Symmetries in Nuclear and Particle Physics

- 1. Symmetries in Physics
- 2. Interacting Boson Model
- 3. Nuclear Supersymmetry
- 4. Quark Model
- 5. Unquenched Quark Model

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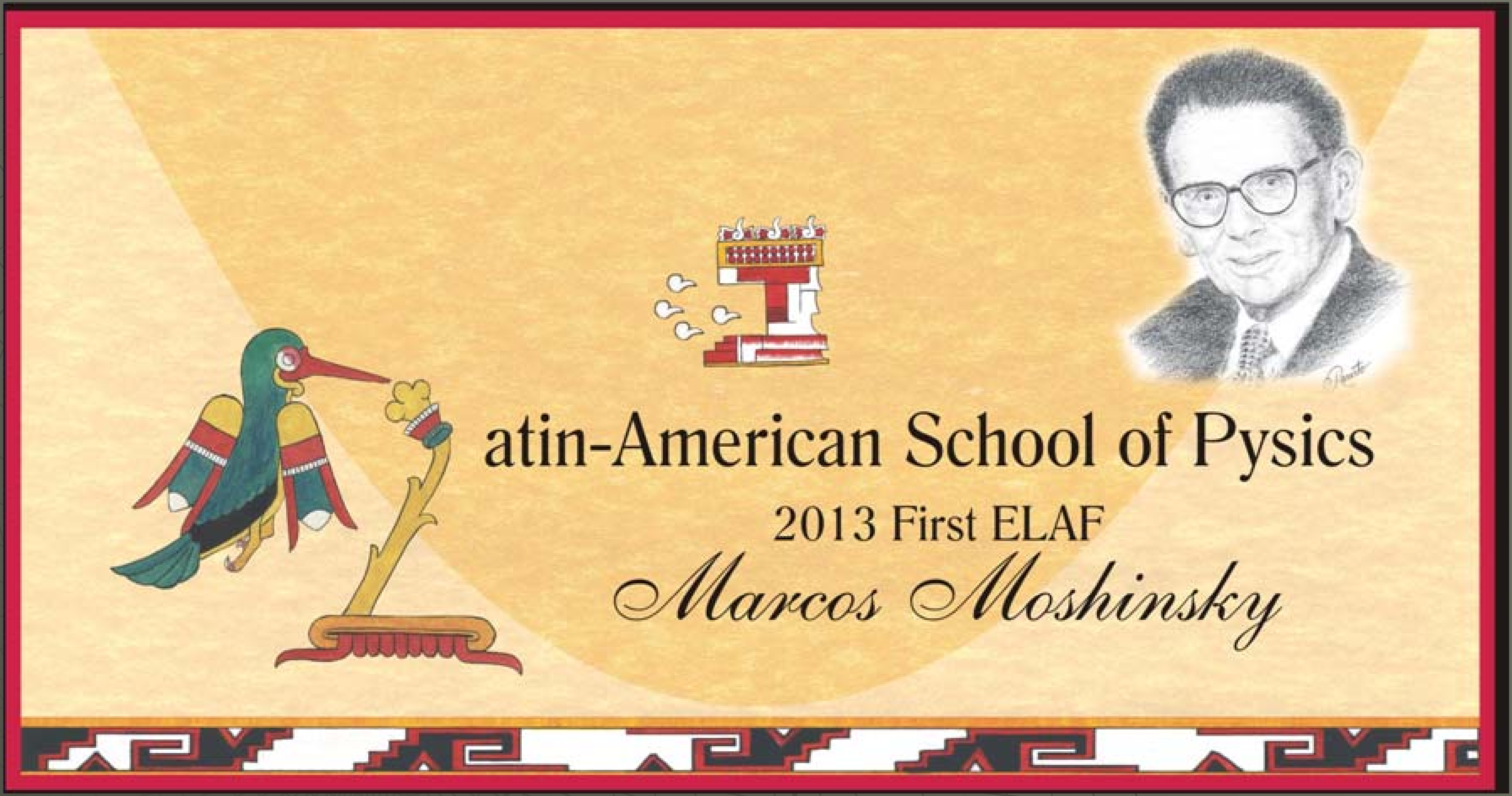
Supersimetría en la Física Nuclear

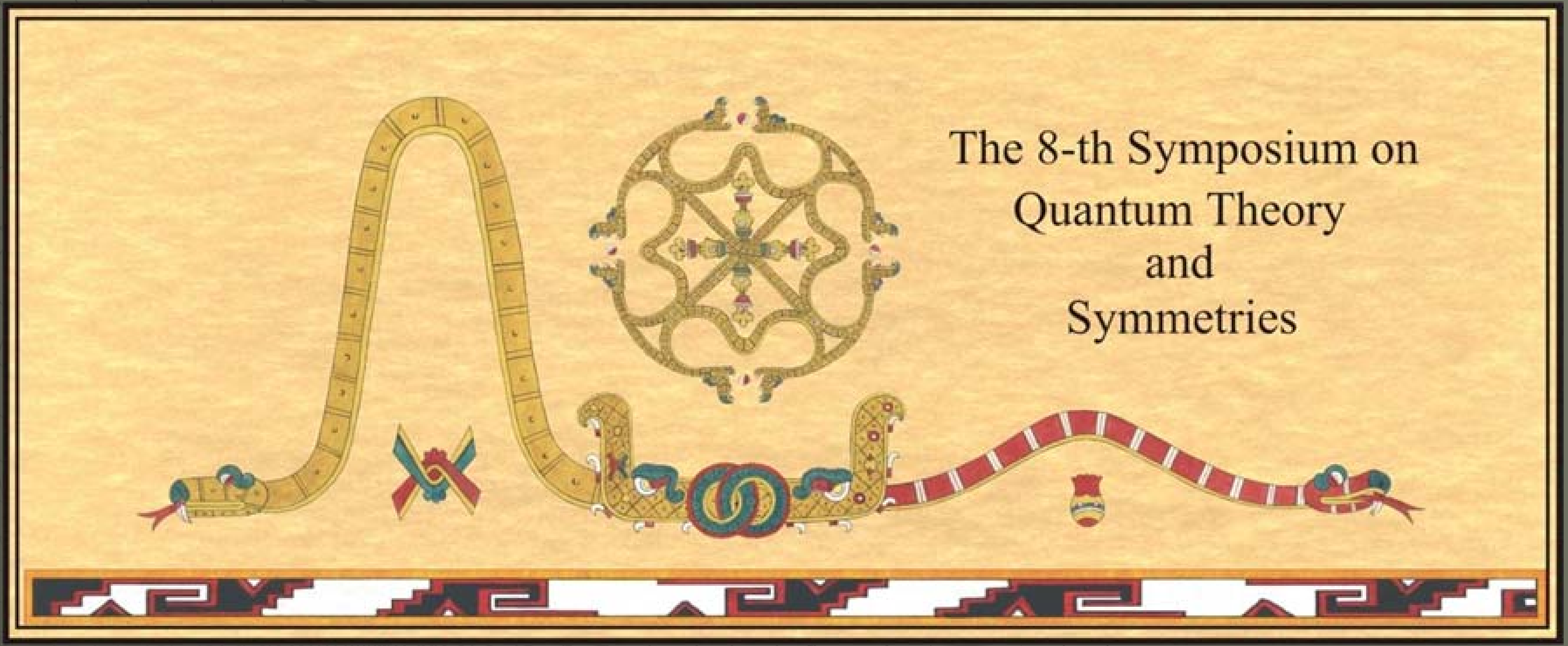
Roelof Bijker
"VI Escuela Mexicana de Física Nuclear"
AIP Conf Proc 1271, 90-132 (2010)

