Nuclear Astrophysics with Radioactive Beams

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Escuela Andina "Física Nuclear en el siglo 21"
(26-30 November 2012)

Part III

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Experiments and theories for radioactive beams

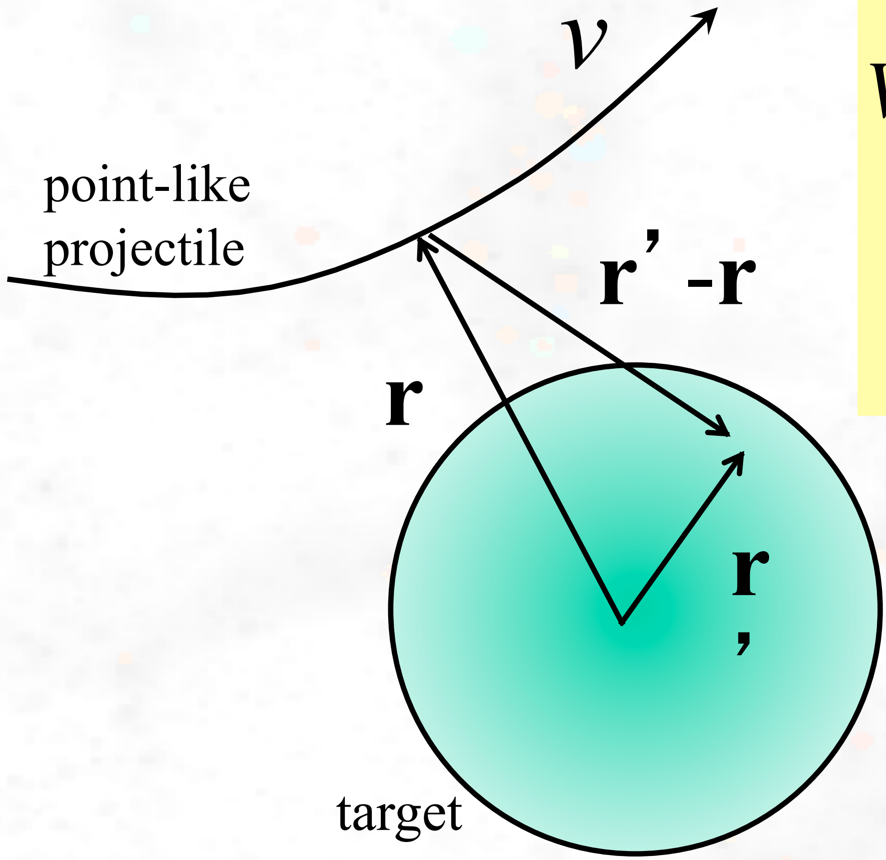
RIB Facilities

(Operating or Under Construction)

Coulomb Dissociation

Radiative Capture Reactions

Coulomb Excitation



$$V_C(r,r') = Z_p e \int \frac{\rho(\mathbf{r}')}{|r-r'|} d^3r'$$

$$= \frac{Z_p e}{r} + \frac{\mathbf{p}\cdot\hat{\mathbf{r}}}{r^3} + \frac{1}{2}\frac{Q_{ij} r_i r_j}{r^5} + \cdots$$

$$\mathbf{p} = \int \mathbf{r}' \rho(\mathbf{r}') d^3r' \quad \text{(dipole)}$$

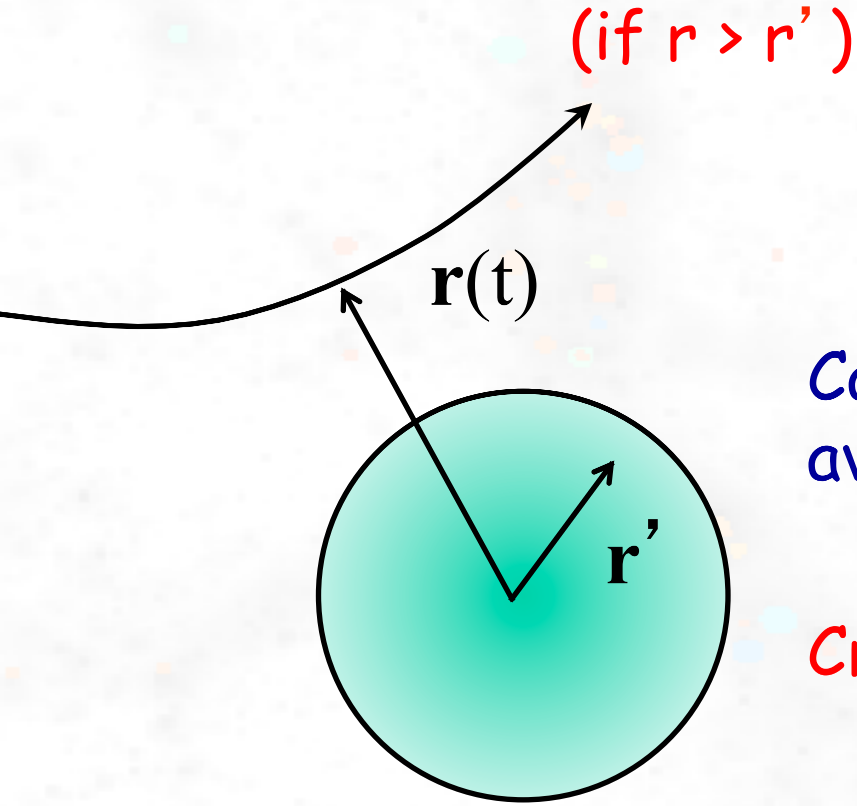
$$Q_{ij} = \int \left(3r'_i r'_j - r'^2 \delta_{ij}\right) \rho(\mathbf{r}') d^3r'$$

(Quadrupole)

Semiclassical method: r = r(t)

Validity: $\eta = \frac{\text{distance of closest approach}}{\text{wavelength}} = \frac{Z_1 Z_2 e^2}{\hbar v} >> 1$

General multipole expansion



$$\frac{1}{|\mathbf{r}(t)-\mathbf{r}'|} = \sum_{L,M} \frac{4\pi}{2L+1} \frac{r'}{r^{L+1}(t)} Y_{LM}(\hat{\mathbf{r}}(t)) Y_M^*(\hat{\mathbf{r}}')$$

Calculate a_{fi} and average over spins:

$$w_{fi} = \frac{1}{2J_i+1} \sum_{M_i M_f} |a_{fi}|^2$$

Cross section:

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma_R}{d\Omega} \cdot w_{fi} = \sum_{L>0} \frac{d\sigma_L}{d\Omega}$$

orbital integral

$$\frac{d\sigma_L}{d\Omega} \sim Z_P^2 B(EL) \left| I_L(\omega_{fi}) \right|^2$$

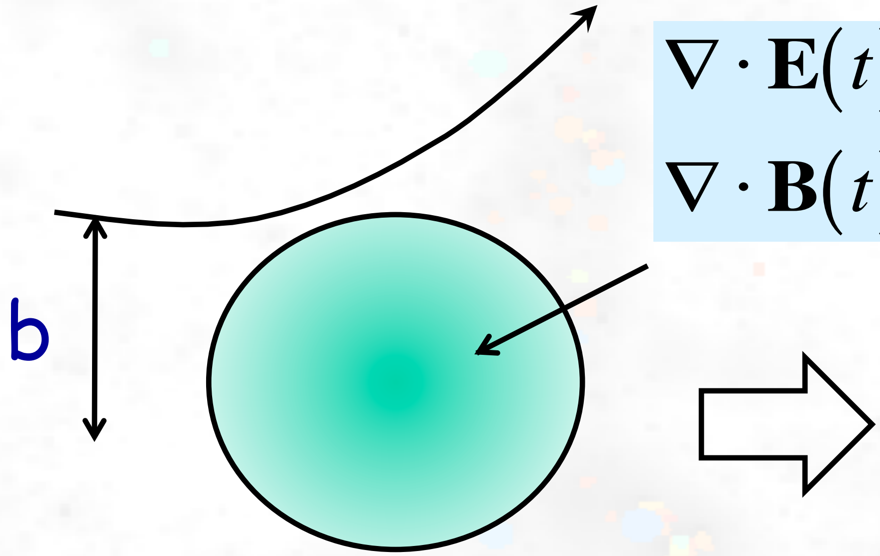
$$I_L(\omega) = \int_{-\infty}^{\infty} dt \frac{1}{r^{L+1}(t)} Y_{LM}(\hat{\mathbf{r}}(t)) e^{i\omega t}$$

$$\omega_{fi} = \frac{E_f - E_i}{\hbar}$$

$$B(EL) \sim \left| \int r^L \delta\rho_{fi} \, d^3r \right|^2$$

reduced transition strength

Virtual photon numbers



$$\nabla \cdot \mathbf{E}(t) = 0$$

$$\nabla \cdot \mathbf{B}(t) = 0$$

E, B-field of projectile divergence free

$$\frac{d\sigma_L}{d\Omega} = \int \frac{dE_\gamma}{E_\gamma} \frac{dn_L}{d\Omega}\left(E_\gamma, \theta\right) \sigma_L^\gamma\left(E_\gamma\right)$$

photonuclear X-section: $\sigma_L^\gamma \sim E_\gamma^{2L+1} B(EL)$

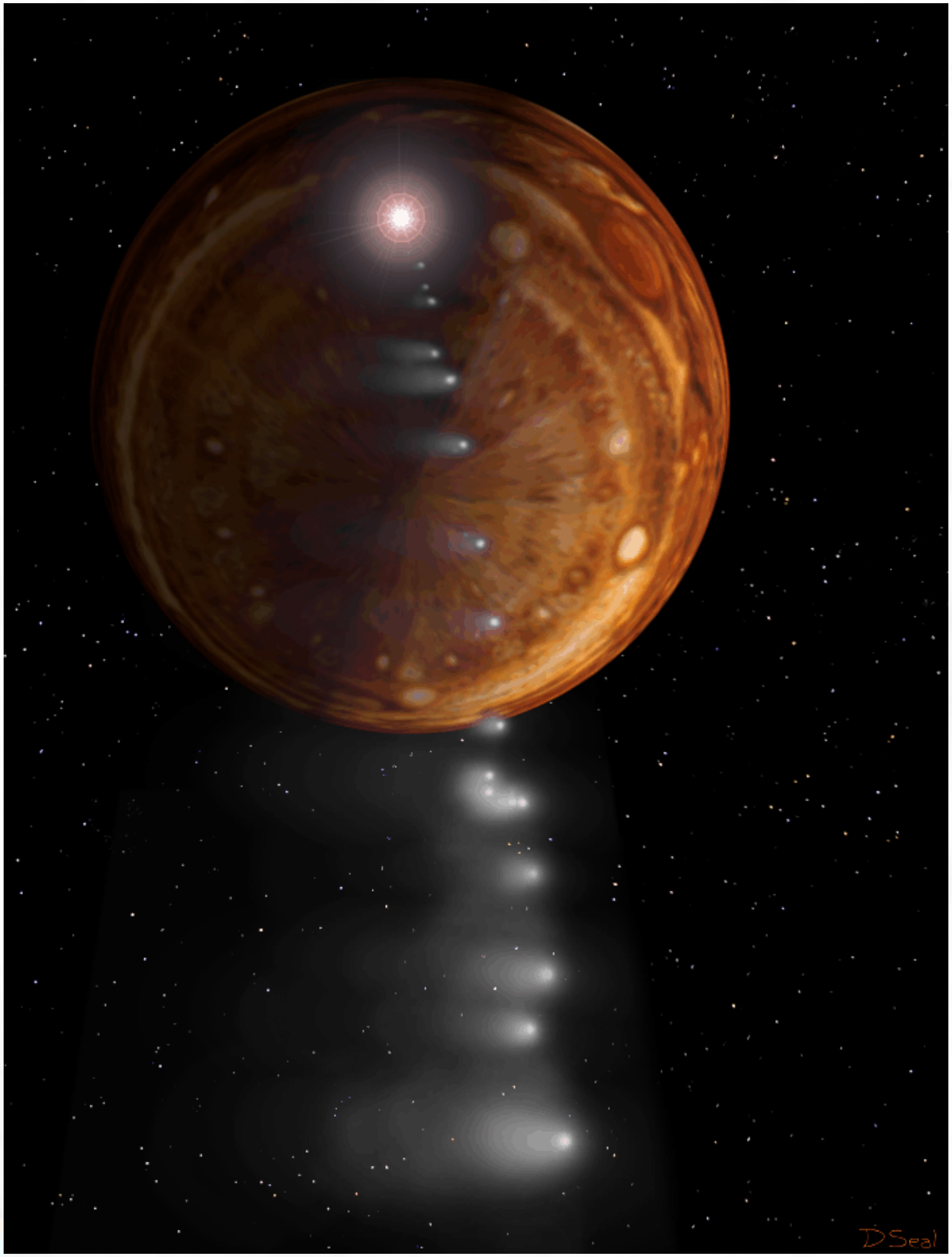
$$E_\gamma = E_f - E_i$$

virtual photon numbers: $\frac{dn_L}{d\Omega} \sim Z_P^2 \left| I_L\left(\omega_{fi}, \theta\right)\right|^2$

impact parameter dependence:

$$n_L\left(E_\gamma, b\right) \equiv \frac{dn_L}{2\pi b db} \sim \sin^4(\theta/2) \frac{dn_L}{d\Omega}$$

$1/r^2$ force



Comet Shoemaker-Levy 9 disintegrating as it approaches Jupiter in July 1994.

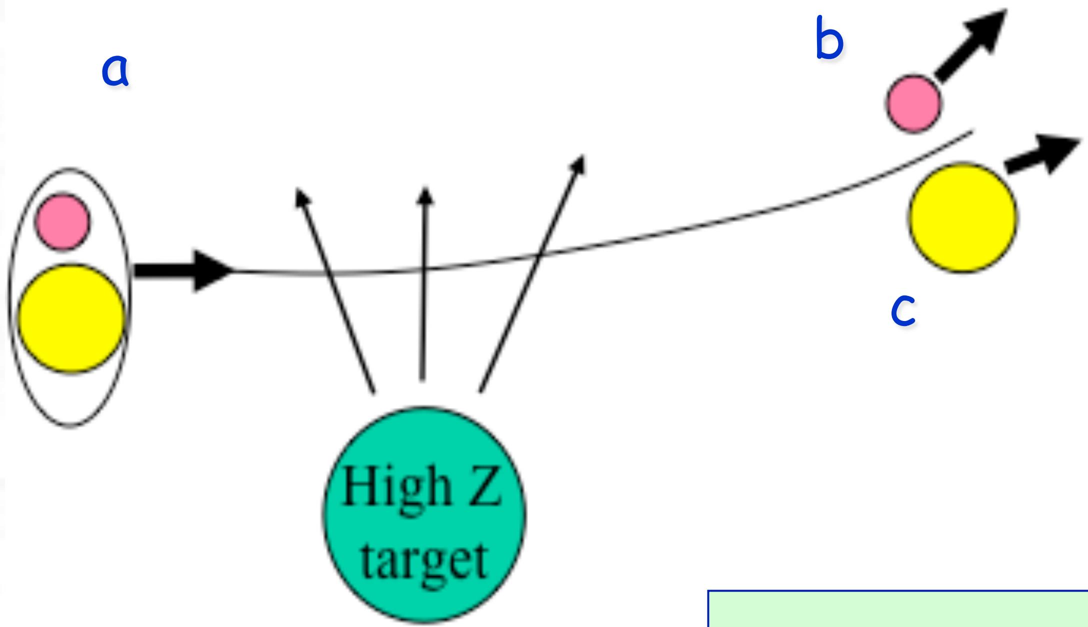
Coulomb dissociation and nuclear astrophysics

Baur, Bertulani, Rebel
NPA 458 (1986) 188

$$\frac{d\sigma}{dE_\gamma d\Omega} = \frac{1}{E_\gamma} \sum_l \frac{dn_l(E_\gamma, \Omega)}{dE_\gamma d\Omega} \sigma_{\gamma + a \to b + c}(E_\gamma)$$

Theory

detailed balance



$$\sigma_{b+c \to a+\gamma} = \frac{2(2j_a+1)}{(2j_b+1)(2j_c+1)} \frac{k_{bc}^2}{k_\gamma^2} \sigma_{\gamma+a \to b+c}$$

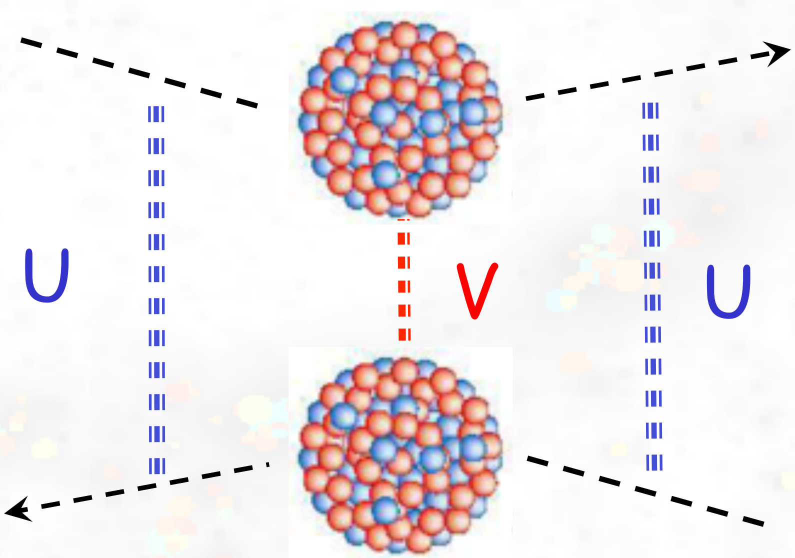
Applications to radiative capture (n,γ) and (p,γ) reactions in nuclear astrophysics.

