + \mathbf{v}_{GT}^{(p)} \right)^2 + \rho_n \left(\mathbf{v}_{SJ}^{(n)} + \mathbf{v}_{GT}^{(n)} \right)^2$$



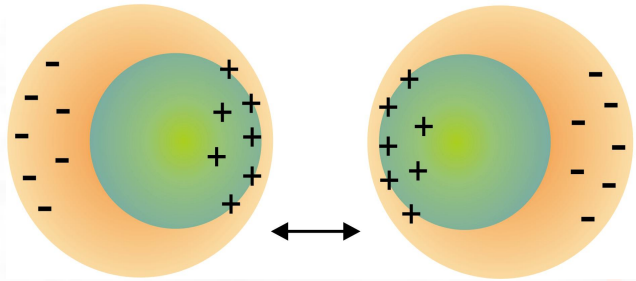
$$V = -\kappa \int d^3r \frac{(\rho_p - \rho_n)^2}{\rho_p + \rho_n} + \text{surf. terms}$$

$$\kappa \cong 30 - 40 \text{ MeV}$$

Transition densities for pygmy resonances



SJ



GT

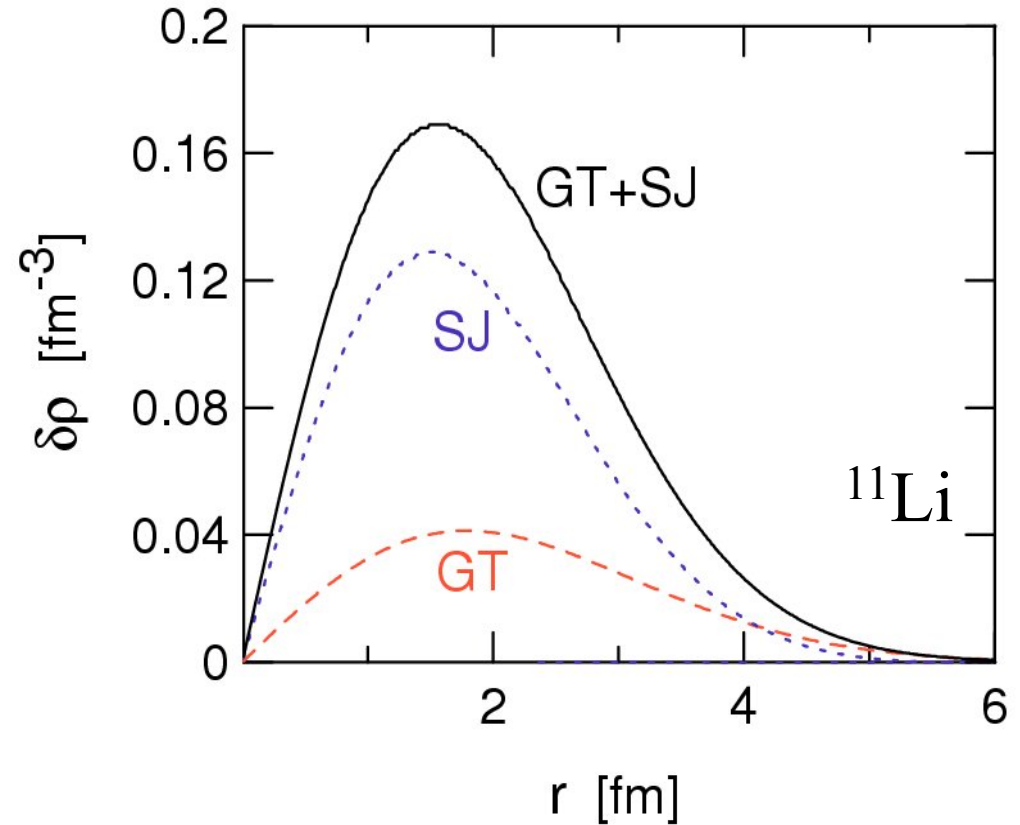
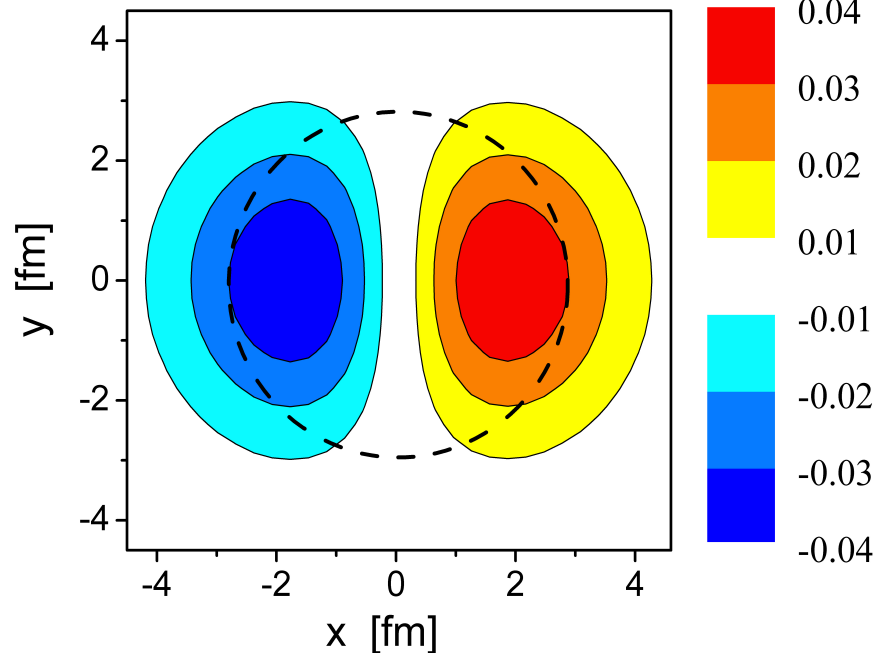
GT