"VIII Escuela Mexicana de Física Nuclear"
México DF, verano de 2013



México DF, 22 de julio – 2 de agosto de 2013

http://www.nucleares.unam.mx/~bijker/elaf2013.html



The 8-th Symposium on
Quantum Theory
and
Symmetries

México DF, 5 – 9 de agosto de 2013

http://www.fis.unam.mx/symposiaqts

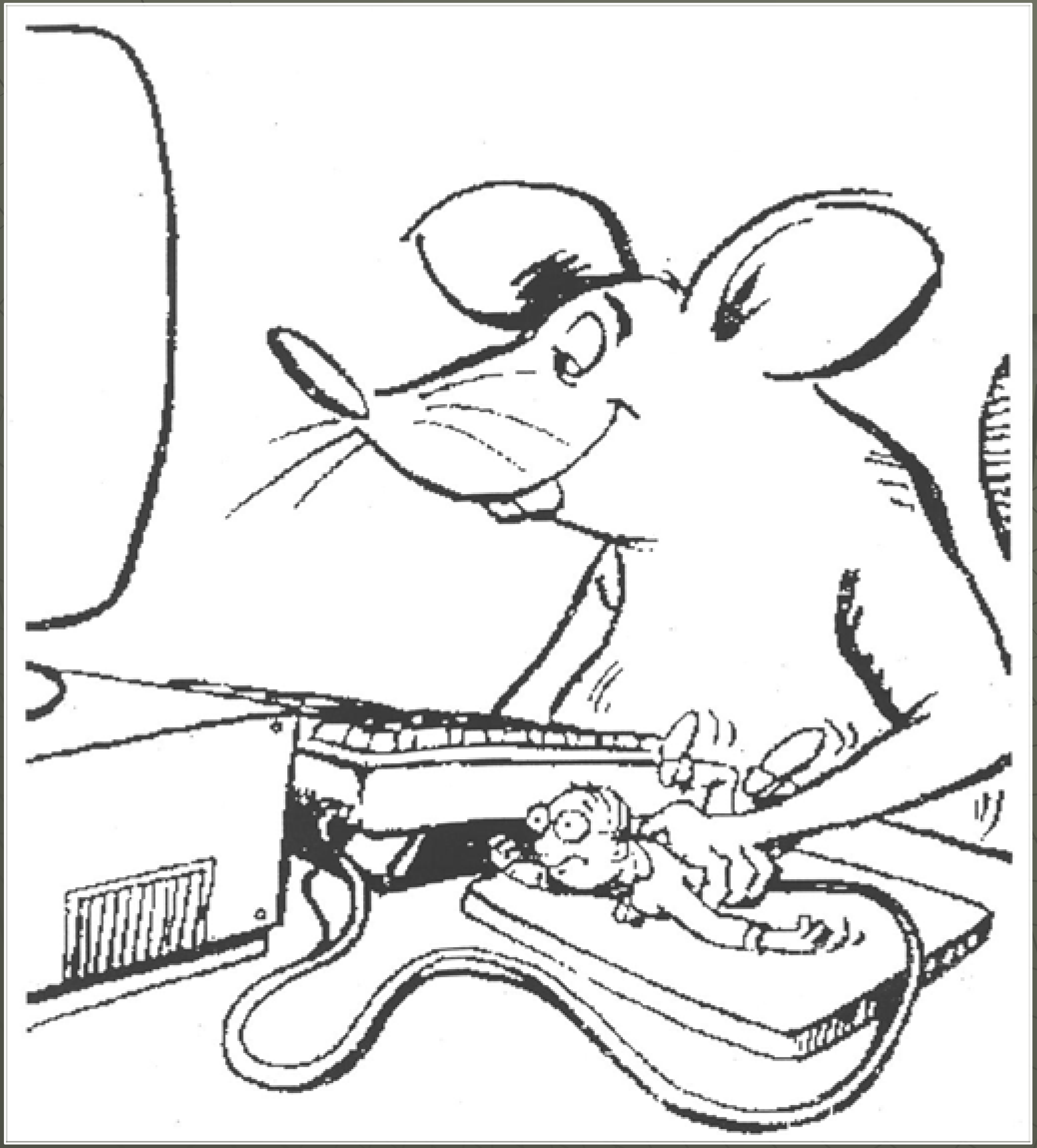
Temario

- Interacting Boson Model (IBM)
- Simetrías dinámicas del IBM
- Aplicaciones
- Interacting Boson-Fermion Model (IBFM)
- Simetrías dinámicas del IBFM
- Aplicaciones

Smaller and smaller
M.C. Escher

Motivación

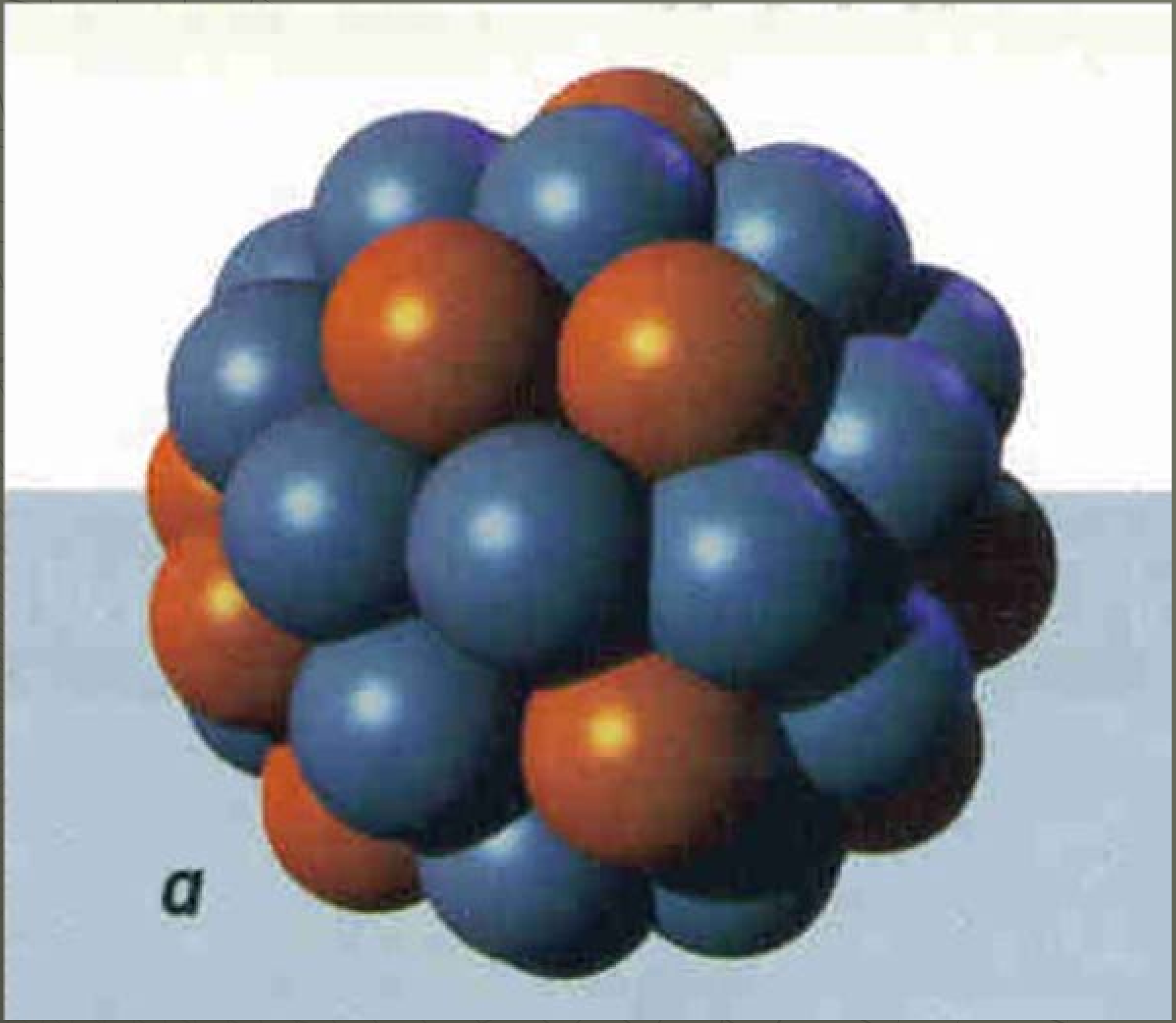
- Estudios numéricos
  - Ab initio
  - Modelo de capas
  - Campo medio
- Simetrías
  - Iso-espín, senioridad
  - Simetrías dinámicas del IBM y sus extensiones
  - Supersimetría nuclear



I am very happy to learn that the computer understands the problem, but I would like to understand it too

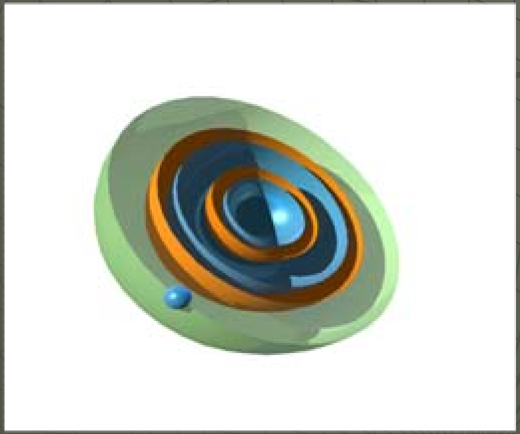
Eugene Wigner

El Núcleo

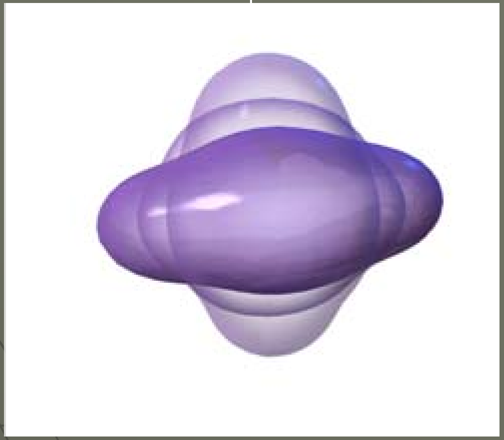


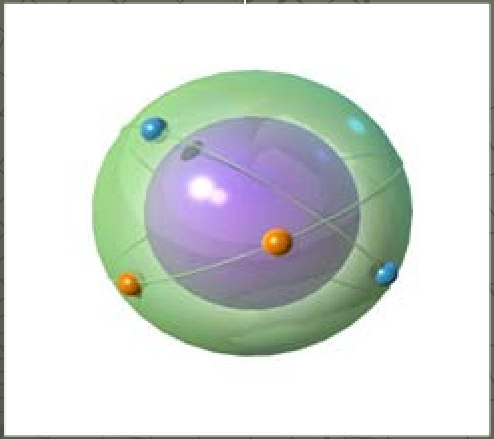
Modelos Nucleares

Estructura de capas:
Nucleones de valencia



Movimiento colectivo:
Formas nucleares





Pares de Cooper:
Sistema de bosones s y d

Interacting Boson Model

- El IBM describe excitaciones colectivas en núcleos par-par como un sistema de pares correlacionados de nucleones con momento angular L=0 y L=2 que se tratan como bosones (s y d) (Arima e Iachello, 1974)

$$\left[b_i, b_j^\dagger\right] = \delta_{ij}, \qquad \left[b_i^\dagger, b_j^\dagger\right] = [b_i, b_j] = 0$$