Including nuclear conribution: DWBA

$$f_{inel}(\theta) = -\frac{4\pi^2\mu}{\hbar^2}\int d^3r\, \chi_{\mathbf{k}'}^{(-)*}(\mathbf{r})\, V(\mathbf{r})\, \Psi_{\mathbf{k}}^{(+)}(\mathbf{r})$$

$$\Psi^{\pm} \sim \chi^{\pm} \quad \Rightarrow \quad f_{DWBA}(\mathbf{k}',\mathbf{k}) = -\frac{4\pi^2\mu}{\hbar^2}\left\langle \chi_{\mathbf{k}'}^{(-)} \middle| V \middle| \chi_{\mathbf{k}}^{(+)} \right\rangle$$

$$T_{DWBA}(\mathbf{k}',\mathbf{k}) = \left\langle \chi_{\mathbf{k}'}^{(-)} \middle| V \middle| \chi_{\mathbf{k}}^{(+)} \right\rangle$$



Distorted: all orders in U

Born: only first order in V

Coulomb + Nuclear excitation

$$f^{C}_{inel}(\theta) \approx \int d^3r\, d^3r'\, \Psi^{(-)*}_{\mathbf{k}'}(\mathbf{r})\, \varphi_f(\mathbf{r}')\, V_C(\mathbf{r},\mathbf{r}')\, \Psi^{(+)}_{\mathbf{k}}(\mathbf{r})\, \varphi_i(\mathbf{r}')$$

nice, well known, angel

$$f^{N}_{inel}(\theta) \approx \int d^3r\, d^3r'\, \Psi^{(-)*}_{\mathbf{k}'}(\mathbf{r})\, \varphi_f(\mathbf{r}')\, V_N(\mathbf{r},\mathbf{r}')\, \Psi^{(+)}_{\mathbf{k}}(\mathbf{r})\, \varphi_i(\mathbf{r}')$$

bad, not well known, a true monster

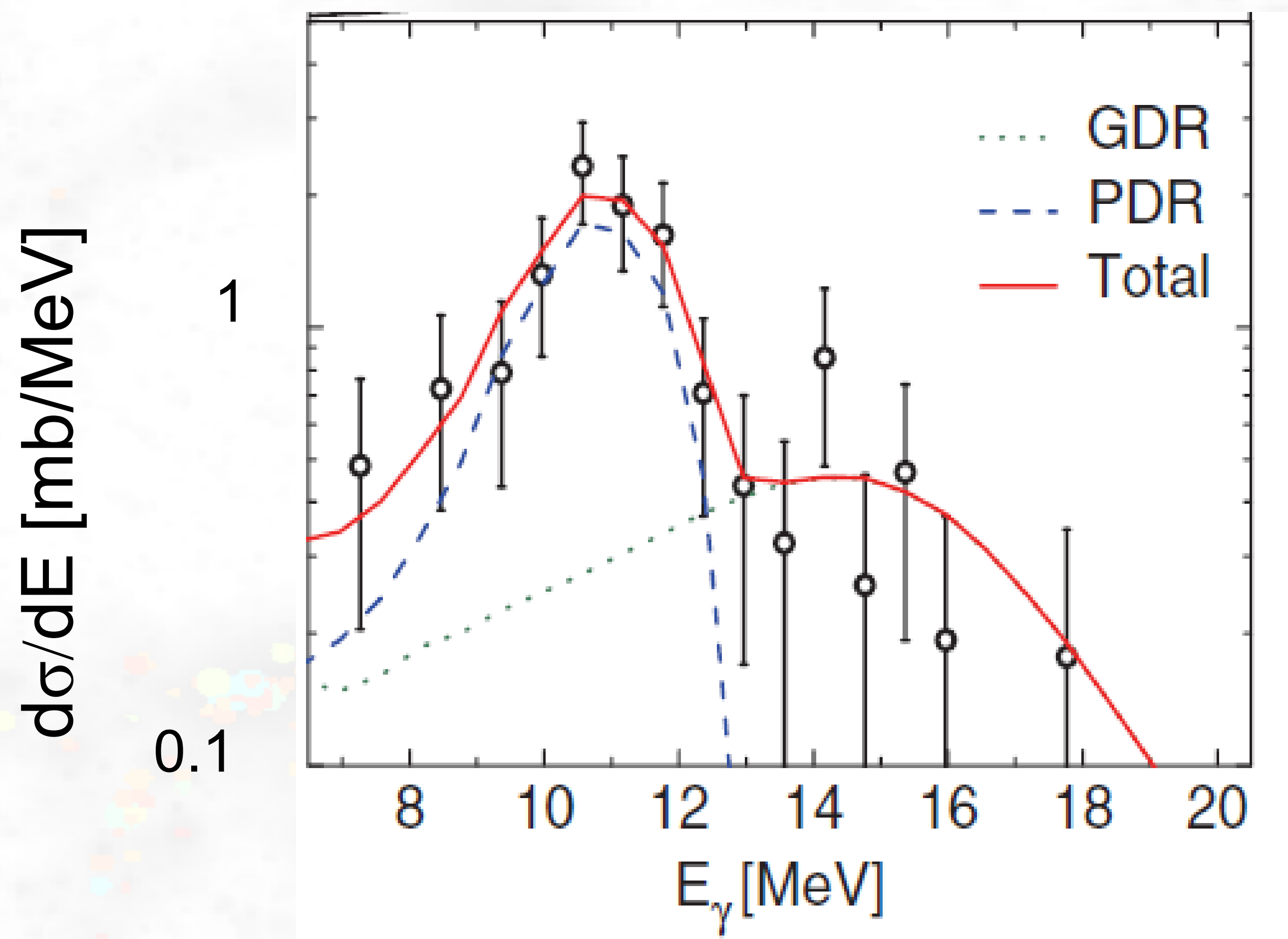
$$\frac{d\sigma}{d\Omega} = \left| f^{N}_{inel}(\theta) + f^{C}_{inel}(\theta) \right|^2$$

Example: Pigmy resonance in ^{68}Ni

PRL **102,** 092502 (2009) PHYSICAL REVIEW LETTERS week ending 6 MARCH 2009

Search for the Pygmy Dipole Resonance in ^{68}Ni at 600 MeV/nucleon

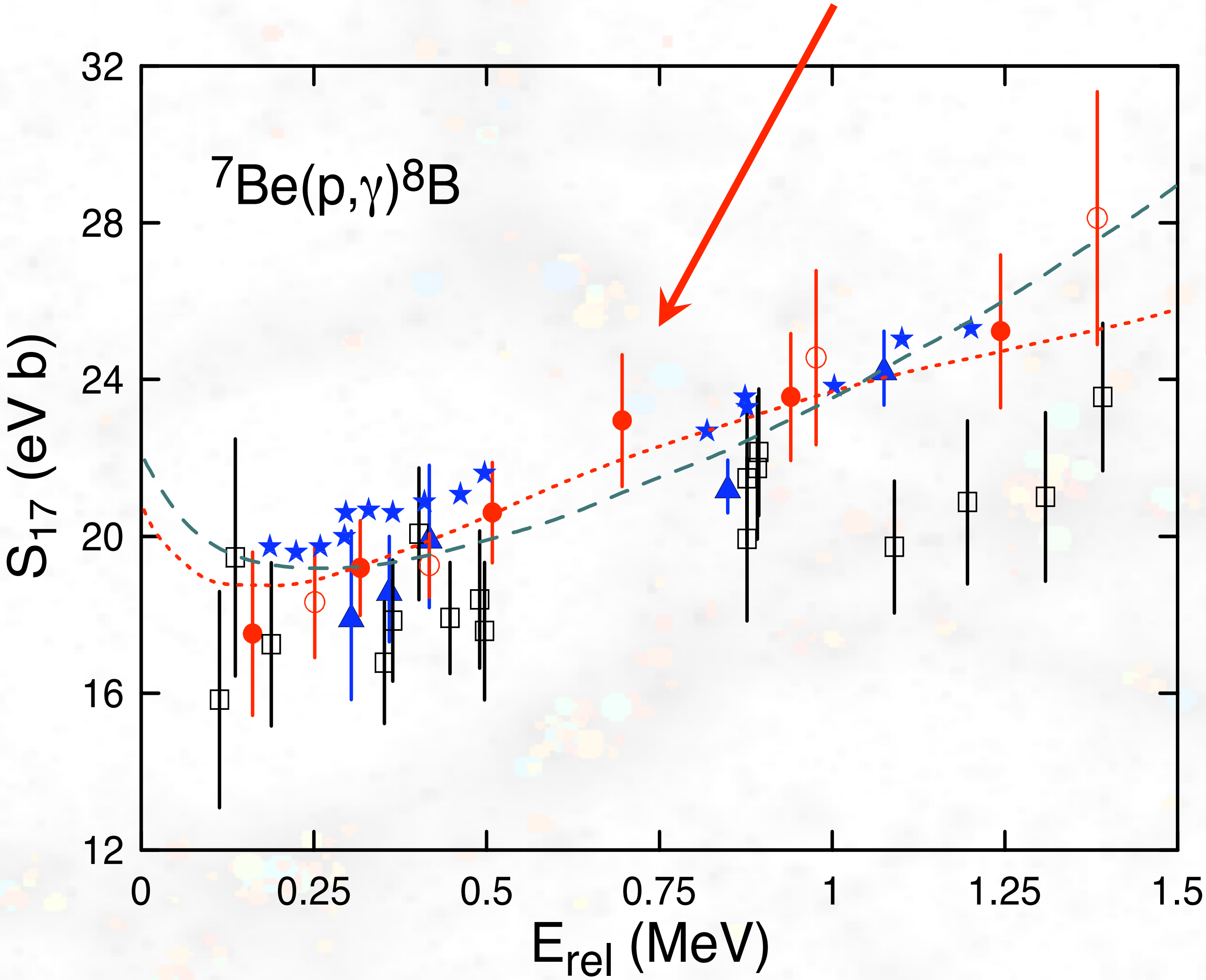
O. Wieland,[1] A. Bracco,[1,2] F. Camera,[1,2] G. Benzoni,[1] N. Blasi,[1] S. Brambilla,[1] F. C. L. Crespi,[1,2] S. Leoni,[1,2] B. Million,[1] R. Nicolini,[1,2] A. Maj,[3] P. Bednarczyk,[3] J. Grebosz,[3] M. Kmiecik,[3] W. Meczynski,[3] J. Styczen,[3] T. Aumann,[4] A. Banu,[4] T. Beck,[4] F. Becker,[4] L. Caceres,[4,*] P. Doornenbal,[4,†] H. Emling,[4] J. Gerl,[4] H. Geissel,[4] M. Gorska,[4] O. Kavatsyuk,[4] M. Kavatsyuk,[4] I. Kojouharov,[4] N. Kurz,[4] R. Lozeva,[4] N. Saito,[4] T. Saito,[4] H. Schaffner,[4] H. J. Wollersheim,[3] J. Jolie,[5] P. Reiter,[5] N. Warr,[5] G. deAngelis,[6] A. Gadea,[6] D. Napoli,[6] S. Lenzi,[7,8] S. Lunardi,[7,8] D. Balabanski,[9,10] G. LoBianco,[9,10] C. Petrache,[9,‡] A. Saltarelli,[9,10] M. Castoldi,[11] A. Zucchiatti,[11] J. Walker,[12] and A. Bürger[13,§]



TRK percentage for the PDR:
5% +/-1.5

Example: Coulomb breakup of ^8B

obtained with
Coulomb dissociation



Solar neutrino problem is due to ν-oscillations

But this reaction needs to be known more accurately

- J. Bahcall

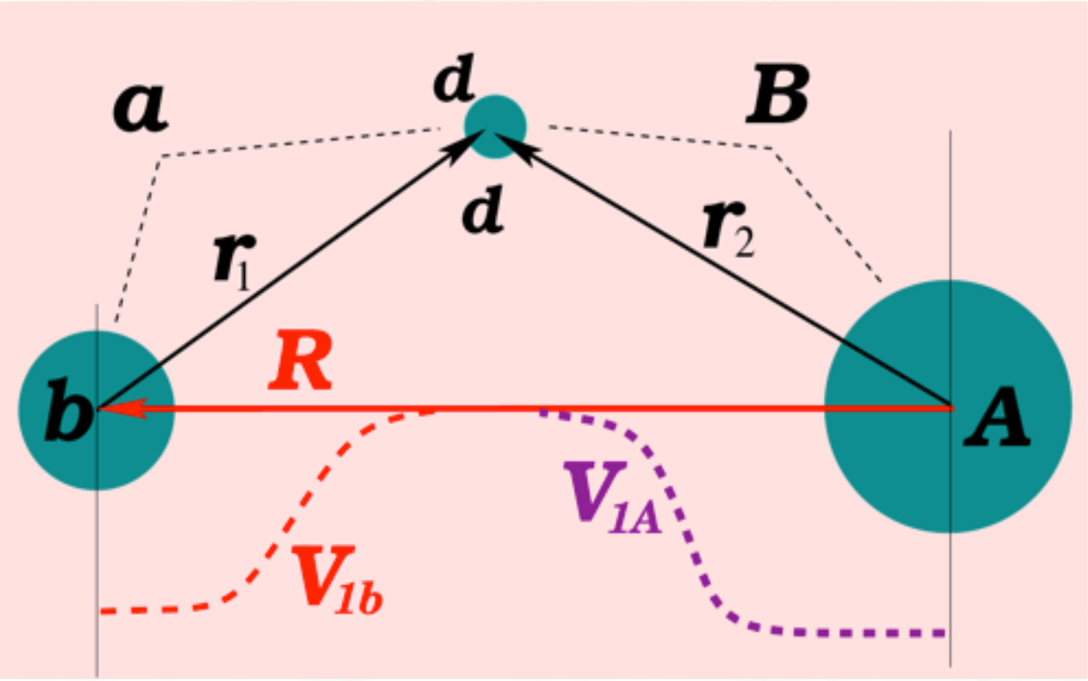
Transfer Reactions

One-nucleon transfer (Born approximation)

$$P_\beta = \left| \frac{i}{\hbar} \int_{-\infty}^{\infty} dt\, F_{\beta\alpha}(\mathbf{R})\, e^{i(E_\beta - E_\alpha)t/\hbar + (\dots)} \right|^2 \sim \tau_{coll} \left| F_{\beta\alpha}(D) \right|^2 g(Q_{\beta\alpha})$$