SJ

$$\delta\rho = \sqrt{\frac{4\pi}{3}} R \left[Z_{eff}^{(GT)} \alpha_{GT} \frac{d}{dr} + Z_{eff}^{(SJ)} \alpha_{SJ} \frac{K}{R} j_1(kr) \right] \rho_0(r)$$

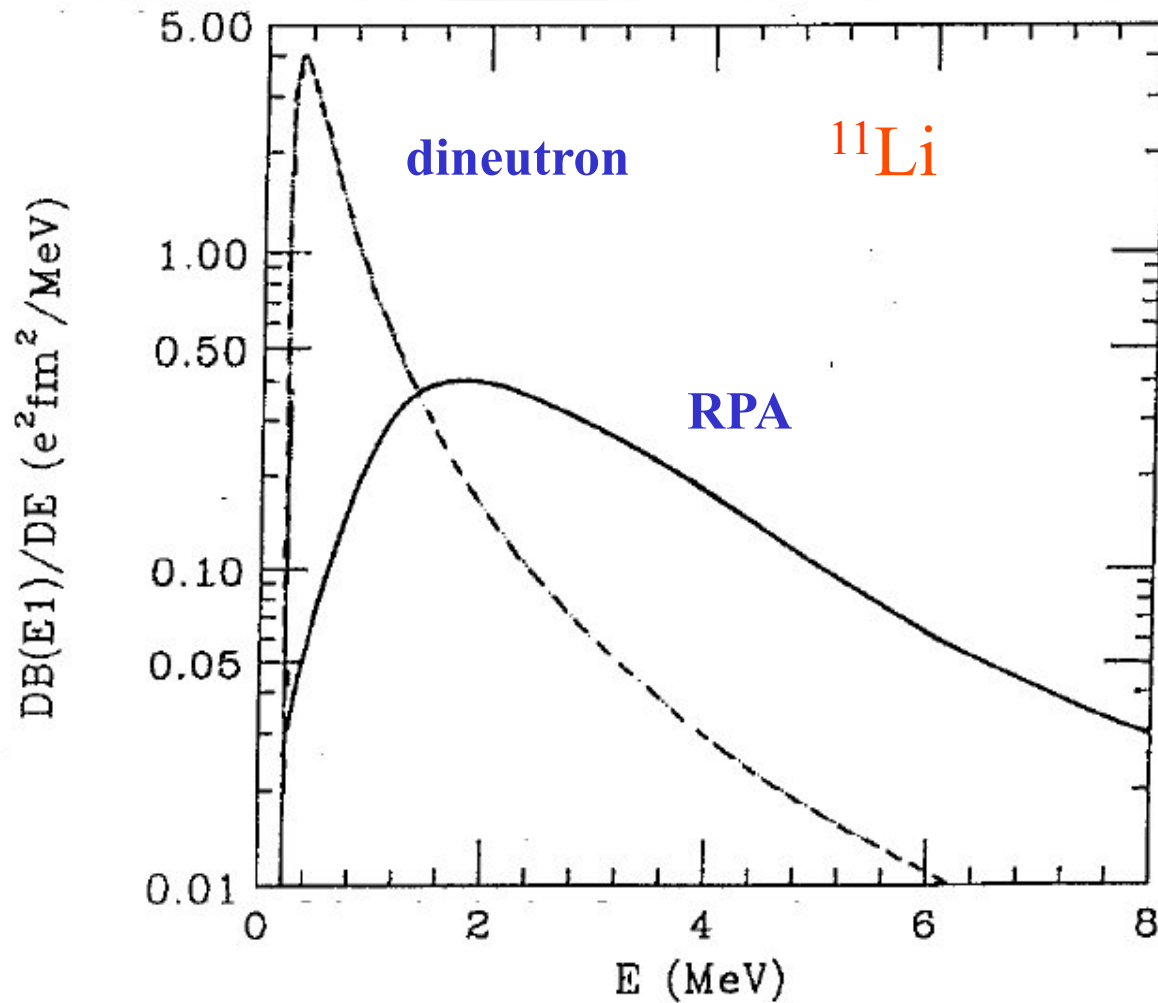
$kR = 2.081, \quad K = 9.93$