- El número de bosones N es la mitad del número de los nucleones de valencia
- El Hamiltoniano conserva el número de bosones N y el momento angular L

$$\begin{aligned} H &= H(B_{ij}) \\ B_{ij} &= b_i^\dagger b_j \end{aligned}$$

$$[B_{ij}, B_{kl}] = B_{il}\delta_{jk} - B_{kj}\delta_{il}$$

Hamiltoniano

- Invariancia rotacional, operadores tensoriales

$$\begin{aligned} & b^\dagger_{lm} \\ \tilde{b}_{lm} &= (-)^{l-m} b_{l,-m} \\ (b^\dagger_l \tilde{b}_{l'})^{(\lambda)}_\mu &= \sum_{mm'} \langle l,m,l',m'|\lambda,\mu\rangle \, b^\dagger_{lm}\tilde{b}_{l'm'} \end{aligned}$$

- Hamiltoniano

$$\begin{aligned} H &= \sum_l \epsilon_l \sum_m b^\dagger_{lm} b_{lm} \\ & +\sum_\lambda \sum_{l_1l_2l_3l_4} u^{(\lambda)}_{l_1l_2l_3l_4} \left[(b^\dagger_{l_1}\tilde{b}_{l_2})^{(\lambda)} \cdot (b^\dagger_{l_3}\tilde{b}_{l_4})^{(\lambda)} + h.c. \right] \end{aligned}$$

Simetría Dinámica: SO(6)

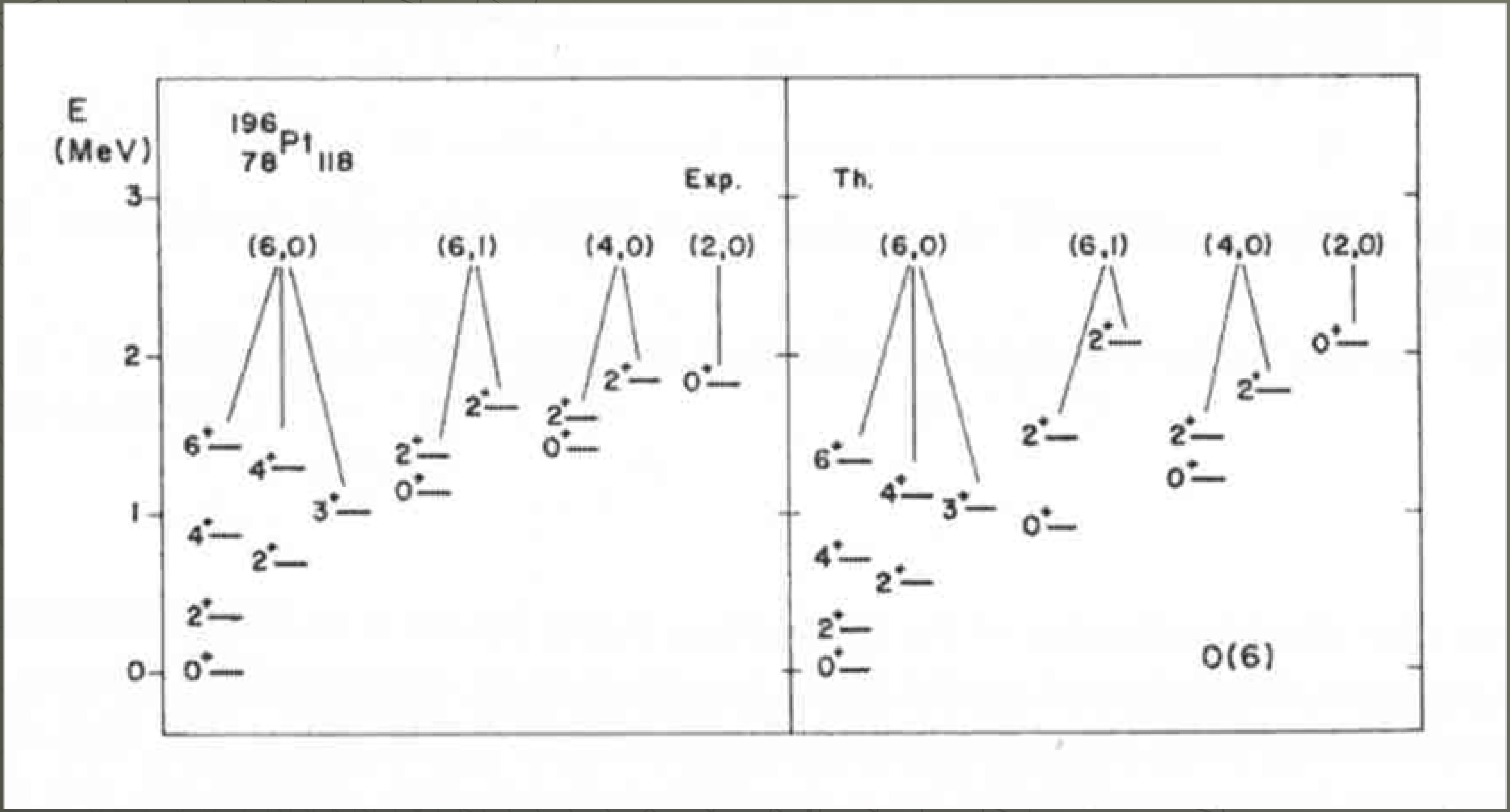
$U(6)$	$\supset$	$SO(6)$	$\supset$	$SO(5)$	$\supset$	$SO(3)$
$\mid [N]$	$,$	$(\sigma,0,0)$	$,$	$(\tau,0)$	$,$	$L\,\rangle$
$(d^\dagger\tilde{d})^{(1)}$		$(d^\dagger\tilde{d})^{(1)}$		$(d^\dagger\tilde{d})^{(1)}$		$(d^\dagger\tilde{d})^{(1)}$
$(d^\dagger\tilde{d})^{(3)}$		$(d^\dagger\tilde{d})^{(3)}$		$(d^\dagger\tilde{d})^{(3)}$		
$(s^\dagger\tilde{d}+d^\dagger\tilde{s})^{(2)}$		$(s^\dagger\tilde{d}+d^\dagger\tilde{s})^{(2)}$				
$(s^\dagger\tilde{s})^{(0)}$						
$(d^\dagger\tilde{d})^{(0)}$						
$(d^\dagger\tilde{d})^{(2)}$						
$(d^\dagger\tilde{d})^{(4)}$						
$i(s^\dagger\tilde{d}-d^\dagger\tilde{s})^{(2)}$						
36		15		10		3

Límite SO(6)

$$\begin{aligned}
H &= -A\,C_{2SO(6)} + B\,C_{2SO(5)} + C\,C_{2SO(3)} \\
&= -\kappa_2\,(s^\dagger\tilde{d} + d^\dagger\tilde{s})^{(2)} \cdot (s^\dagger\tilde{d} + d^\dagger\tilde{s})^{(2)} \\
&\quad +\kappa_3\,(d^\dagger\tilde{d})^{(3)} \cdot (d^\dagger\tilde{d})^{(3)} \\
&\quad +\kappa_1\,(d^\dagger\tilde{d})^{(1)} \cdot (d^\dagger\tilde{d})^{(1)} \\
\\
E &= -A\,\sigma(\sigma+4) + B\,\tau(\tau+3) + C\,L(L+1)
\end{aligned}$$

$$\begin{aligned}
\kappa_2 &= A \\
\kappa_3 &= -2A + 2B \\
\kappa_1 &= -2A + 2B + 10C
\end{aligned}$$

Límite SO(6)



Dynamical Symmetries

$$\begin{array}{c} U(6) \\ N \end{array} \supset \left\{ \begin{array}{ccccc} U(5) & \supset & SO(5) & \supset & SO(3) \\ n_d & & \tau & & L \\ & & & & \\ SU(3) & \supset & SO(3) & & \\ (\lambda,\mu) & & L & & \\ & & & & \\ SO(6) & \supset & SO(5) & \supset & SO(3) \\ \sigma & & \tau & & L \end{array} \right.$$

Schematic Hamiltonian:

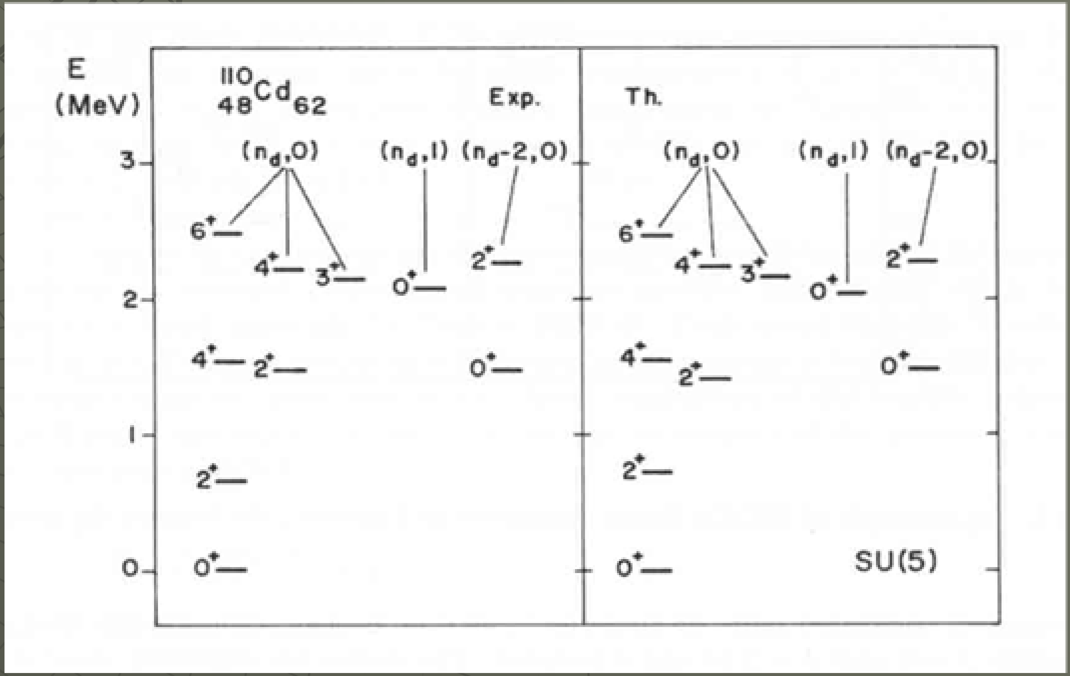
$$\begin{aligned} H &= \epsilon\,\hat{n}_d - \kappa\,\hat{Q}(\chi)\cdot\hat{Q}(\chi) \\ \hat{n}_d &= \sum_m d_m^\dagger d_m \\ \hat{Q}(\chi) &= (s^\dagger\tilde{d} + d^\dagger\tilde{s} + \chi d^\dagger\tilde{d})^{(2)} \end{aligned}$$

U(5) limit

For κ=0 :

$$H_1 = \epsilon\,\hat{n}_d = \epsilon\,C_{1U(5)}$$
$$E_1 = \epsilon\,n_d$$

Spectrum of a harmonic oscillator associated with quadrupole oscillations of the nuclear surface: spherical nuclei

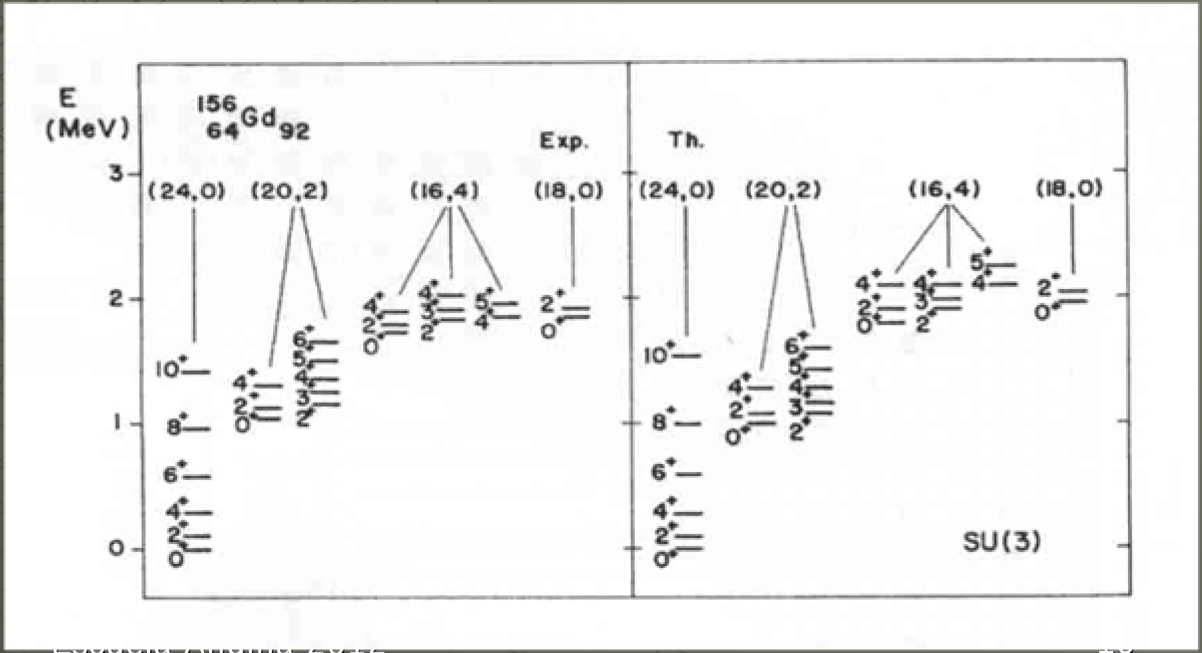


SU(3) limit

For ε=0 and χ= $\pm\frac{1}{2}\sqrt{7}$

$$\begin{aligned} H_2 &= -\kappa\,\hat{Q}(\pm\sqrt{7}/2)\cdot\hat{Q}(\pm\sqrt{7}/2) \\ &= -\frac{1}{2}\kappa\left[C_{2SU(3)}-\frac{3}{4}C_{2SO(3)}\right] \\ E_2 &= -\frac{1}{2}\kappa\,[\lambda(\lambda+3)+\mu(\mu+3)+\lambda\mu]+\frac{3}{8}\kappa\,L(L+1) \end{aligned}$$

Rotation-vibration spectrum with β- and γ-vibrational bands: axially-deformed nuclei