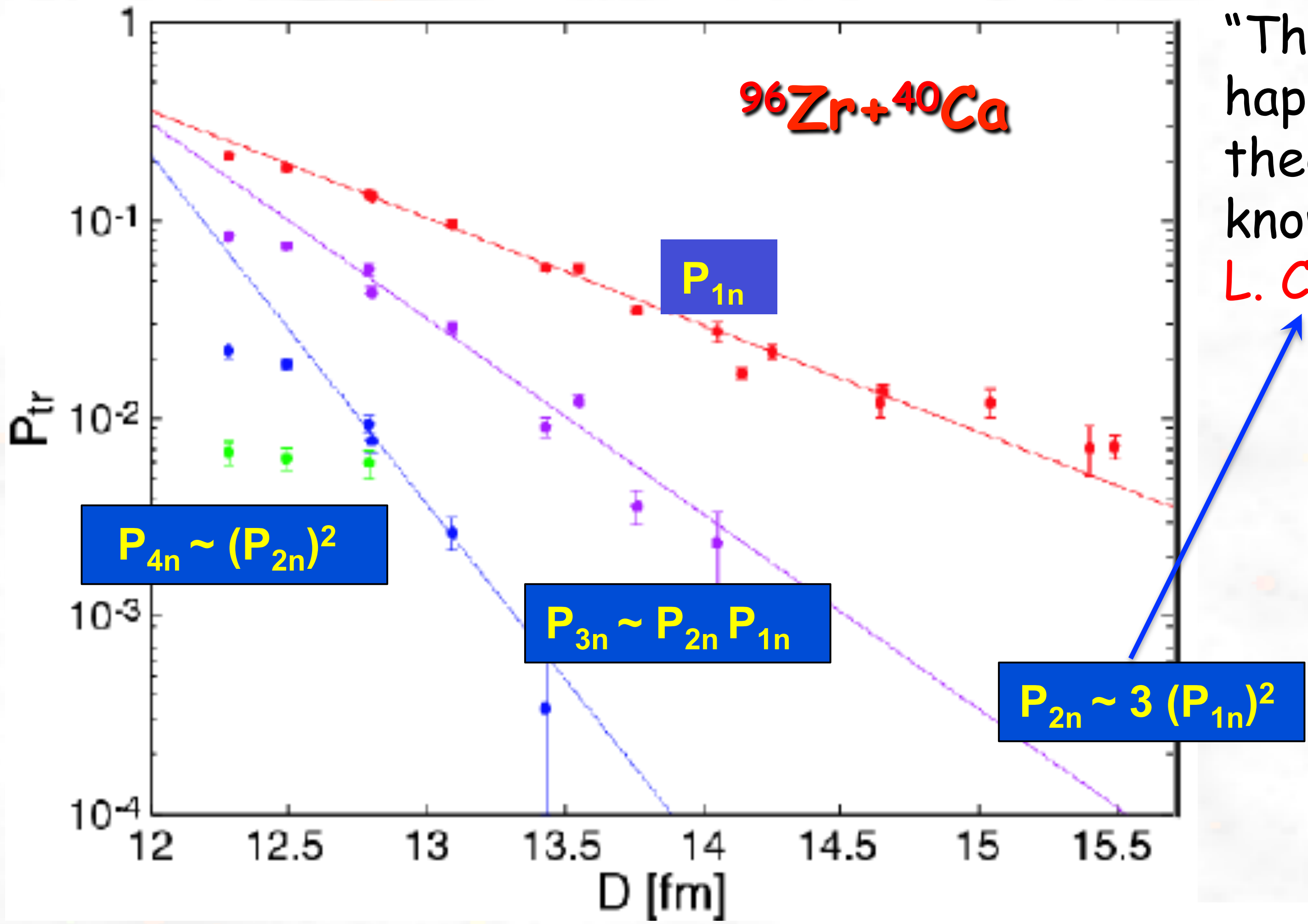
$$F_{\beta\alpha}(\mathbf{R}) \sim \int d^3\mathbf{r}_1\, e^{i\mathcal{Q}\bullet\mathbf{r}_1} \phi_{a_n}^{(A)}(\mathbf{R}+\mathbf{r}_1)\left(V_{1A} - \langle U \rangle\right)\phi_{a_n'}^{(b)}(\mathbf{r}_1)$$

$\mathcal{Q}$ = momentum transfer
V_{1A} transfer interaction.
Why not V_{1b} ??
POST-PRIOR representation



Multi-nucleon transfer (Born approximation)

$$\frac{P_{tr}}{\sin(\theta_{c.m.}/2)} \propto \exp(-2\alpha D)$$



"That is what happens when theorists do not know what to do"
L. Corradi - Legnaro

$$D = \frac{Z_1 Z_2 e^2}{2E_{c.m.}}\left(1 + \frac{1}{\sin(\theta_{c.m.}/2)}\right)$$

Transfer Reactions
Asymptotic Normalization Coefficients

Spectroscopic factors

- What is the amplitude for $^{12}C + n$ in ^{13}C?
- Define overlap function:

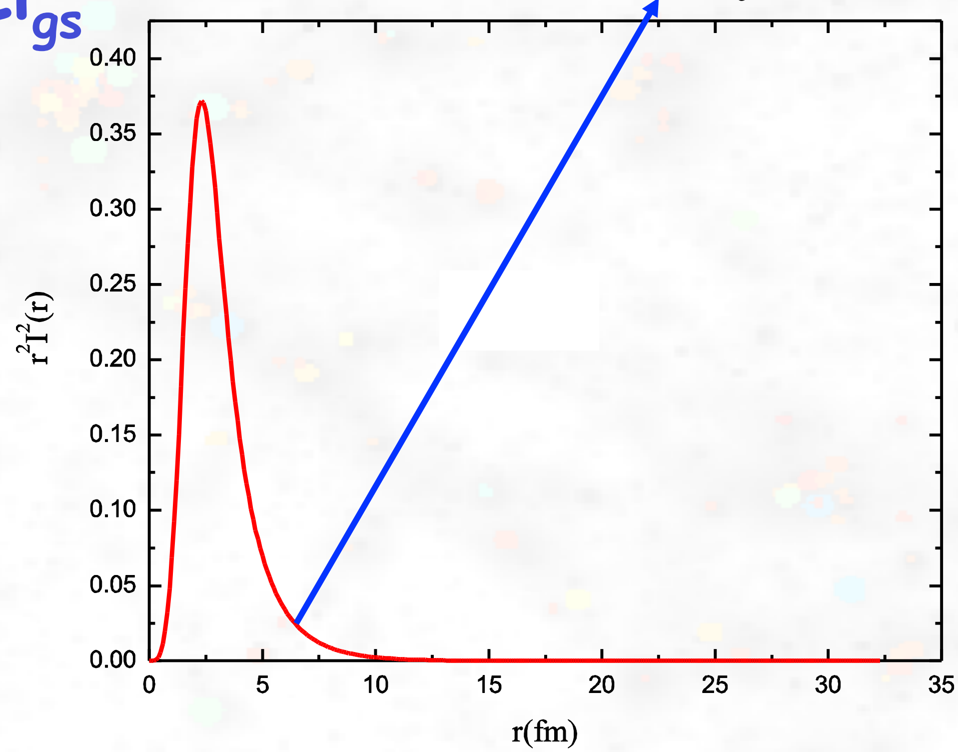
$$I(\boldsymbol{r}) = <\varphi_A(\zeta_A)\varphi_n(\zeta_N)\,|\,\varphi_B(\zeta_A,\zeta_N;\boldsymbol{r})>$$

And the spectroscopic factor is

$$\int d^3r \; |\, I^c_{\ell j}(\mathbf{r})|^2 = S(\ell j)$$

An example: $(^7Li_{gs} + n)_{2+} \leftrightarrow {}^8Li_{gs}$

tail is controlled by **ANC**



Single particle approach:

$$I^c_{\ell j}(\mathbf{r}) = \sqrt{S(\ell j)}\;\phi(\mathbf{r})$$

$$0 \le r \le 6\,\text{fm}$$

96%

Asymptotic Region – I (neutron)

- Single particle overlap function for $r > R_N$

$$I_{(lj)}(r) \xrightarrow{r>R_N} K_{(lj)}\varphi_{(lj)}(r)$$

$$\varphi_{(lj)}(r) \xrightarrow{r>R_N} b_{(lj)}\, i\kappa\, h_l^{(1)}(i\kappa r)$$

- Model independent definition:

$$I_{(lj)}(r) \xrightarrow{r>R_N} C_{(lj)}\, ik\, h_l^{(1)}(ikr)$$

$$k = \sqrt{2m_{An}e_{An}^{B}}, \qquad e_{An}^{B} = m_A + m_n - m_B$$

Asymptotic Region – II (neutron)

- Asymptotic Normalization Coefficient

$$C_{(lj)} = K_{(lj)}\, b_{(lj)}$$

- Typical approach, assume for all r

$$I_{lj}(r) = K_{lj}\varphi_{n(lj)}(r)$$

$$\Rightarrow \quad S_{lj} = \int_0^\infty dr r^2 I_{lj}^2(r) = K_{(lj)}^2 \int_0^\infty dr\, r^2 \varphi_{lj}^2(r) = K_{(lj)}^2$$

DWBA Again

- Cross section for ***A(d,p)B***

$$\sigma^{DW} = |M|^2 = \left| \langle \psi_f^{(-)} I_{An}^{B} | V | \phi_{pn} \psi_i^{(+)} \rangle \right|^2$$

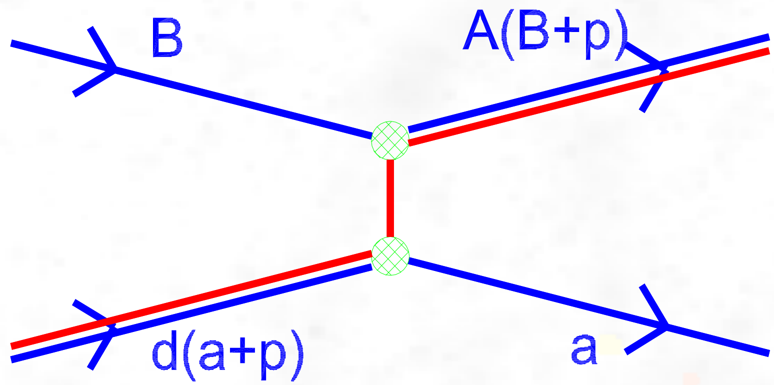
- With the single particle approximation

$$\sigma^{DW} = S \left| \langle \psi_f^{(-)} \phi_{An}(n_r lj) | V | \phi_{pn} \psi_i^{(+)} \rangle \right|^2$$

S is the normalization (i.e. 'spectroscopic') factor

Transfer Reaction (proton)

Transition amplitude:



Peripheral transfer:

$$M = \langle \psi_f^{(-)} I_{An}^B | V | \phi_{pn} \psi_i^{(+)} \rangle$$

$$I_{Bp}^A \overset{r_{Bp} > R_N}{\approx} C_{Bp}^A \frac{W_{-\eta_A, l+\frac{1}{2}}(2\kappa_{Bp} r_{Bp})}{r_{Bp}}$$

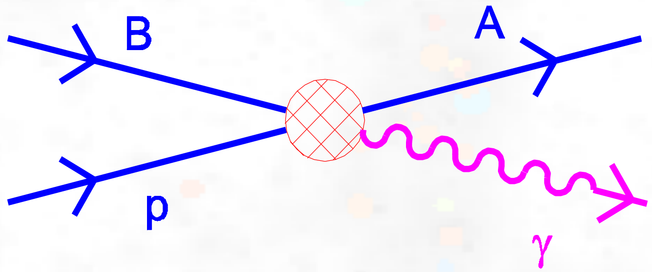
$[S = C^2/b^2]$

$$\frac{d\sigma}{d\Omega} = (C_{Bpl_A j_A}^A)^2 (C_{apl_d j_d}^d)^2 \frac{\sigma_{l_A j_A l_d j_d}^{DW}}{b_{Bpl_A j_A}^2 \, b_{apl_d j_d}^2}$$

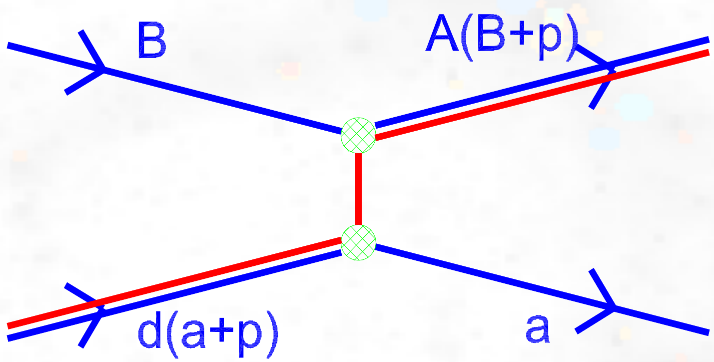
Use of ANCs

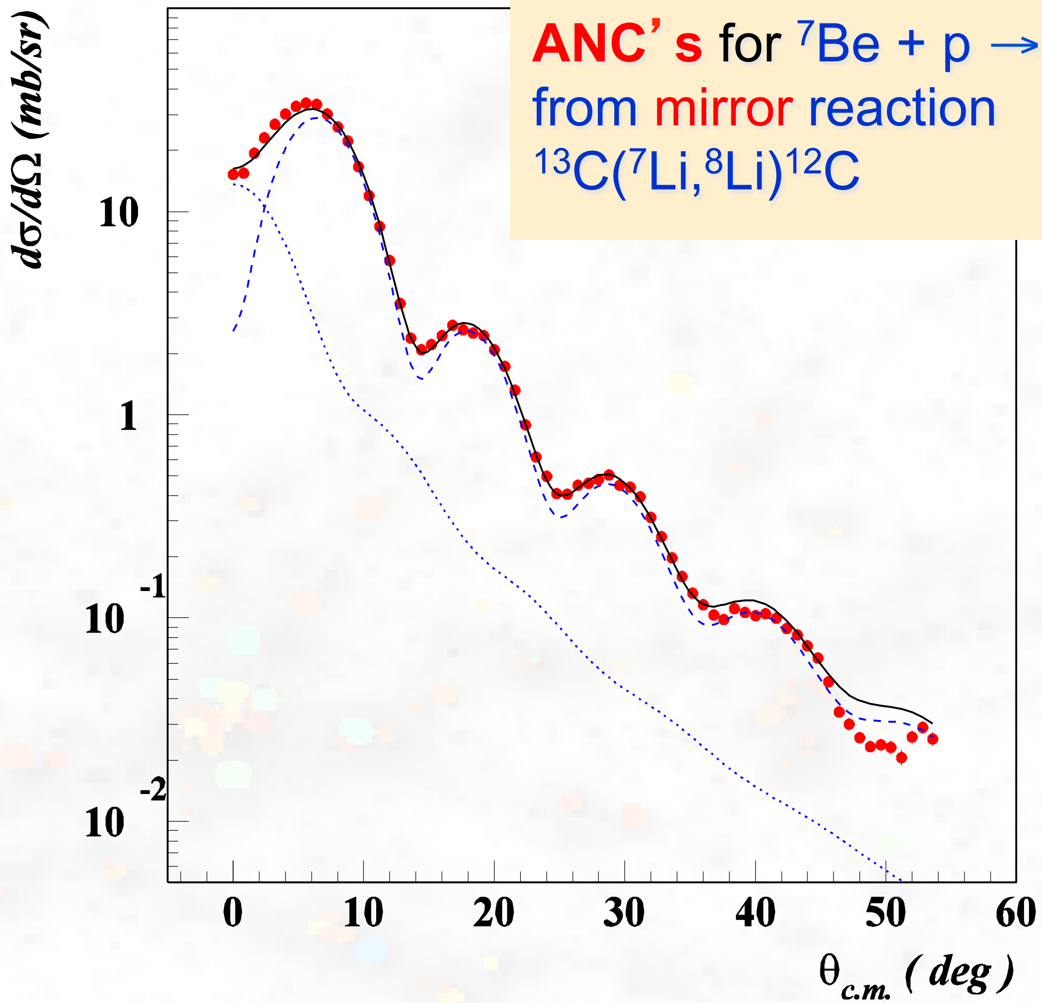
- Find a peripheral transfer reaction
- Measure angular distribution (abs. c.s.)
- DWBA calculation (optical model parameters)
- Determine single particle ANCs
-
- Use the information (ANCs) obtained for the wavefunctions to calculate matrix elements of astrophysical interest

Asymptotic normalization coefficients



$$\sigma_{capture} \propto (C^{A}_{Bp})^2$$





ANC's for $^{7}Be + p \rightarrow {}^{8}B$ from mirror reaction $^{13}C(^{7}Li,^{8}Li)^{12}C$

$$\frac{d\sigma}{d\Omega} = \frac{(C^{A}_{Bpl_A j_A})^2 (C^{d}_{apl_d j_d})^2}{b^2_{Bpl_A j_A}\, b^2_{apl_d j_d}} \sigma^{DW}_{l_A j_A l_d j_d}$$