Bertulani, PRC 75, 024606 (2007)

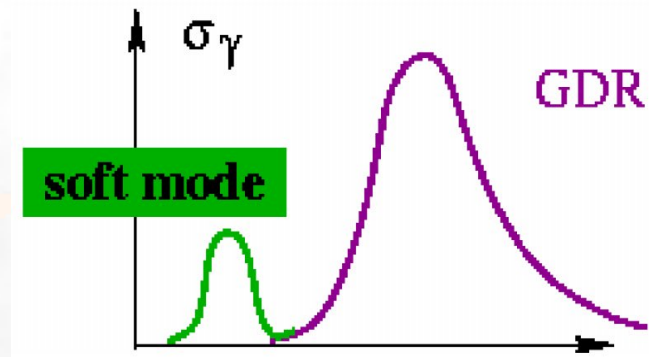


Collective response in light neutron rich nuclei

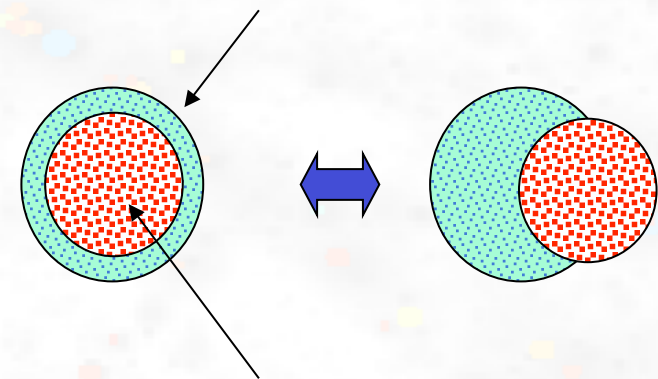
N. Teruya et al,
PRC 43 (1991) 2049 1991