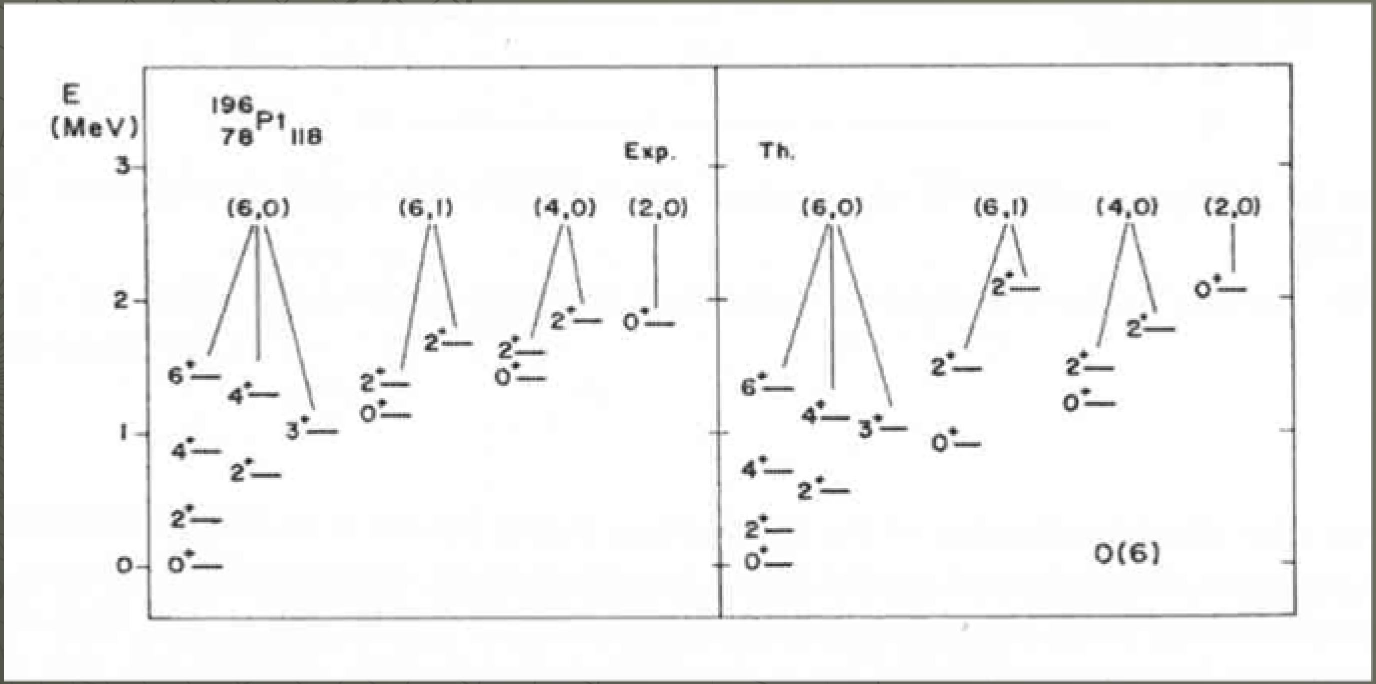


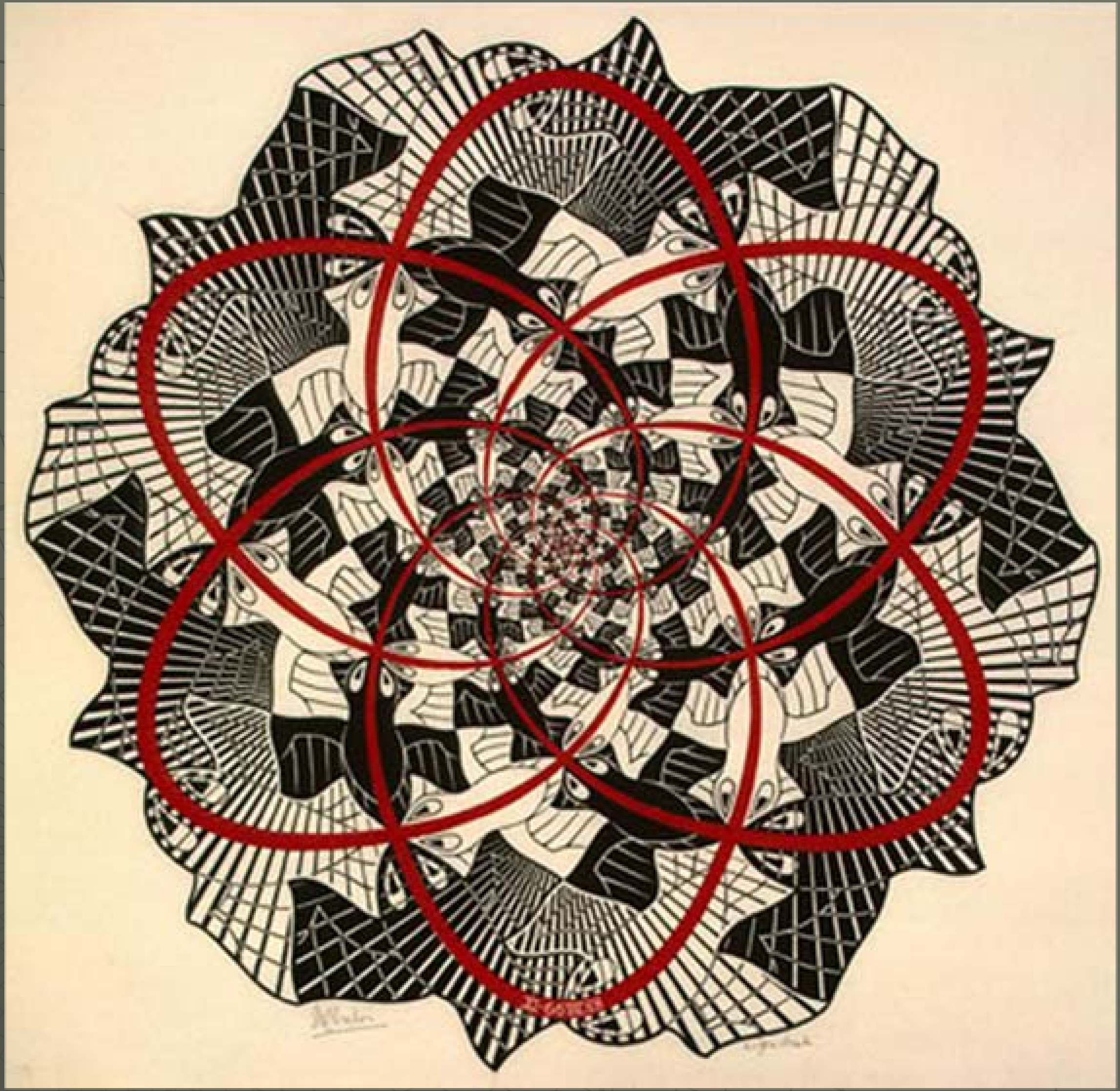
SO(6) limit

For ε=0 and χ=0 :

$$
\begin{aligned}
H_3 &= -\kappa\, \hat{Q}(0) \cdot \hat{Q}(0) \\
&= -\kappa \left[C_{2SO(6)} - C_{2SO(5)} \right] \\
E_3 &= -\kappa \left[\sigma(\sigma+4) - \tau(\tau+3) \right]
\end{aligned}
$$

Rotation-vibration spectrum of a γ-unstable nucleus





Interacting Boson-Fermion Model

- Considera una extensión del IBM a núcleos impares que incluye, además de los grados de libertad colectivos (bosones), a los grados de libertad de partícula independiente del nucleón extra (fermión con momento angular j=j$_1$, j$_2$, ...)

$$\left[b_i, b_j^\dagger\right] = \delta_{ij} \;, \qquad \left[b_i^\dagger, b_j^\dagger\right] = [b_i, b_j] = 0$$
$$\{a_\mu, a_\nu^\dagger\} = \delta_{\mu\nu} \;, \qquad \{a_\mu^\dagger, a_\nu^\dagger\} = \{a_\mu, a_\nu\} = 0$$

$$\begin{aligned} H &= H(B_{ij}, A_{\mu\nu}) \\ B_{ij} &= b_i^\dagger b_j \\ A_{\mu\nu} &= a_\mu^\dagger a_\nu \end{aligned}$$

$$\begin{aligned} [B_{ij}, B_{kl}] &= B_{il}\delta_{jk} - B_{kj}\delta_{il} \\ [A_{\mu\nu}, A_{\rho\sigma}] &= A_{\mu\sigma}\,\delta_{\nu\rho} - A_{\rho\nu}\,\delta_{\mu\sigma} \\ [B_{ij}, A_{\mu\nu}] &= 0 \end{aligned}$$

Building Blocks

bosons	$l = 0, 2$	$\sum_l (2l+1) = 6$
fermions	$j = j_1, j_2, \ldots$	$\sum_j (2j+1) = \Omega$

Model	Generators	Invariant	Algebra
IBM	$b_i^\dagger b_j$	N	$U(6)$
IBFM	$b_i^\dagger b_j,\ a_\mu^\dagger a_\nu$	$N,\ M$	$U(6) \otimes U(\Omega)$

$N = \sum_i b_i^\dagger b_i$	total number of bosons
$M = \sum_\mu a_\mu^\dagger a_\mu$	total number of fermions

Invariancia Rotacional

$$b^\dagger_{lm} \,, \qquad \tilde{b}_{lm} = (-)^{l-m} b_{l,-m}$$

$$a^\dagger_{jm} \,, \qquad \tilde{a}_{jm} = (-)^{j-m} a_{j,-m}$$

$$(b^\dagger_l \tilde{b}_{l'})^{(\lambda)}_\mu = \sum_{mm'} \langle l, m, l', m' | \lambda, \mu \rangle \, b^\dagger_{lm} \tilde{b}_{l'm'}$$

$$(a^\dagger_j \tilde{a}_{j'})^{(\lambda)}_\mu = \sum_{mm'} \langle j, m, j', m' | \lambda, \mu \rangle \, a^\dagger_{jm} \tilde{a}_{j'm'}$$

Hamiltoniano

$$H = H_B + H_F + V_{BF}$$

$$\begin{aligned}
H_B &= \sum_l \epsilon_l \sum_m b^\dagger_{lm} b_{lm} \\
&\quad + \sum_\lambda \sum_{l_1 l_2 l_3 l_4} u^{(\lambda)}_{l_1 l_2 l_3 l_4} \left[(b^\dagger_{l_1} \tilde{b}_{l_2})^{(\lambda)} \cdot (b^\dagger_{l_3} \tilde{b}_{l_4})^{(\lambda)} + h.c. \right] \\
H_F &= \sum_j \eta_j \sum_m a^\dagger_{jm} a_{jm} \\
&\quad + \sum_\lambda \sum_{j_1 j_2 j_3 j_4} v^{(\lambda)}_{j_1 j_2 j_3 j_4} \left[(a^\dagger_{j_1} \tilde{a}_{j_2})^{(\lambda)} \cdot (a^\dagger_{j_3} \tilde{a}_{j_4})^{(\lambda)} + h.c. \right] \\
V_{BF} &= \sum_\lambda \sum_{l_1 l_2 j_1 j_2} w^{(\lambda)}_{l_1 l_2 j_1 j_2} \left[(b^\dagger_{l_1} \tilde{b}_{l_2})^{(\lambda)} \cdot (a^\dagger_{j_1} \tilde{a}_{j_2})^{(\lambda)} + h.c. \right]
\end{aligned}$$

SO(6) más j=3/2

$SO^B(6)$	$\supset$	$SO^B(5)$	$\supset$	$SO^B(3)$
$(d^\dagger\tilde{d})^{(1)}$		$(d^\dagger\tilde{d})^{(1)}$		$(d^\dagger\tilde{d})^{(1)}$
$(d^\dagger\tilde{d})^{(3)}$		$(d^\dagger\tilde{d})^{(3)}$		
$(s^\dagger\tilde{d}+d^\dagger\tilde{s})^{(2)}$				