$S_{17}(0)$ = 17.6 ± 1.7 eV.b

Mukhamedzahnov, Trible, Cagliardi, Texas A&M

Transfer Reactions
Trojan Horse Method

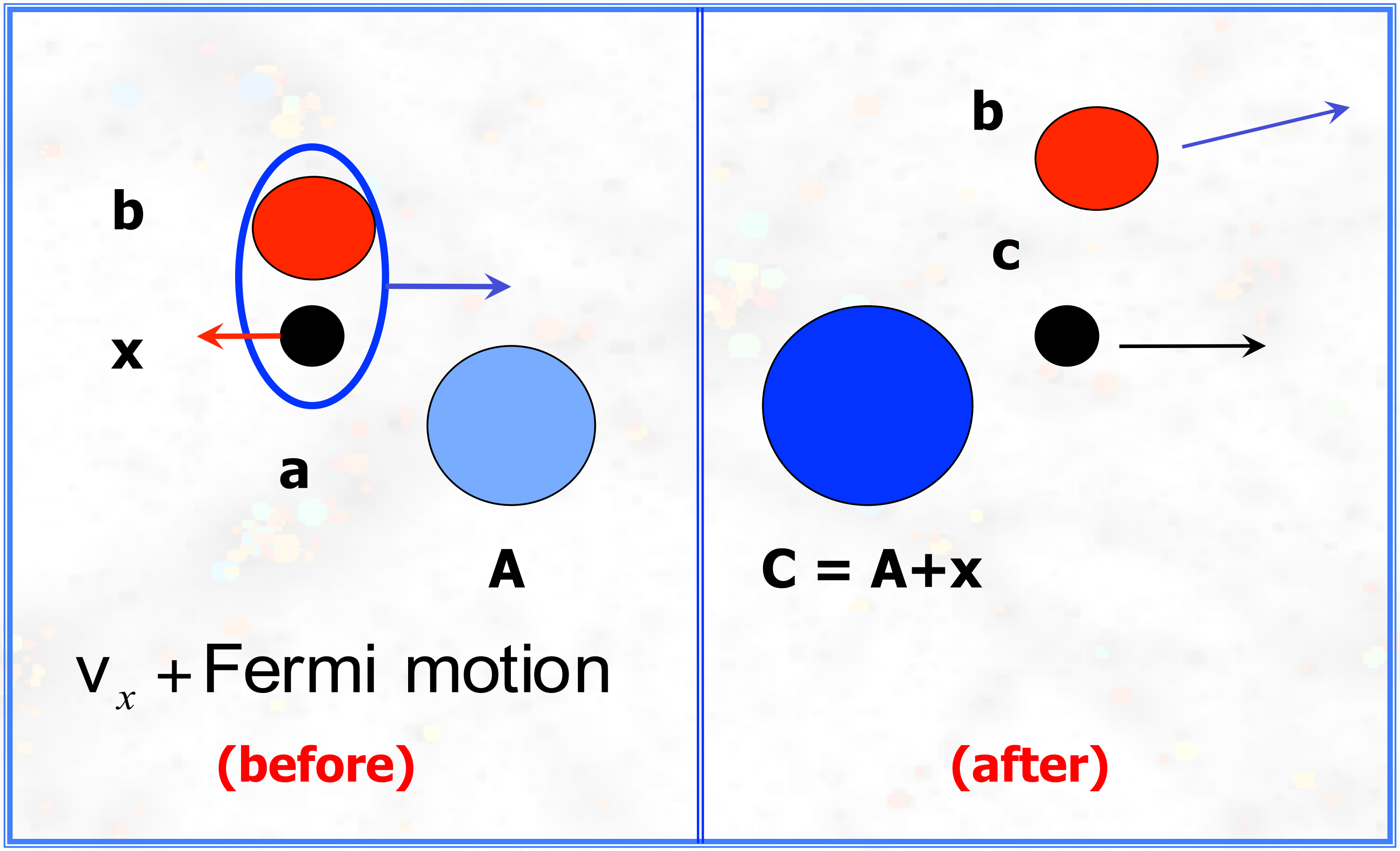
Trojan horse method

$$\text{Measuring} \quad A + a \rightarrow b + c + C$$

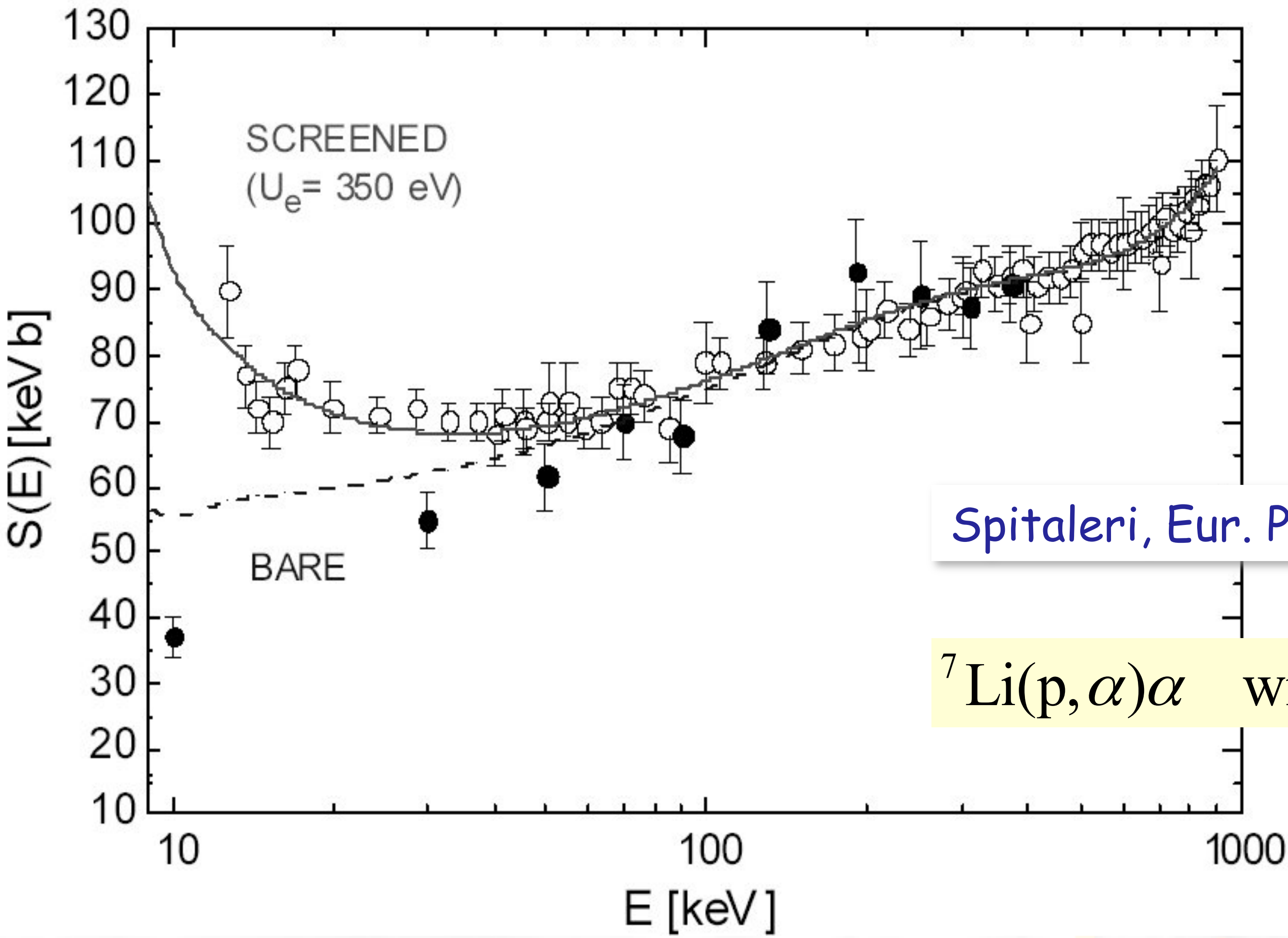
$$\text{with} \quad a = b + x \quad \Rightarrow$$

$$A + x \rightarrow C + c \quad \text{(astrophysics)}$$

G.Baur, PLB 178 (1986) 135



Trojan horse method - examples



Spitaleri, Eur. Phys. J. A 2000

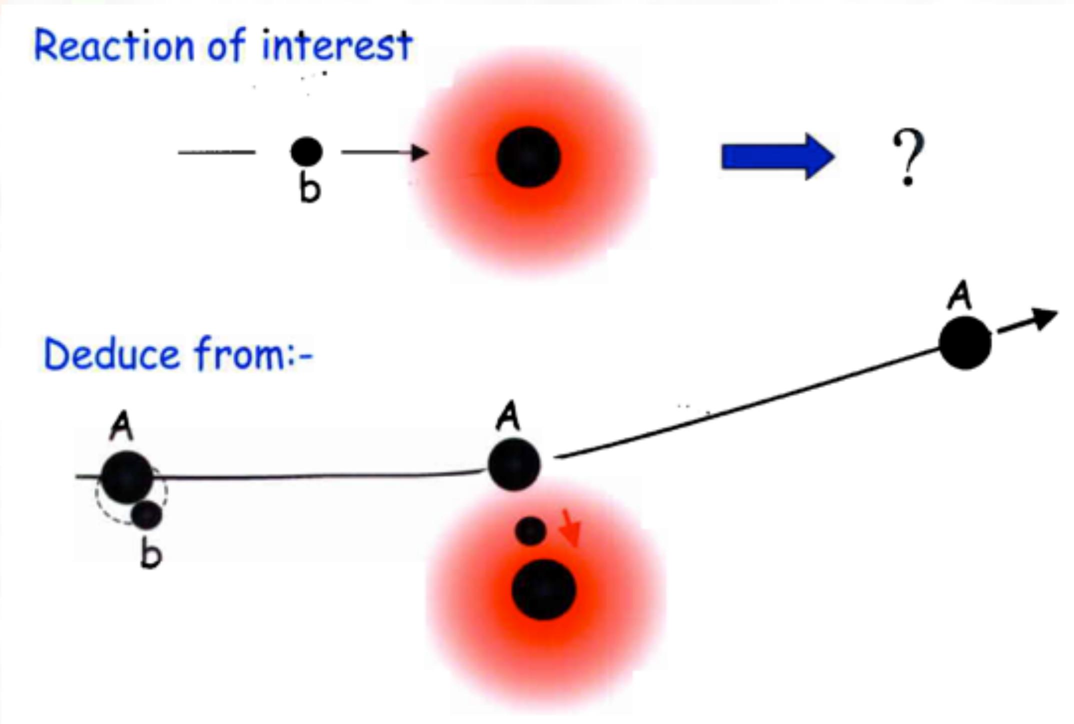
$^{7}\text{Li}(p,\alpha)\alpha$ with $^{2}\text{H}(^{7}\text{Li},\alpha\alpha)n$

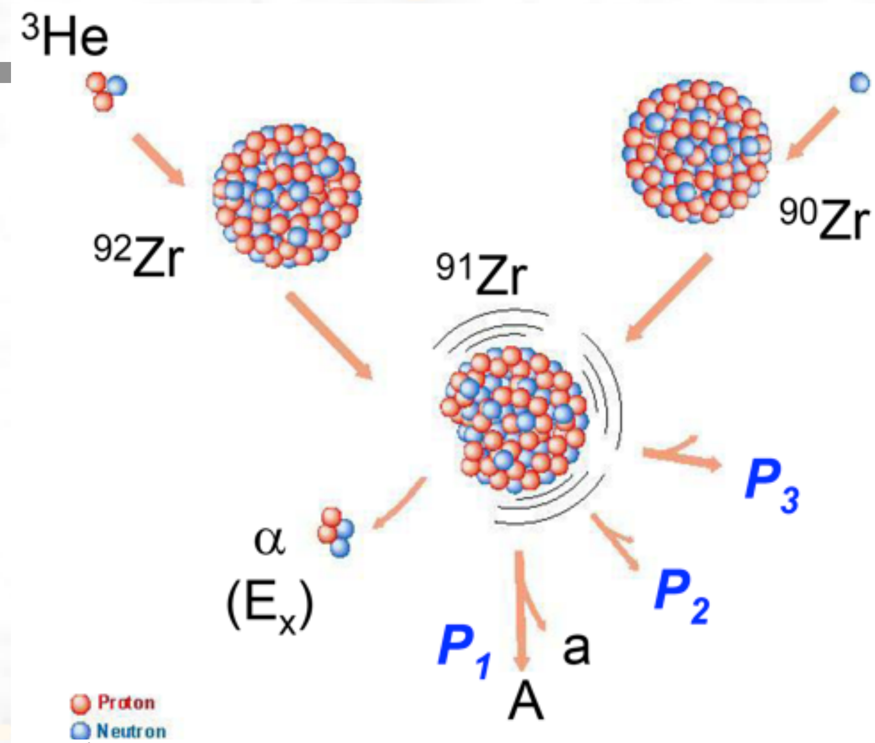
Method extended and applied to several reactions of astrophysics interest by Claudio Spitaleri and collaborators

Transfer Reactions

Surrogate Reactions

Surrogate reactions





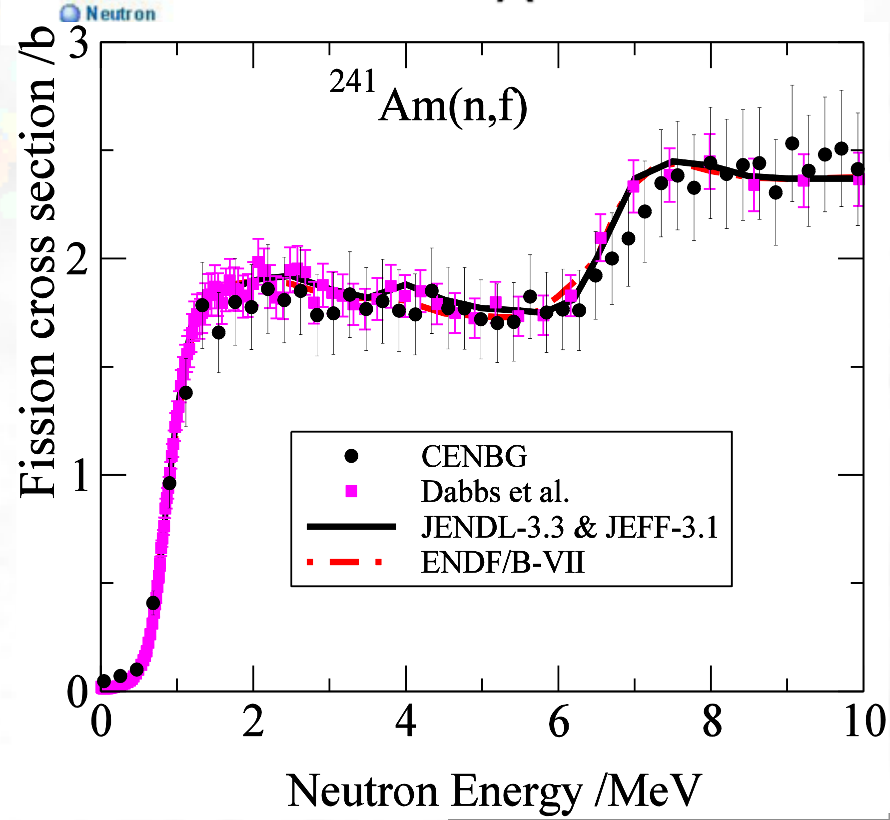
e.g., (n,f) from transfer reactions

Kessedjian, et al., PLB 692 (2010) 297

Fission cross sections not sensitive to differences J^{π} distributions!!!

→ Hauser-Feshbach = Ewing-Weisskopf

→ Surrogate reactions work

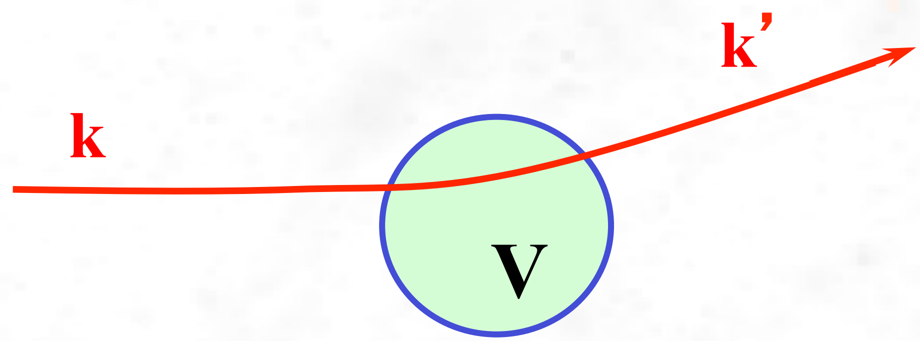


BUT, unfortunately, most often it doesn't work.