RPA + $2n_p-2n_h$ excitations



excess neutrons

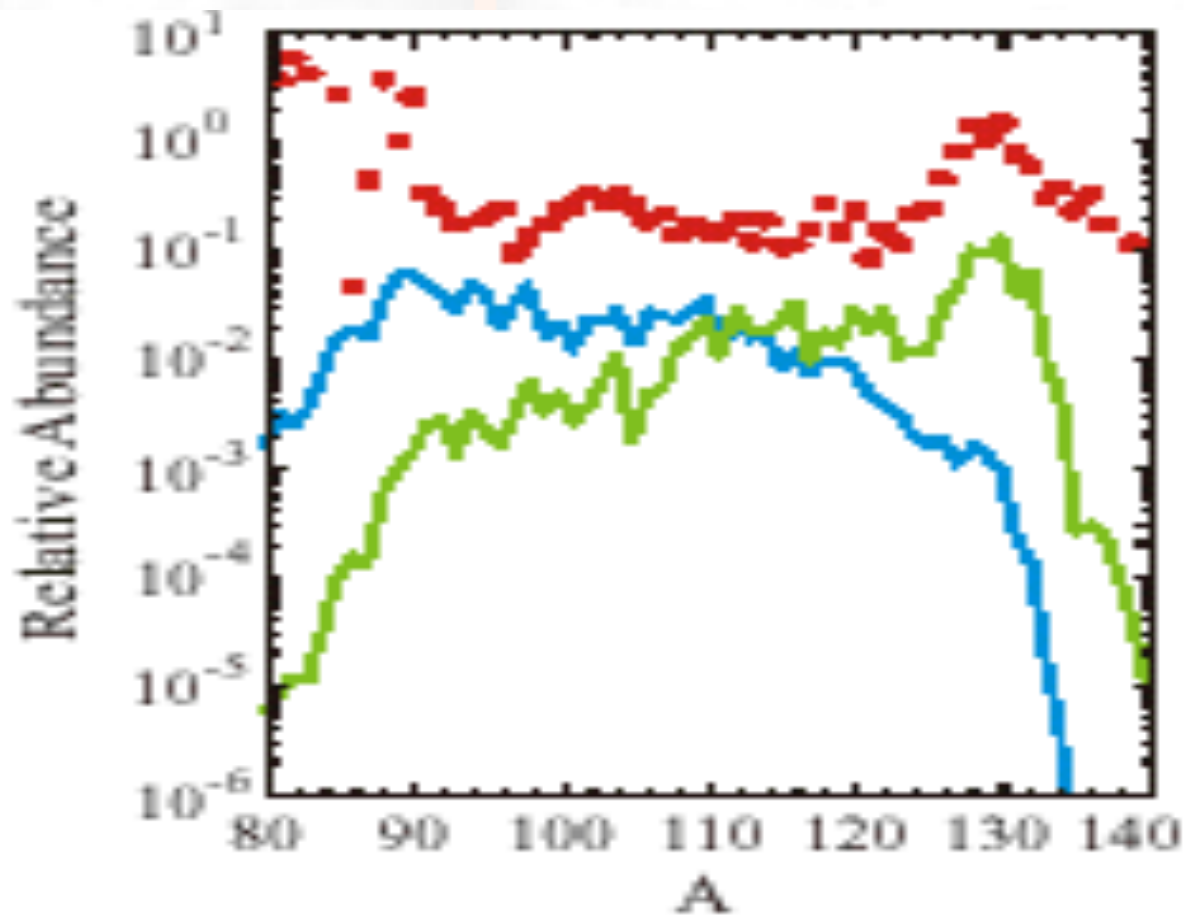


“core” with p and n

Is there a soft dipole ?
Only in light nuclei ?

Relevance for nuclear astrophysics (medium A)

Nucleosynthesis: (γ, n) or (n, γ) cross sections in the r-process
Importance of the "pygmy" states



Red: empirical

Blue: no pygmy

Green: with pygmy

S. Goriely, PLB 2000

It is important to have reliable measurements and model predictions

End of part II