$U^F(4)$	$\supset$	$SU^F(4)$	$\supset$	$Sp^F(4)$	$\supset$	$SU^F(2)$
$(a^\dagger_{3/2}\tilde{a}_{3/2})^{(1)}$		$(a^\dagger_{3/2}\tilde{a}_{3/2})^{(1)}$		$(a^\dagger_{3/2}\tilde{a}_{3/2})^{(1)}$		$(a^\dagger_{3/2}\tilde{a}_{3/2})^{(1}$
$(a^\dagger_{3/2}\tilde{a}_{3/2})^{(3)}$		$(a^\dagger_{3/2}\tilde{a}_{3/2})^{(3)}$		$(a^\dagger_{3/2}\tilde{a}_{3/2})^{(3)}$		
$(a^\dagger_{3/2}\tilde{a}_{3/2})^{(2)}$		$(a^\dagger_{3/2}\tilde{a}_{3/2})^{(2)}$				
$(a^\dagger_{3/2}\tilde{a}_{3/2})^{(0)}$						

Simetría Bose-Fermi I

$$\begin{array}{ccccc}
Spin(6) & \supset & Spin(5) & \supset & Spin(3) \\
 & & & & \\
G^{(1)} & & G^{(1)} & & G^{(1)} \\
G^{(3)} & & G^{(3)} & & \\
G^{(2)} & & & &
\end{array}$$

$$\begin{aligned}
G^{(1)} &= (d^\dagger \tilde{d})^{(1)} - \frac{1}{\sqrt{2}}(a^\dagger_{3/2}\tilde{a}_{3/2})^{(1)} \\
G^{(3)} &= (d^\dagger \tilde{d})^{(3)} + \frac{1}{\sqrt{2}}(a^\dagger_{3/2}\tilde{a}_{3/2})^{(3)} \\
G^{(2)} &= (s^\dagger \tilde{d} + d^\dagger \tilde{s})^{(2)} + (a^\dagger_{3/2}\tilde{a}_{3/2})^{(2)}
\end{aligned}$$

$$\begin{tikzcd}
\begin{array}{c} U^B(6) \\ {[N]} \end{array} \arrow[d] & \otimes & \begin{array}{c} U^F(4) \\ \{M=1\} \end{array} \arrow[d] \\
\begin{array}{c} SO^B(6) \\ (\Sigma,0,0) \end{array} \arrow[dr] & \otimes & \begin{array}{c} SU^F(4) \\ (\frac{1}{2},\frac{1}{2},\frac{1}{2}) \end{array} \arrow[dl] \\
 & \begin{array}{c} Spin(6) \\ (\sigma_1,\sigma_2,\sigma_3) \end{array} \arrow[d] & \\
 & \begin{array}{c} Spin(5) \\ (\tau_1,\tau_2) \end{array} \arrow[d] & \\
 & \begin{array}{c} Spin(3) \\ J \end{array} &
\end{tikzcd}$$

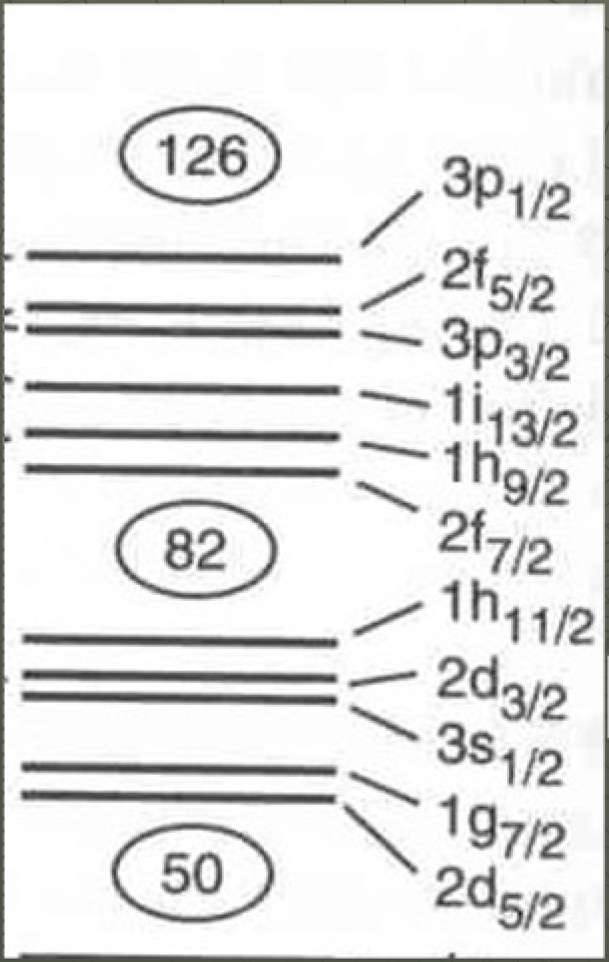
$$\left| [N], \{M=1\}, (\Sigma,0,0), (\frac{1}{2},\frac{1}{2},\frac{1}{2}); (\sigma_1,\sigma_2,\sigma_3), (\tau_1,\tau_2), J \right\rangle$$

Límite Spin(6)

$$
\begin{aligned}
H &= -A\,C_{2Spin(6)} + B\,C_{2Spin(5)} + C\,C_{2Spin(3)} \\
&= -\kappa_2\,G^{(2)}\cdot G^{(2)} + \kappa_3\,G^{(3)}\cdot G^{(3)} + \kappa_1\,G^{(1)}\cdot G^{(1)} \\
\\
E &= -A\left[\sigma_1(\sigma_1+4)+\sigma_2(\sigma_2+4)+\sigma_3^2\right] \\
&\quad +B\,[\tau_1(\tau_1+3)+\tau_2(\tau_2+1)] + C\,J(J+1)
\end{aligned}
$$

$$
\begin{aligned}
\kappa_2 &= A \\
\kappa_3 &= -2A+2B \\
\kappa_1 &= -2A+2B+10C
\end{aligned}
$$

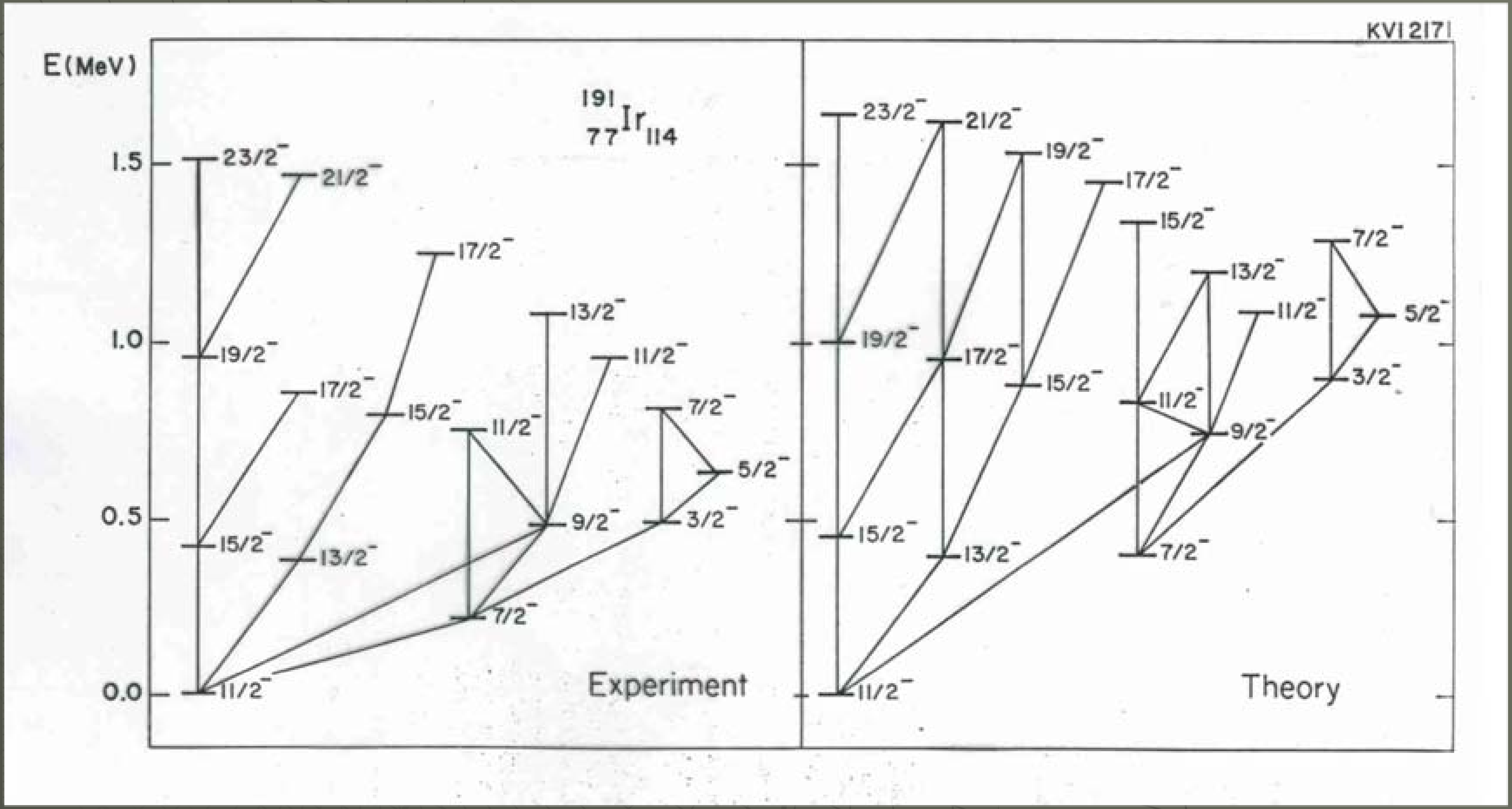
Núcleos A~190



Niveles de protón, capa 50-82

Núcleos Ir, Au
con 77, 79 protones

Ejemplo Spin(6)