Direct Reactions at High Energies

Quantum scattering: Low energy

$$\left[-\frac{\hbar^2}{2\mu}\nabla^2 + V\right]\Psi = E\Psi$$



Partial wave expansion:

$$u_l(r) \xrightarrow[r\to\infty]{} \frac{i}{2}\left\{H_l^{(-)}(kr) - S_l H_l^{(+)}(kr)\right\}$$

Incoming wave

Outgoing wave

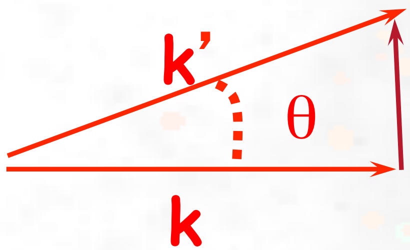
"Survival" amplitude (S-matrix)

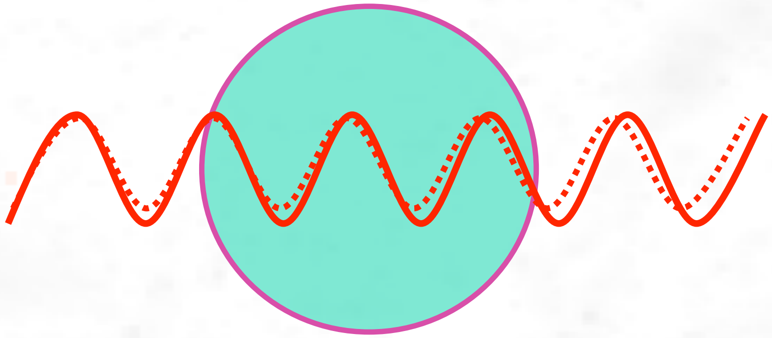
$$S_l = e^{2i\delta_l}$$ (δ_l = Phase shift)

$$\left|S_l\right|^2 = \text{"Survival" probability} \leq 1$$

High energy collisions (E_{lab} > 50 MeV/nucleon) Eikonal Waves

$\Delta E \ll E$, $\theta \ll 1$ radian , $|\Delta\psi/\psi|_{\Delta r=\lambda} \ll 1$





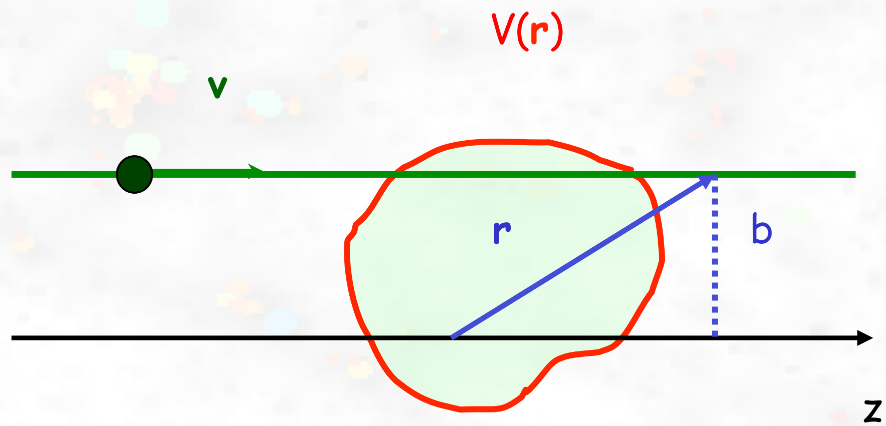
$$\Psi(\mathbf{r}) = S(\mathbf{b},z)\, e^{i\mathbf{k}\cdot\mathbf{r}}$$

$$S(\mathbf{b},z) = \exp\left\{-\frac{i}{\hbar v}\int_{-\infty}^{z} V(\mathbf{r}')\,dz'\right\}$$

$$\mathbf{r}' = (\mathbf{b},z')$$

$z \longrightarrow \infty$ after the collision:

$$\Psi(\mathbf{r}) = S(\mathbf{b})\, e^{i\mathbf{k}\cdot\mathbf{r}}$$



$$S(\mathbf{b}) = e^{i\chi(\mathbf{b})}$$
$$= \exp\left\{-\frac{i}{\hbar v}\int_{-\infty}^{\infty} V(\mathbf{r}')\,dz'\right\}$$

Eikonal waves (reactions)

Harmonic oscillator (structure)

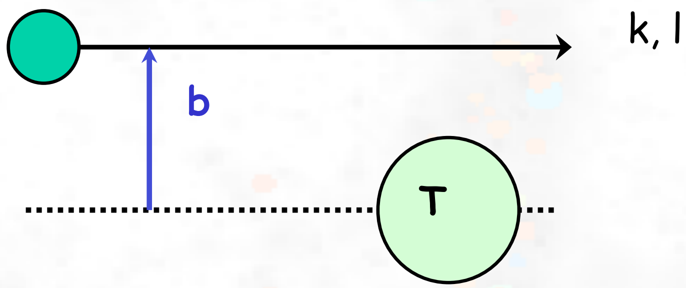
Pearls of quantum mechanics

Eikonal Waves: Applications

(sometimes called "Glauber theory")

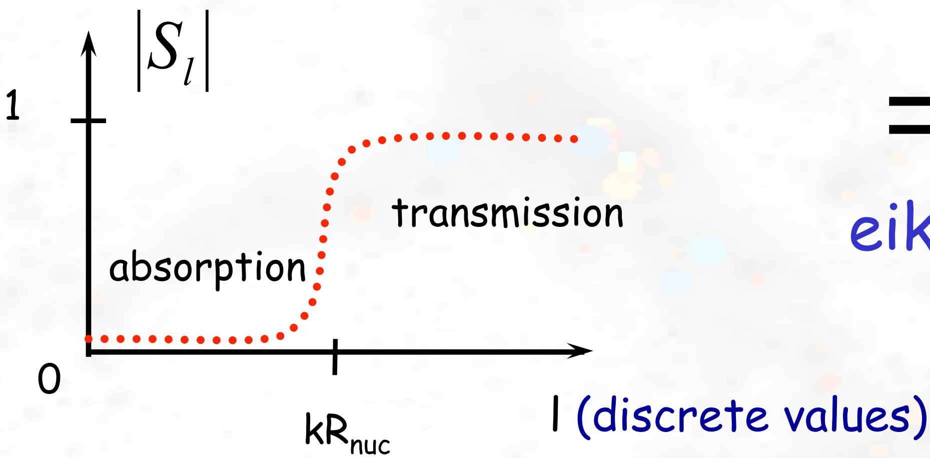
Roy Glauber
2005 Nobel prize
(for another "Glauber theory")

S-matrices ("Survival" Amplitudes)



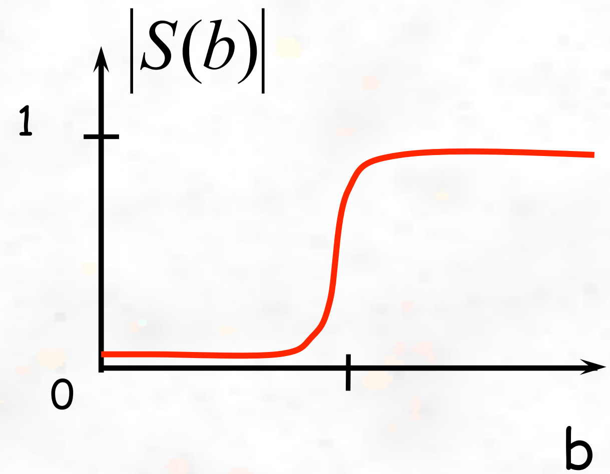
b = impact parameter

l = kb (actually l + 1/2 = kb)



$\Longrightarrow$

eikonal



Ex: Elastic Scattering

$$f(\theta) = \frac{i}{k}\sum_l (l+\tfrac{1}{2})(1-S_l)P_l(\cos\theta)$$

$$\frac{d\sigma}{d\Omega} = |f(\theta)|^2$$

$$f(\theta) = ik\int db\, b\, J_0(kb)\{1-S(b)\}$$

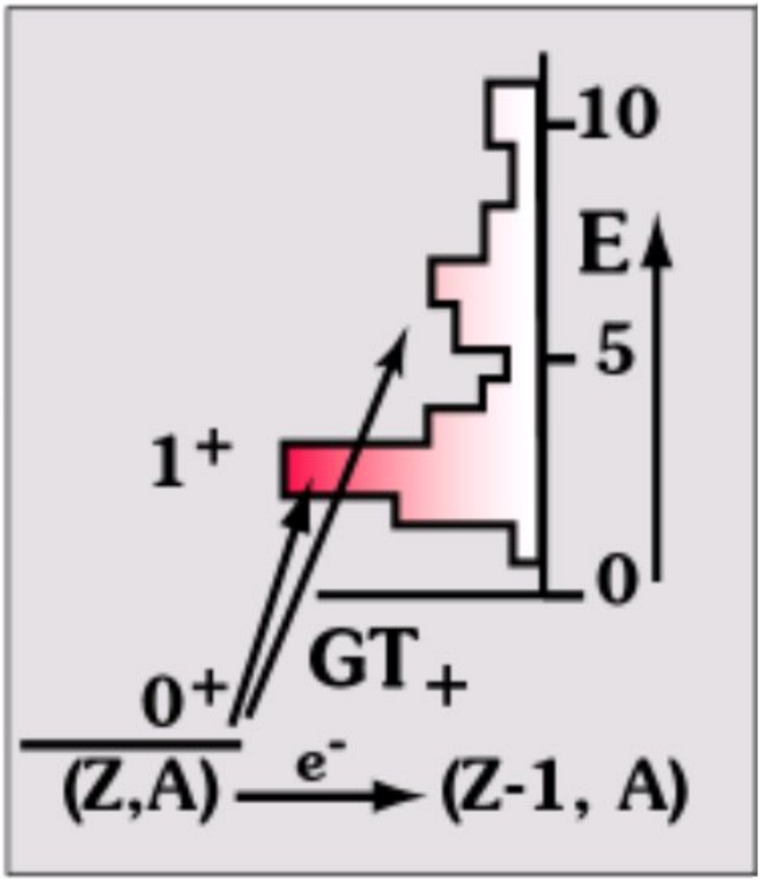
Direct Reactions at High Energies

Supernovae physics

SN-collapse scenario

- Gravitational pressure balanced by degenerate e- gas up to M_{ch} = 1.44
- Electron capture e- + (Z,A) → (Z-1, A) + ν_e
  e- + p → n + ν_e → rates determined by GT-strength
- loss of energy by neutrino cooling
- loss of pressure → collapse at 0.3c
- neutrino trapping, decoupling of the core free fall
- storing gravitational energy in neutrinos
- core bounce and outgoing shock wave
- re-heating shock wave by neutrinos and explosion
- successful explosion ONLY if Ye > 0.43

Neutrinos



e- + (Z,A) ←→ (Z-1, A) + ν_e

Needs $\left|\langle B \| \sigma\tau \| A \rangle\right|^2$ for numerous nuclei

Also the case for neutrino induced reactions

Neutrino detection on Earth difficult

Number of target nuclei

Neutrino flux

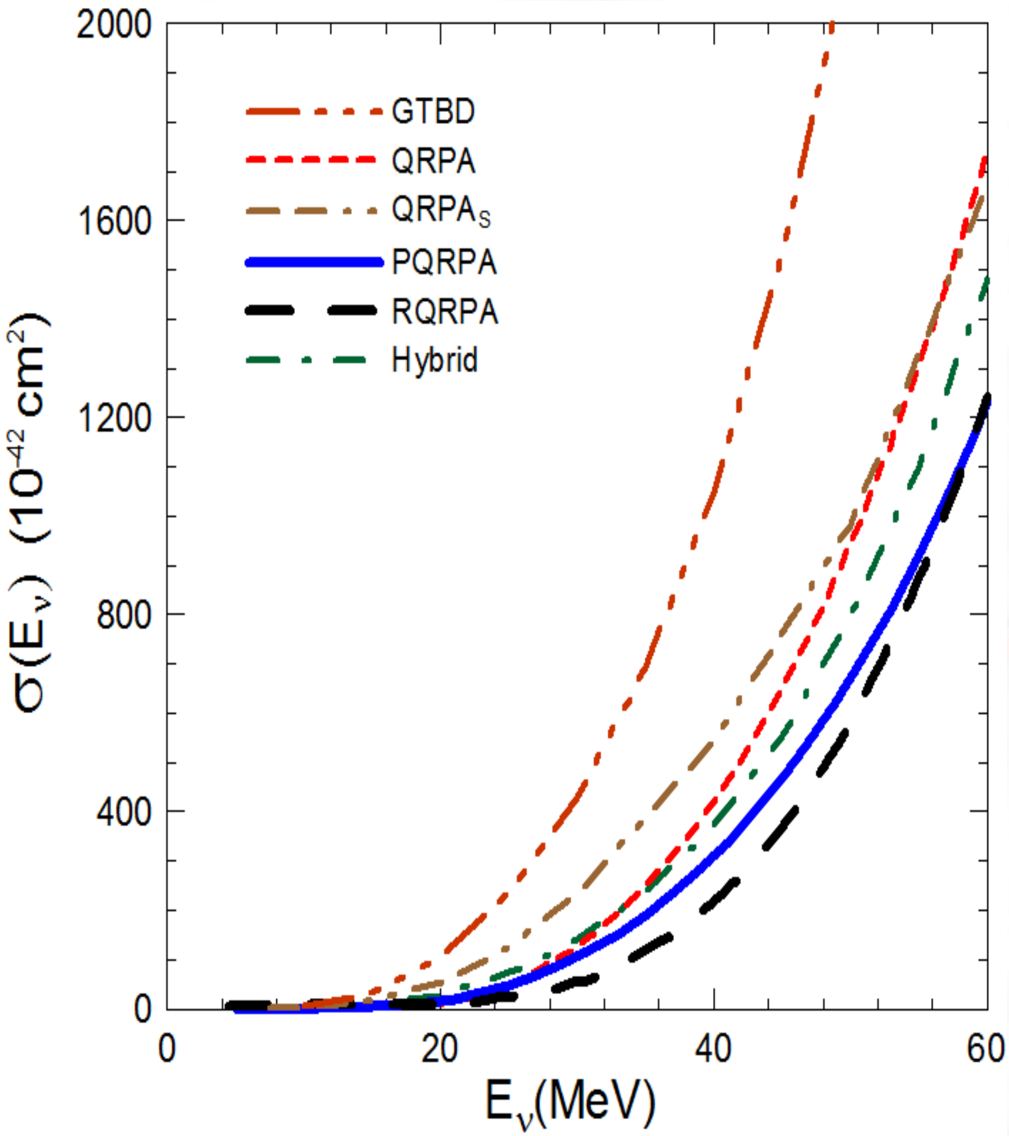
Interaction cross section

Efficiency

$$N_{e\nu} = N_t \int_0^\infty F(E_\nu)\cdot\sigma(E_\nu)\cdot\varepsilon(E_\nu)\,dE_\nu$$

Theoretical neutrino-nucleus calculations unreliable

$$\nu_e + {}^{56}Fe \Rightarrow {}^{56}Co^* + e^-$$



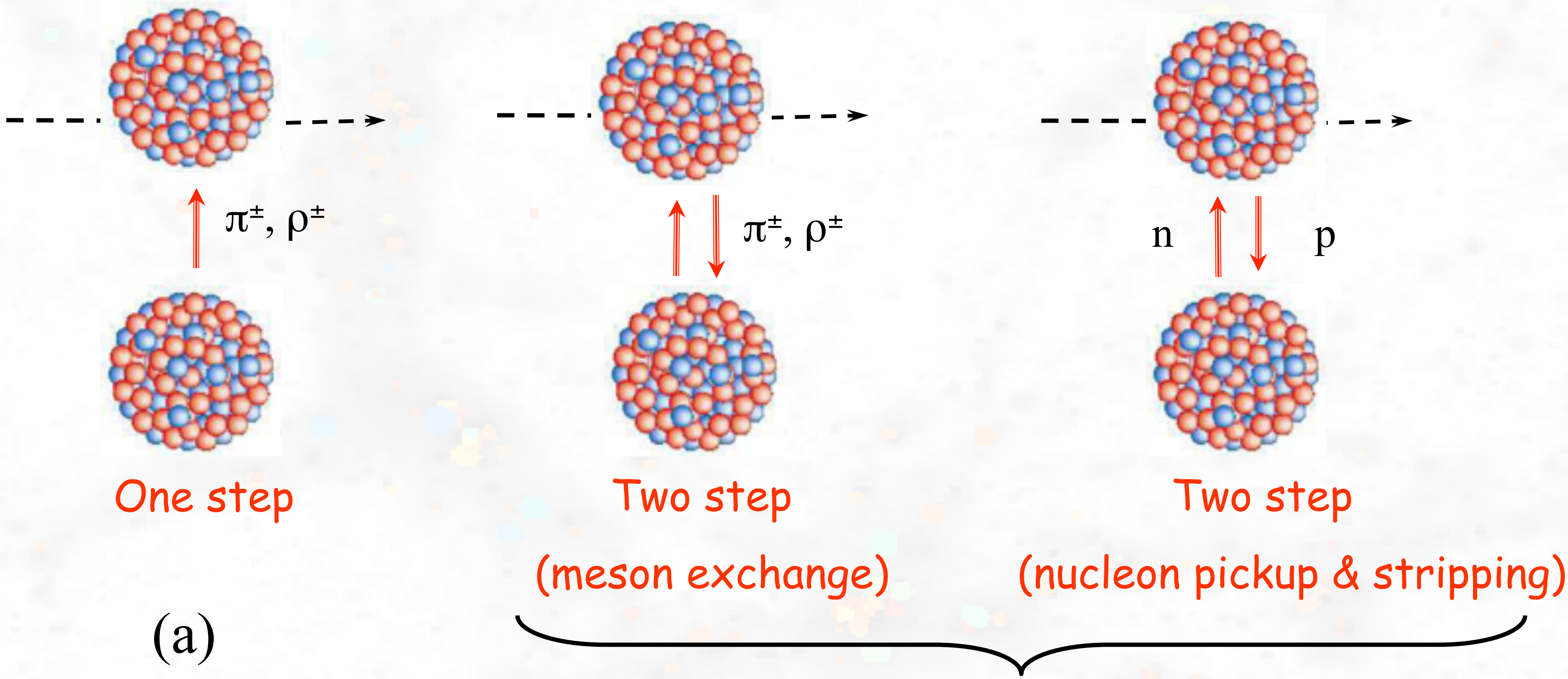
$$\langle \sigma_e \rangle = \int dE_\nu \sigma(E_\nu) n(E_\nu),$$

$$n(E_\nu) = \frac{96 E_\nu^2}{M_\mu^4} (M_\mu - 2E_\nu),$$

Model	$\langle \sigma_e \rangle$
QRPA	264.6
PQRPA	197.3
Hybrid$^{(a)}$ [14]	228.9
Hybrid$^{(b)}$ [14]	238.1
TM [26]	214
RPA [27]	277
QRPA$_S$ [15]	352
RQRPA [16]	140
Exp[5]	$256 \pm 108 \pm 43$

Samana, Bertulani,
PRC 78, 024312 (2008)

Solution with charge-exchange reactions



(a) $$T_{DWBA}(\mathbf{k'},\mathbf{k}) = \left\langle \chi_{\mathbf{k'}}^{(-)} \middle| U \middle| \chi_{\mathbf{k}}^{(+)} \right\rangle$$

(b) $$T_{DWA}(\mathbf{k'},\mathbf{k}) = \sum_{\gamma=0} C_\gamma \left\langle \chi_{\mathbf{k'}}^{(-)} \middle| U\left(G^{(+)}U\right)^\gamma \middle| \chi_{\mathbf{k}}^{(+)} \right\rangle$$

Effective interaction V_{NN} (phenomenological)

$$V_{NN}(\mathbf{r}) = V^C(r) + V^C_\sigma(r)(\sigma_1\cdot\sigma_2) + \left[V^C_\tau(r) + V^C_{\sigma\tau}(r)(\sigma_1\cdot\sigma_2)\right](\tau_1\cdot\tau_2)$$
$$+\left[V^T(r) + V^T_\tau(r)(\tau_1\cdot\tau_2)\right]S_{12}(\hat{\mathbf{r}}) + V^{LS}(r)\ \boldsymbol{l}\cdot(\sigma_1+\sigma_2)$$

Antisimetrization: $V_{NN}(\mathbf{r}) = \left[1-(-)^l P_x\right]V_{12}(\mathbf{r})$ $\quad P_x : \mathbf{r}\rightarrow -\mathbf{r}$

$V^{LS}(r)\,\boldsymbol{l}\cdot(\sigma_1+\sigma_2)$ small and usually neglected

Notation:

$$V^C(r) = V^0_{00}(r),\quad V^C_\sigma(r) = V^0_{10}(r),\quad V^C_\tau(r) = V^0_{01}(r)$$
$$V^C_{\sigma\tau}(r) = V^0_{11}(r),\quad V^T(r) = V^2_{10}(r),\quad V^T_\tau(r) = V^2_{01}(r)$$

$$V_{12}(\mathbf{r}) = \sum_{\substack{K=0,2\\ ST}} V^K_{ST}(r)\,C^K_S\ Y_K(\hat{\mathbf{r}})\left[\sigma_1\otimes\sigma_2\right]^K\left[\tau_1\cdot\tau_2\right]^T$$

K = 0 : central force

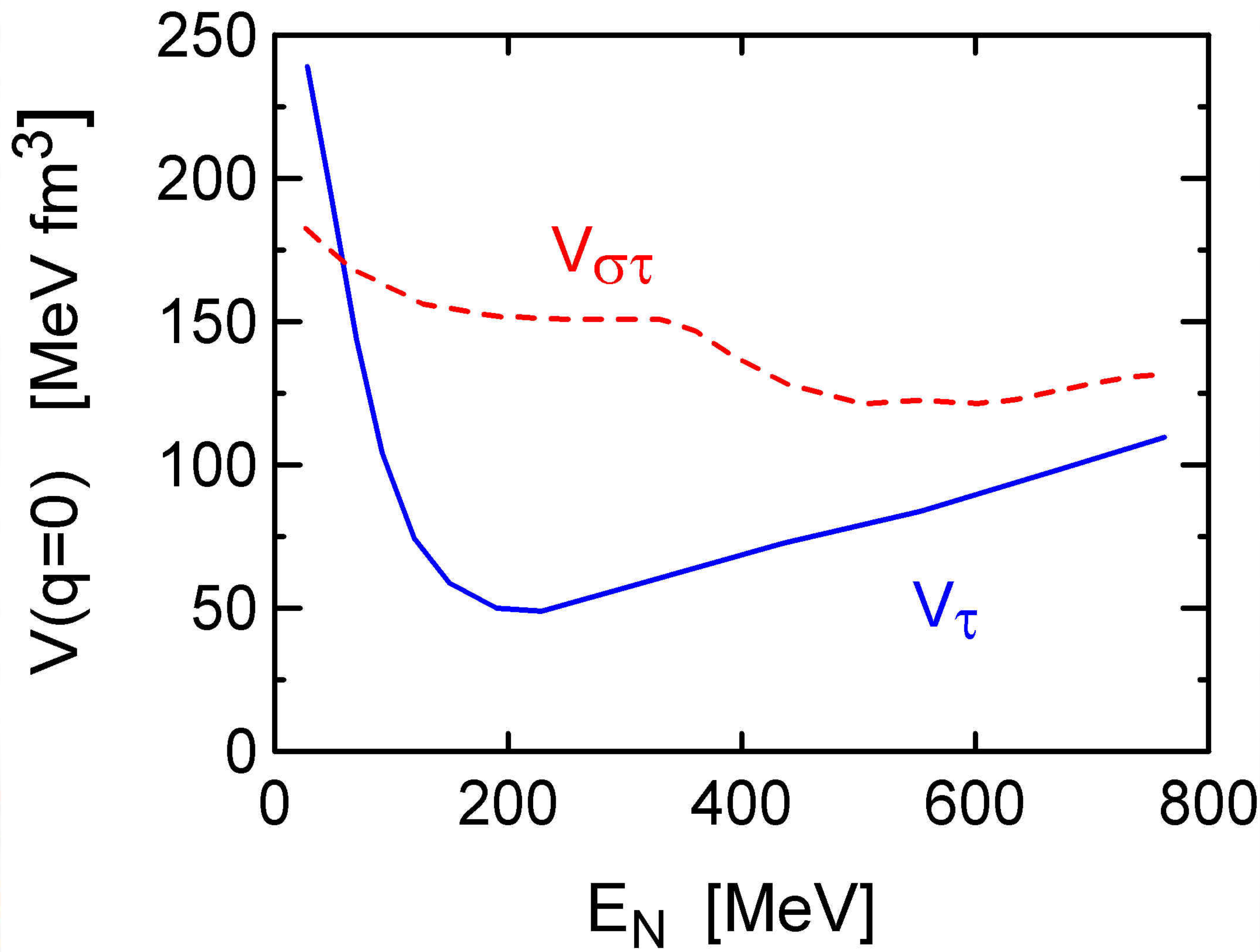
K = 2 : tensor force

$$\sigma^{S=0} = 1,\quad \sigma^{S=1} = \sigma$$
$$\tau^{T=0} = 1,\quad \tau^{T=1} = \tau$$

$$C^0_0 = \sqrt{4\pi},\quad C^0_1 = -\sqrt{12\pi}$$
$$C^2_0 = 0,\quad C^2_1 = \sqrt{25\pi/5}$$

Effective interaction V_{NN} (phenomenological)

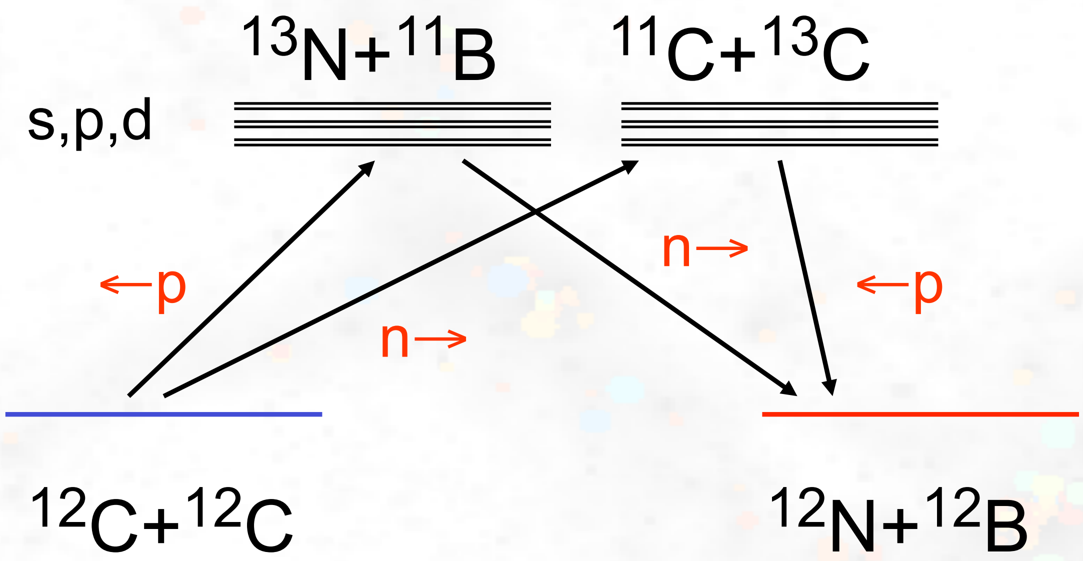
Love, Franey, NPA 1981, 1985

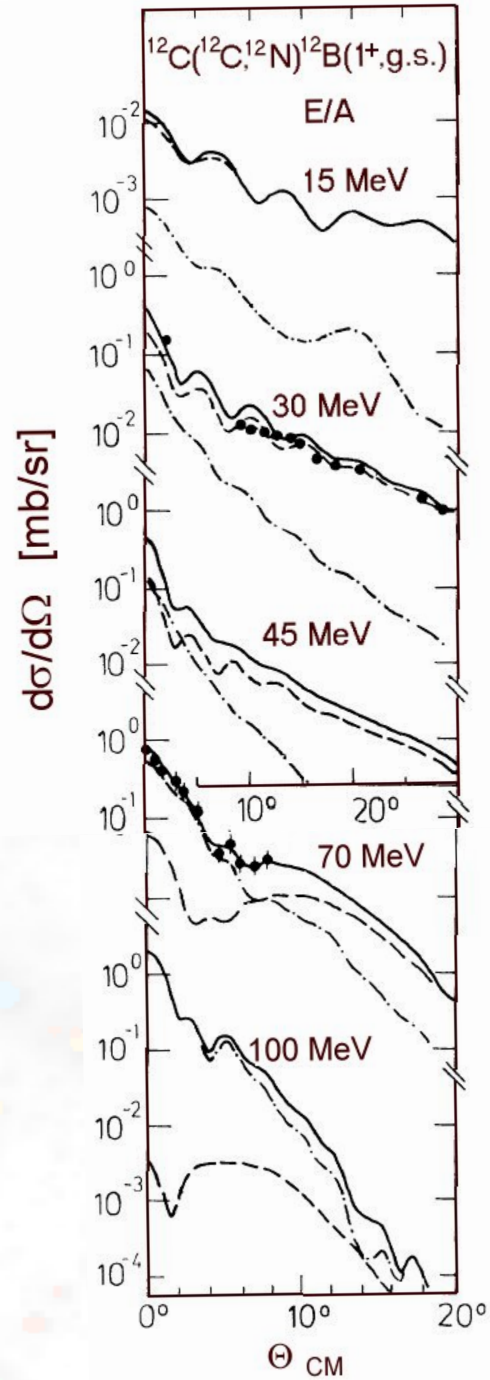


Two step (proton pickup & neutron-stripping)

Lenske, Wolter, Bohlen, PRL 1989 →

$^{12}C(^{12}C,^{12}N)^{12}B(1^+,gs)$





Two step (double $\pi+\rho$ exchange)

Bertulani, NPA 1993

$^{13}C(^{13}N,^{13}C)^{13}N$ 70 MeV/nuc.

$$\sigma_{2nd} \sim 10^{-4} \times \sigma_{1st}$$

DBWA again!

$$T_{ch.exch.}(\mathbf{k}',\mathbf{k}) = \int d^3r\, S(b)\, \exp[i\mathbf{q}\cdot\mathbf{r}]\, \langle bB | U(\mathbf{r}) | aA \rangle$$

$$|aA\rangle = |aA; J_a M_a T_a N_a; J_A M_A T_A N_A\rangle$$

eikonal + few pages of algebra

Bertulani, NPA 554, 493 (1993)

$$T_{ch.exch.}(\mathbf{k}',\mathbf{k}) = \sum_{\substack{K=0,2\\ ST}} \sum_{\substack{LL'JJ'\\ MM'\mu}} C(KS; LL'JJ'MM'\mu) \int db\, b\, S(b) J_0(qb)$$

$$\times \int dp\, p\, J_{M'-M-\mu}(pb)\, \tilde{V}^K_{ST}(p)\, \tilde{\rho}^{aA}_{LJST}(p)\, \tilde{\rho}^{bB}_{L'J'ST}(p)$$

$$\tilde{\rho}^{aA}_{LJST}(p) = \int dr\, r^2\, j_L(pr) \left\langle J_a T_a \left\| \sum_i \frac{\delta(r-r_i)}{r_i^2} \Im^{LSJ}_M \boldsymbol{\tau}^T \right\| J_b T_b \right\rangle$$

$$\Im^{LSJ}_M = \sum_{\mu M_L} \langle L M_L S\mu | JM \rangle i^L Y_{LM_L}(\hat{\mathbf{r}}) \sigma^{S\mu}$$

STRUCTURE INPUT beautifully factorized

Charge exchange at forward angles

$$T_{aA\to bB}(\mathbf{k}',\mathbf{k}) = \sum_{\dots}\sum_{\dots}\dots\int db\, b\, S(b)\, J_0(qb)\int dp\, p\, J_{\dots}(pb)\, \tilde{\rho}^{aA}_{\dots}(p)\, \tilde{\rho}^{bB}_{\dots}(p)$$

$$S(b)\sim 1 \;\longrightarrow\; p\sim q$$

- $S(b)\neq 1$ but largest value of $T_{aA\to bB}$ occurs when

  $J_0(qb)$ oscillates in phase with $J_{\dots}(pb)$

$\Longrightarrow$ $p\sim q$

Forward scattering: $q\sim 0$

Bertulani, NPA 554, 493 (1993)

$$f_{aA\to bB}(\theta\sim 0) = \dots\tilde{\rho}^{aA}_{\dots}(0)\,\tilde{\rho}^{bB}_{\dots}(0)\times\int dp\, p\, V^{K}_{ST}(p)\times\int db\, b\, J_0(qb)\, e^{i\chi(b)}$$