Iachello, PRL 44, 772 (1980)

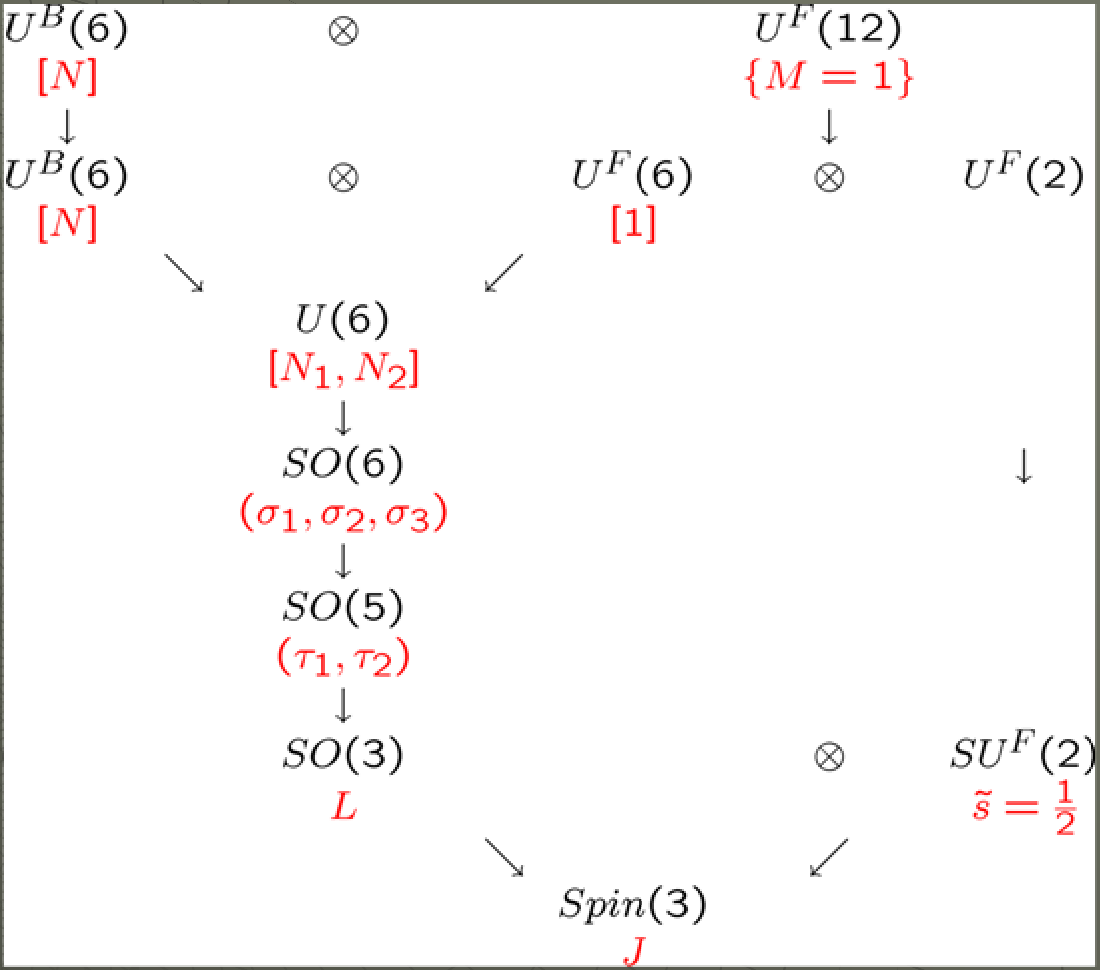
SO(6) más j=1/2, 3/2, 5/2

Simetría pseudo-espín

$$j = \frac{1}{2}, \frac{3}{2}, \frac{5}{2} \qquad (\tilde{l} = 0, 2) \otimes (\tilde{s} = \frac{1}{2})$$

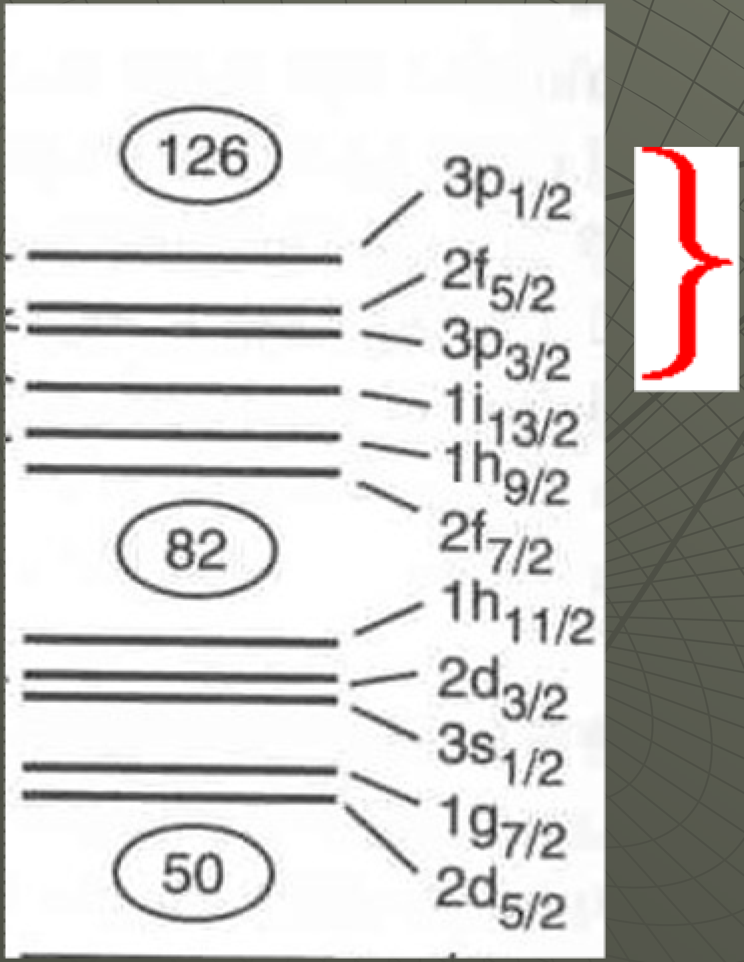
$$\begin{tikzcd}
 & & U^F(12) \arrow[d] & \\
U^B(6) \arrow[d] & U^F(6) \arrow[d] & \otimes & U^F(2) \arrow[ddd] \\
SO^B(6) \arrow[d] & SO^F(6) \arrow[d] & & \\
SO^B(5) \arrow[d] & SO^F(5) \arrow[d] & & \\
SO^B(3) & SO^F(3) & \otimes & SU^F(2)
\end{tikzcd}$$

Simetría Bose-Fermi II



$$\begin{tikzcd}
\begin{array}{c} U^B(6) \\ {[N]} \end{array} \arrow[d] & \otimes & & \begin{array}{c} U^F(12) \\ \{M=1\} \end{array} \arrow[d] & \\
\begin{array}{c} U^B(6) \\ {[N]} \end{array} \arrow[dr] & \otimes & \begin{array}{c} U^F(6) \\ {[1]} \end{array} \arrow[dl] & \otimes & U^F(2) \arrow[dddd] \\
& \begin{array}{c} U(6) \\ {[N_1,N_2]} \end{array} \arrow[d] & & & \\
& \begin{array}{c} SO(6) \\ (\sigma_1,\sigma_2,\sigma_3) \end{array} \arrow[d] & & & \\
& \begin{array}{c} SO(5) \\ (\tau_1,\tau_2) \end{array} \arrow[d] & & & \\
& \begin{array}{c} SO(3) \\ L \end{array} \arrow[dr] & & \otimes & \begin{array}{c} SU^F(2) \\ \tilde{s}=\frac{1}{2} \end{array} \arrow[dll] \\
& & \begin{array}{c} Spin(3) \\ J \end{array} & &
\end{tikzcd}$$