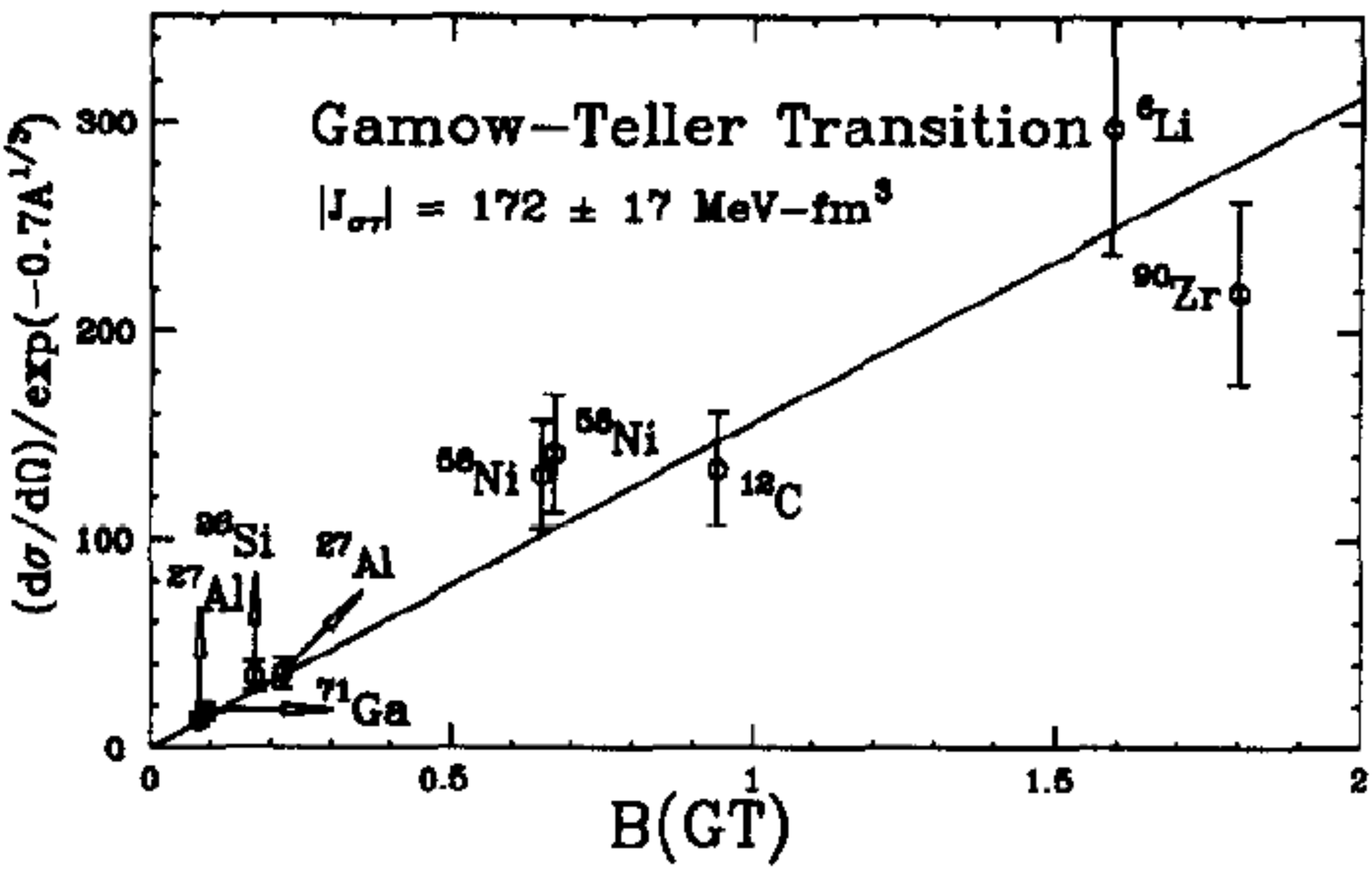
$$\tilde{\rho}^{aA}_{\dots}(0) = \cdots\langle A\|\sigma^S\tau\|a\rangle$$

$$\frac{d\sigma}{d\Omega}(\theta\sim 0^0) = \cdots\left|\langle A\|\sigma^S\tau\|a\rangle\right|^2\left|\langle B\|\sigma^S\tau\|b\rangle\right|^2$$

$\Longrightarrow$ • If $\left|\langle A\|\sigma^S\tau\|a\rangle\right|^2$ well known. E.g. (a,A) = (n,p) then

Fermi and Gamow-Teller m.e. READ DIRECTLY from $\frac{d\sigma}{d\Omega}(\theta\sim 0^0)$

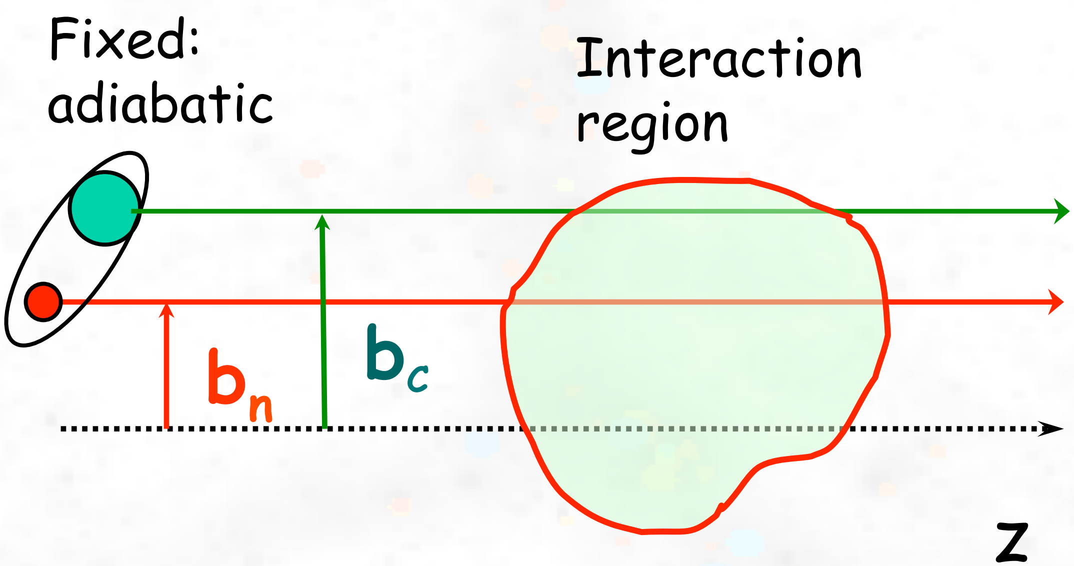
Charge exchange at forward angles - Example



Direct Reactions at High Energies

Knockout Reactions

Applications of Eikonal WFs: elastic breakup



Elastic:

including breakup effects

$$\Psi^{eik}(\mathbf{r}) = S_C(\mathbf{b}_C)\,S_n(\mathbf{b}_n)\,e^{i\mathbf{k}\cdot\mathbf{r}}\,\varphi_0$$

Best possible wfs:

(Spectroscopy)

$$S_{elast}(\mathbf{b}) = \left\langle \varphi_0 \left| S_C(\mathbf{b}_C)\,S_n(\mathbf{b}_n) \right| \varphi_0 \right\rangle$$

Survival amplitude

for projectile at impact parameter b

Survival amplitudes

for particles C and n at impact parameters b_C and b_n

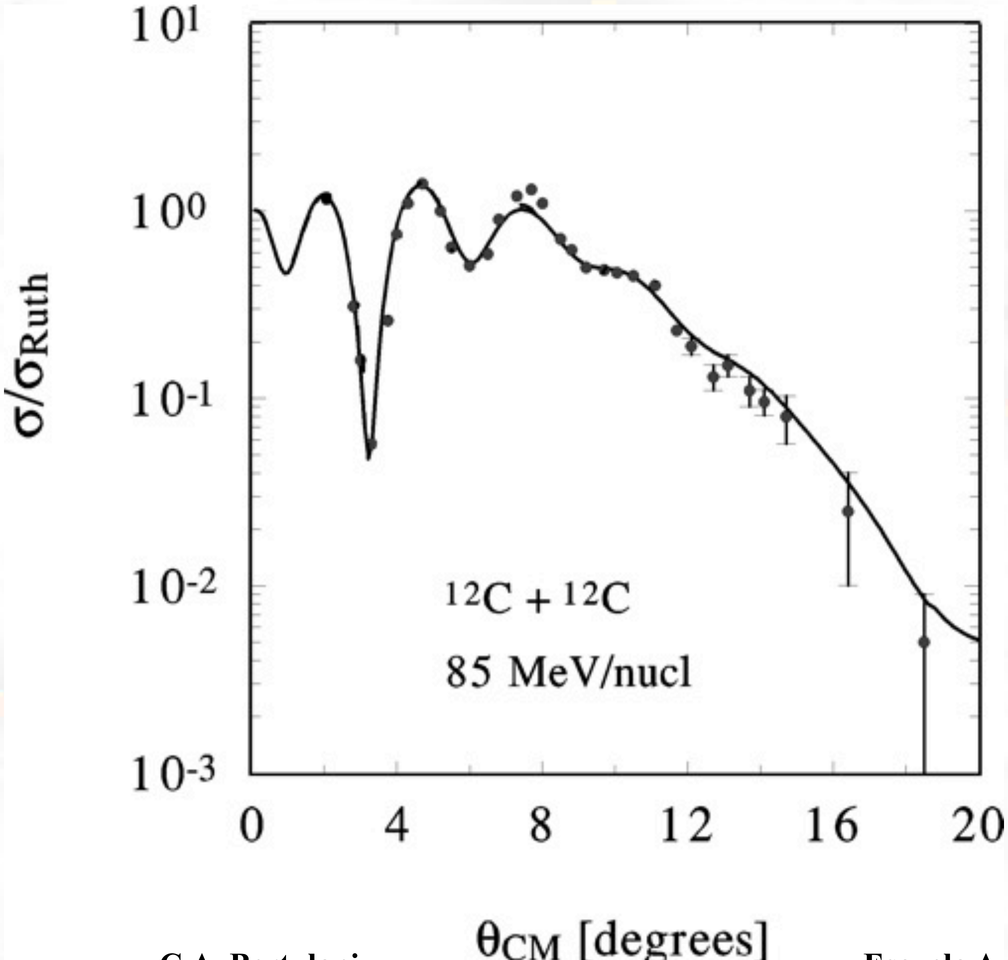
(Dynamics)

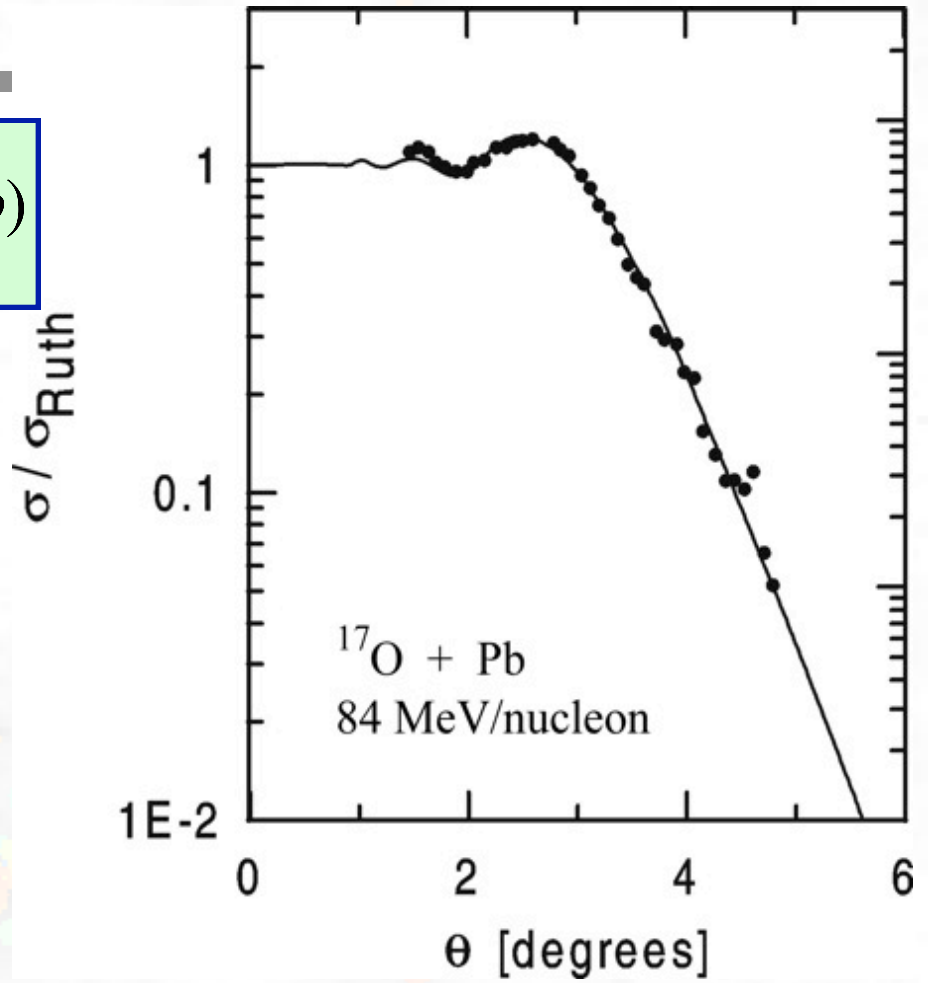
Probing nuclear densities

$$\chi_{AB}^{(N)}(b) = \frac{1}{k_{nn}} \int_0^\infty dq\, q\, \tilde{\rho}_A(q) \tilde{\rho}_B(q) f_{nn}(q) J_0(qb)$$

$$f_{nn}(q) = \frac{k_{nn}}{4\pi} \sigma_{nn} \left(i + \alpha_{nn}\right) e^{-\beta_{nn} q^2}$$

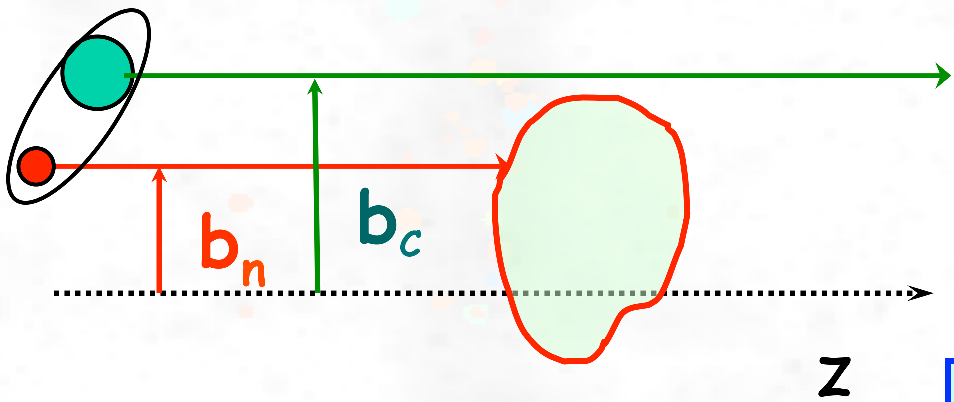
(from nn scattering)





solid curves: Glauber

Stripping

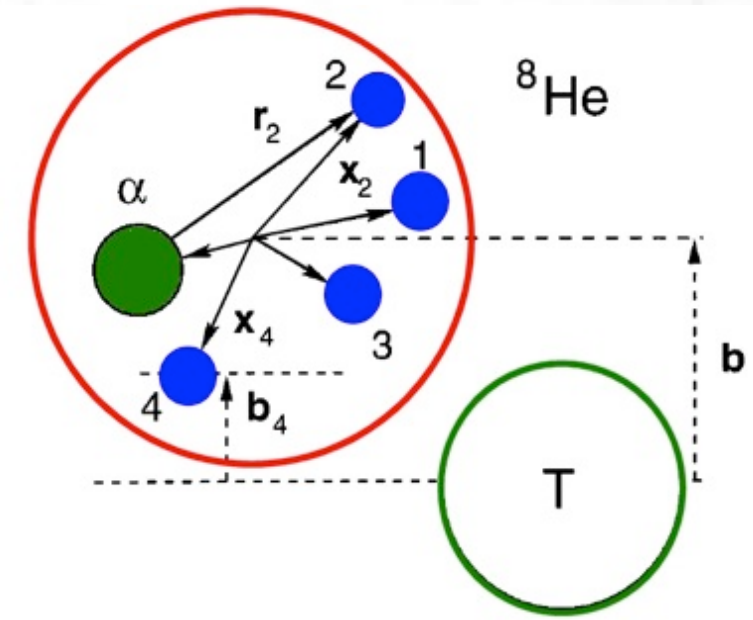


$$\left|S_C(\mathbf{b}_C)\right|^2\left(1-\left|S_n(\mathbf{b}_n)\right|^2\right)$$

C survives, n absorbed

$$\sigma_{strip}(\mathbf{b}) = \int d\mathbf{b}\left\langle \varphi_0 \left| \left|S_C\right|^2\left(1-\left|S_n\right|^2\right) \right| \varphi_0 \right\rangle^2$$

(d) Composite particles:



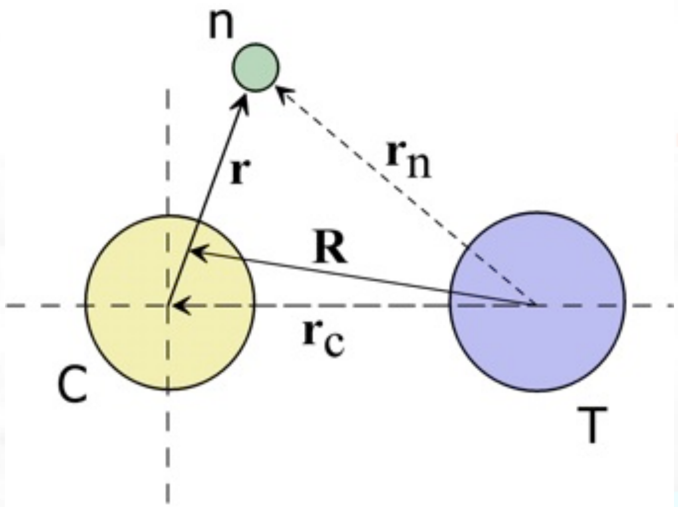
$$S_{dif.dis.}(\mathbf{b}) = \left\langle \varphi_8 \left| S_\alpha(\mathbf{b}_\alpha)\prod_{i=1}^{4} S_i(\mathbf{b}_i) \right| \varphi_8 \right\rangle$$

$$\prod_{j\ survive}\left|S_j(\mathbf{b}_j)\right|^2 \prod_{k\ absorbed}\left(1-\left|S_k(\mathbf{b}_k)\right|^2\right)$$

Momentum distributions

C scatters elastically and C+n breaks up:

$$\left|\left\langle \varphi_{Continuum}(\mathbf{r})\middle| S_C(\mathbf{b}_C)\varphi_{l_0,m_0}(\mathbf{r})\right\rangle\right|^2$$

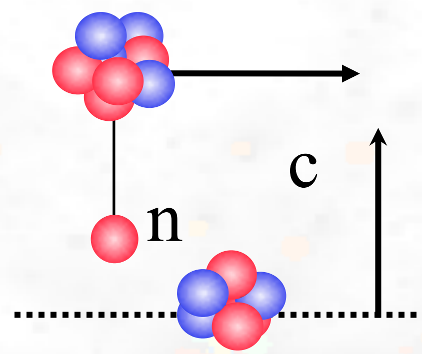


$$\mathbf{r}_n = \mathbf{R} + \frac{m_C}{m}\mathbf{r}, \qquad \mathbf{r}_C = \mathbf{R} - \frac{m_n}{m}\mathbf{r}$$

$$\mathbf{K} = \mathbf{k}_C - \mathbf{k}_n, \qquad \mathbf{k} = \frac{m_n}{m}\mathbf{k}_C - \frac{m_C}{m}\mathbf{k}_n$$

n is absorbed:

$$1 - \left|S_n(\mathbf{b}_n)\right|^2$$

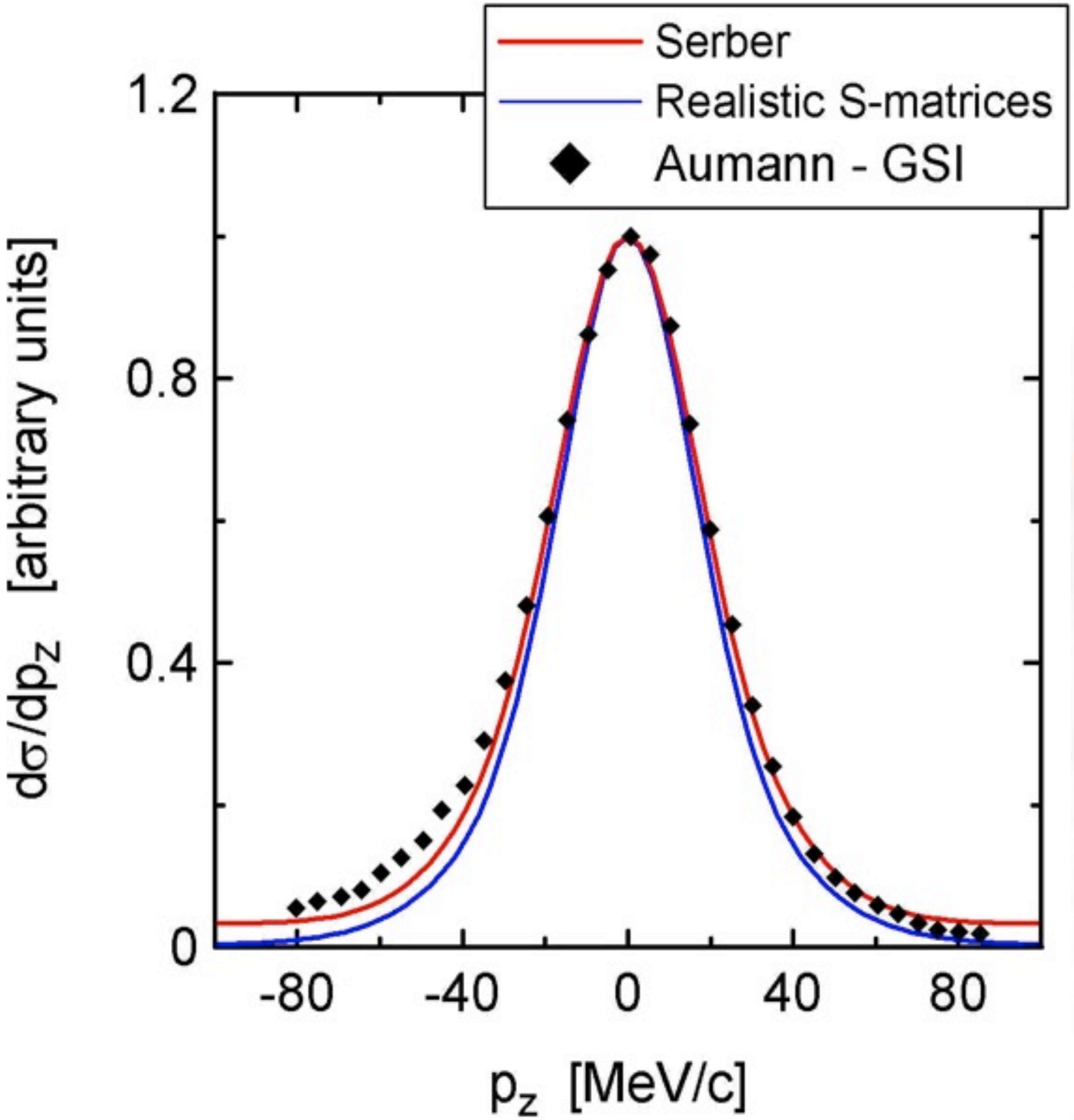


$$\varphi_{Continuum}(\mathbf{r}) \sim e^{i\mathbf{k}.\mathbf{r}}$$

$$\frac{d\sigma_{strip}}{d^3k_C} = \frac{1}{(2\pi)^3}\frac{1}{(2l_0+1)}\sum_{m_0}\int d^2b_n\left[1-\left|S_n(\mathbf{b}_n)\right|^2\right]\left|\int d^3r\, e^{-i\mathbf{k}_C\cdot\mathbf{r}}\, S_C(\mathbf{b}_C)\,\varphi_{l_0,m_0}(\mathbf{r})\right|^2$$

Bertulani, McVoy, PRC 46 (1992) 2638: $d\sigma / dp_z$ best probe

Longitudinal Momentum Dist. - Example



$$^{9}\mathrm{Be}\left(^{11}\mathrm{Be},^{10}\mathrm{Be}+\gamma\right)\mathrm{X}$$

One neutron-removal

60 MeV/nucleon

$1s\frac{1}{2}$ neutron, S_n = 0.503 MeV

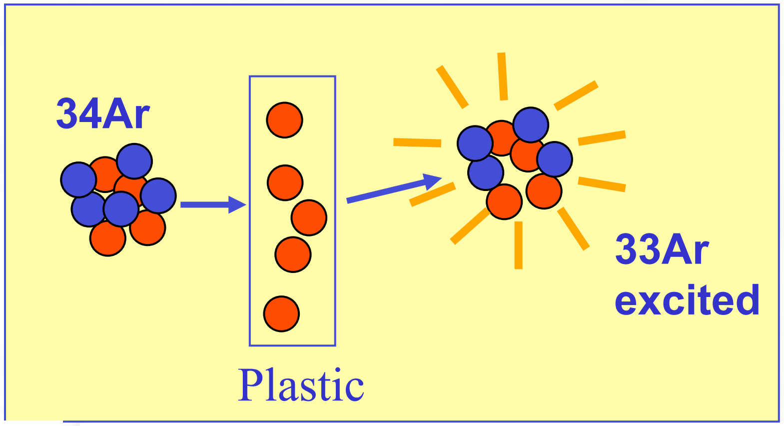
Tails & asymmetry:
higher order corrections

One needs continuum-continuum couplings

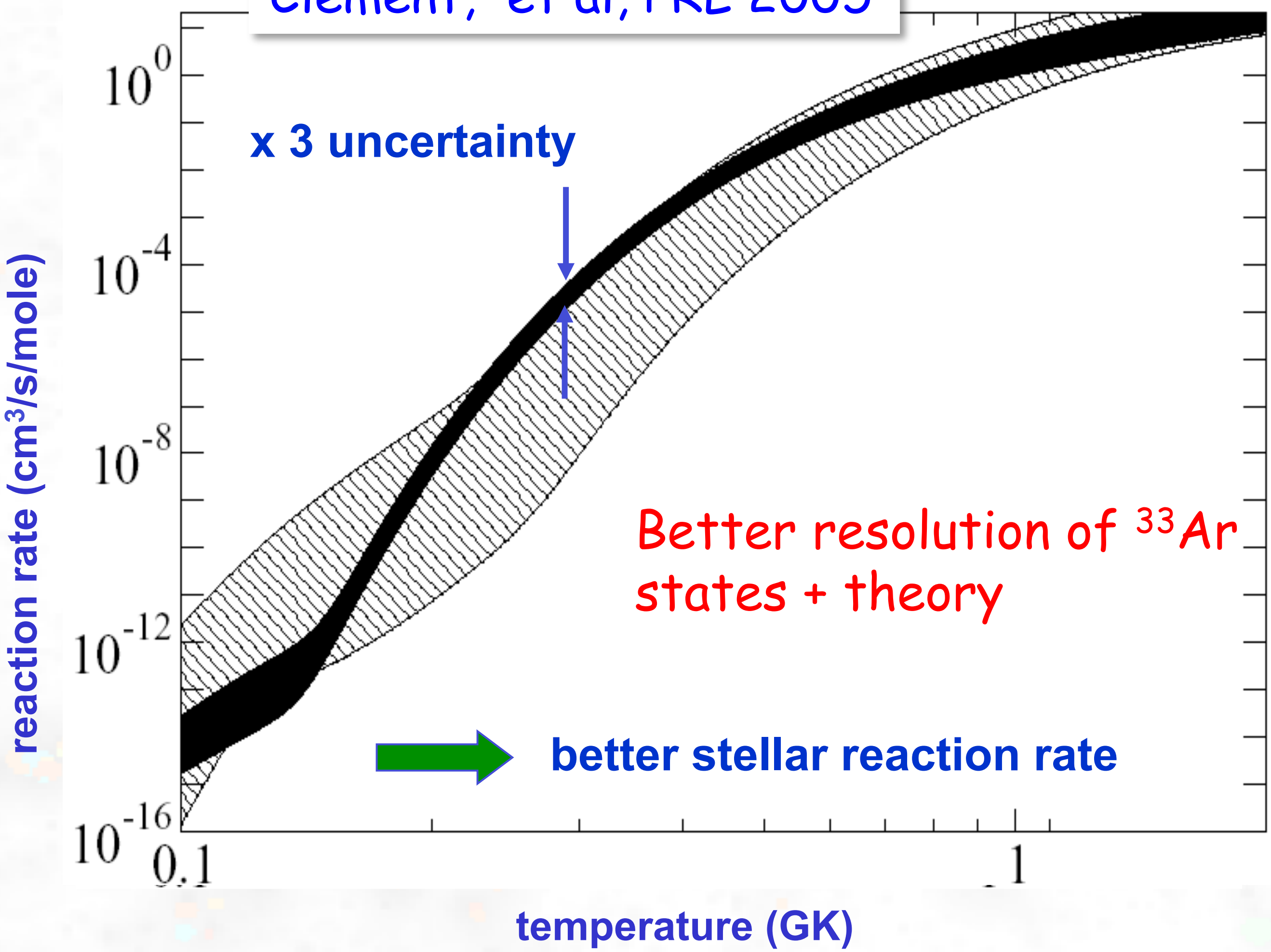
Example: Astrophysical Capture on Excited States

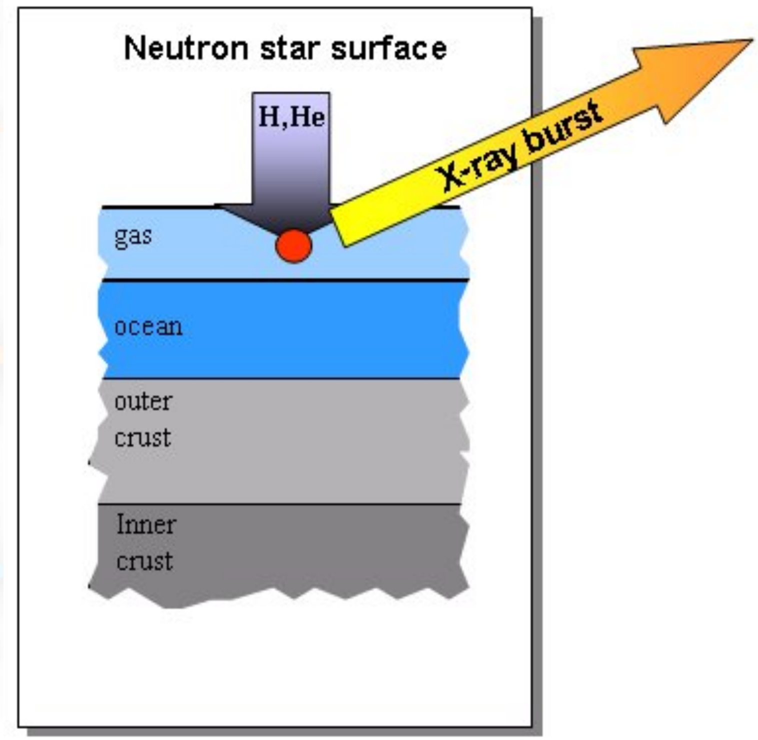
$^{32}Cl^* + p \rightarrow {}^{33}Ar^* \rightarrow {}^{33}Ar + \gamma$

Enhancement through capture on 89 keV state in ^{32}Cl



Clement, et al, PRL 2005





Capture on excited state of ^{32}Cl 4 times larger!

Nuclear structure calculations have absolutely zero value if one does not have a good understanding of nuclear reactions.

There is still lots of problems with reaction theory and consequently with experiments.

End of part III