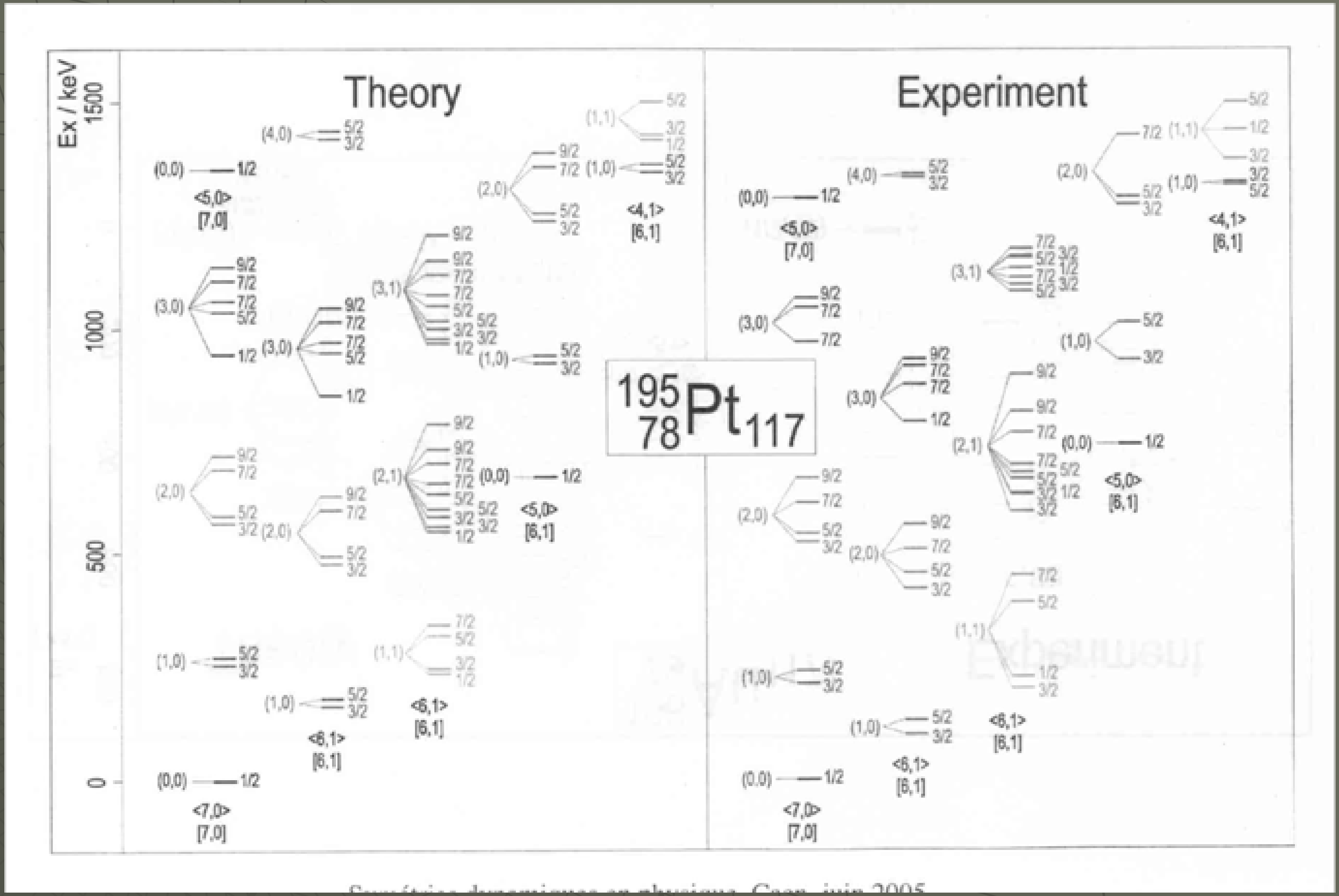
Núcleos A~190



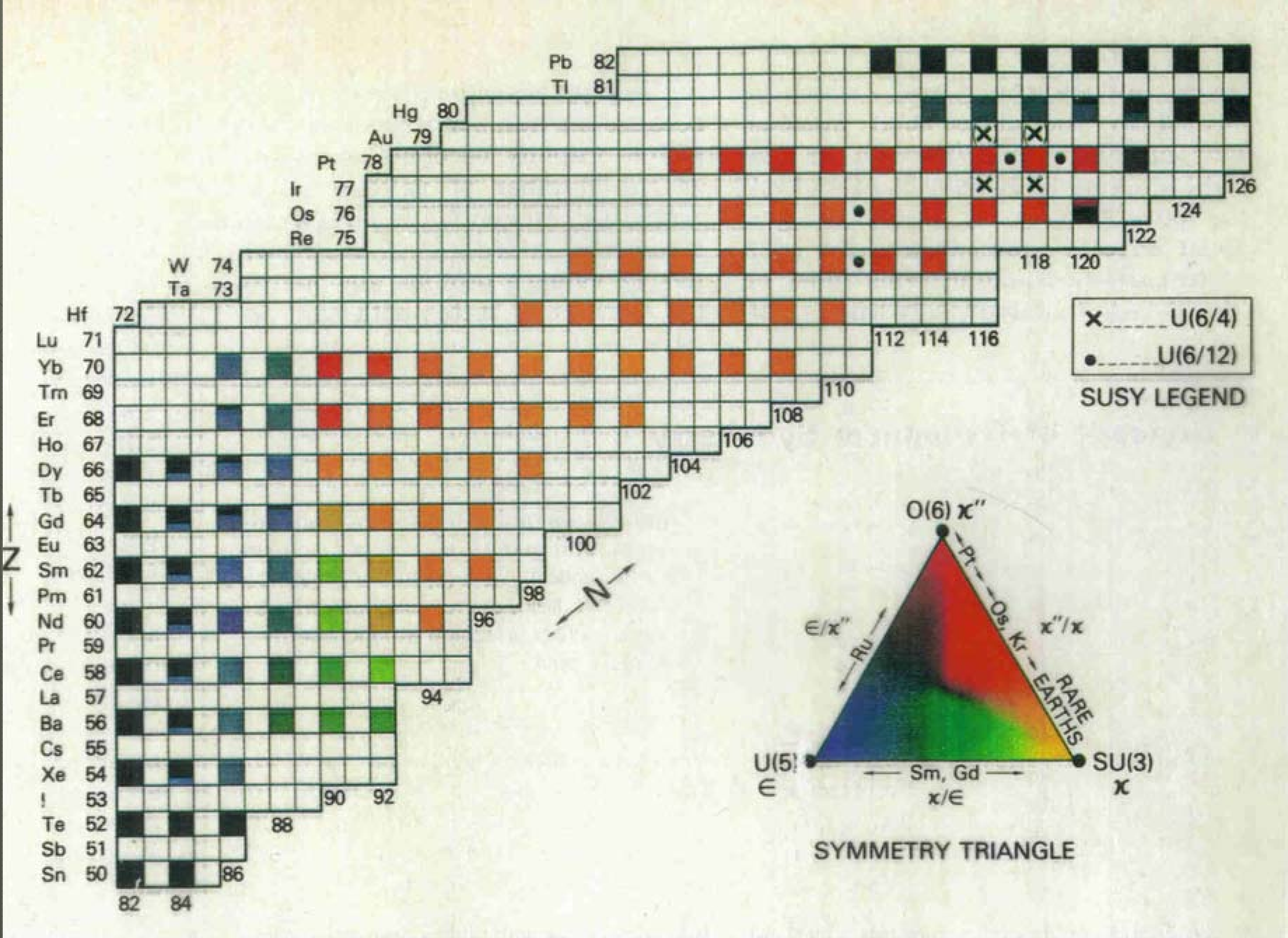
Niveles de neutrón, capa 82-126

Núcleos Os, Pt
con 76, 78 protones

Ejemplo U(6) x U(2)



Balantekin, Bars, Bijker, Iachello, PRC 27, 1761 (1983)


Pb 82
Tl 81
Hg 80
Au 79
Pt 78
Ir 77
Os 76
Re 75
W 74
Ta 73
Hf 72
Lu 71
Yb 70
Tm 69
Er 68
Ho 67
Dy 66
Tb 65
Gd 64
Eu 63
Sm 62
Pm 61
Nd 60
Pr 59
Ce 58
La 57
Ba 56
Cs 55
Xe 54
I 53
Te 52
Sb 51
Sn 50
Z
N
82
84
86
88
90
92
94
96
98
100
102
104
106
108
110
112
114
116
118
120
122
124
126
× U(6/4)
● U(6/12)
SUSY LEGEND
O(6) χ''
U(5) ∈
SU(3) χ
∈/χ''
χ''/χ
χ/∈
Pt
Os, Kr
RARE EARTHS
Ru
Sm, Gd
SYMMETRY TRIANGLE

Resumen

- IBM: núcleos par-par
- IBFM: núcleos impar
- Simetrías dinámicas
- ¿Es posible describir los núcleos par-par e impares en el marco de un modelo unificado?

- La respuesta es sí: SUSY

References

- Iachello and Arima - The interacting boson model (1987)
- Iachello and Van Isacker - The interacting boson-fermion model (1991)

- Frank, Barea and Bijker - Lecture Notes in Physics 652, 285-324 (2004) [arXiv:nucl-th/0402058]
- Bijker, AIP 1271, 90-132 (2010)