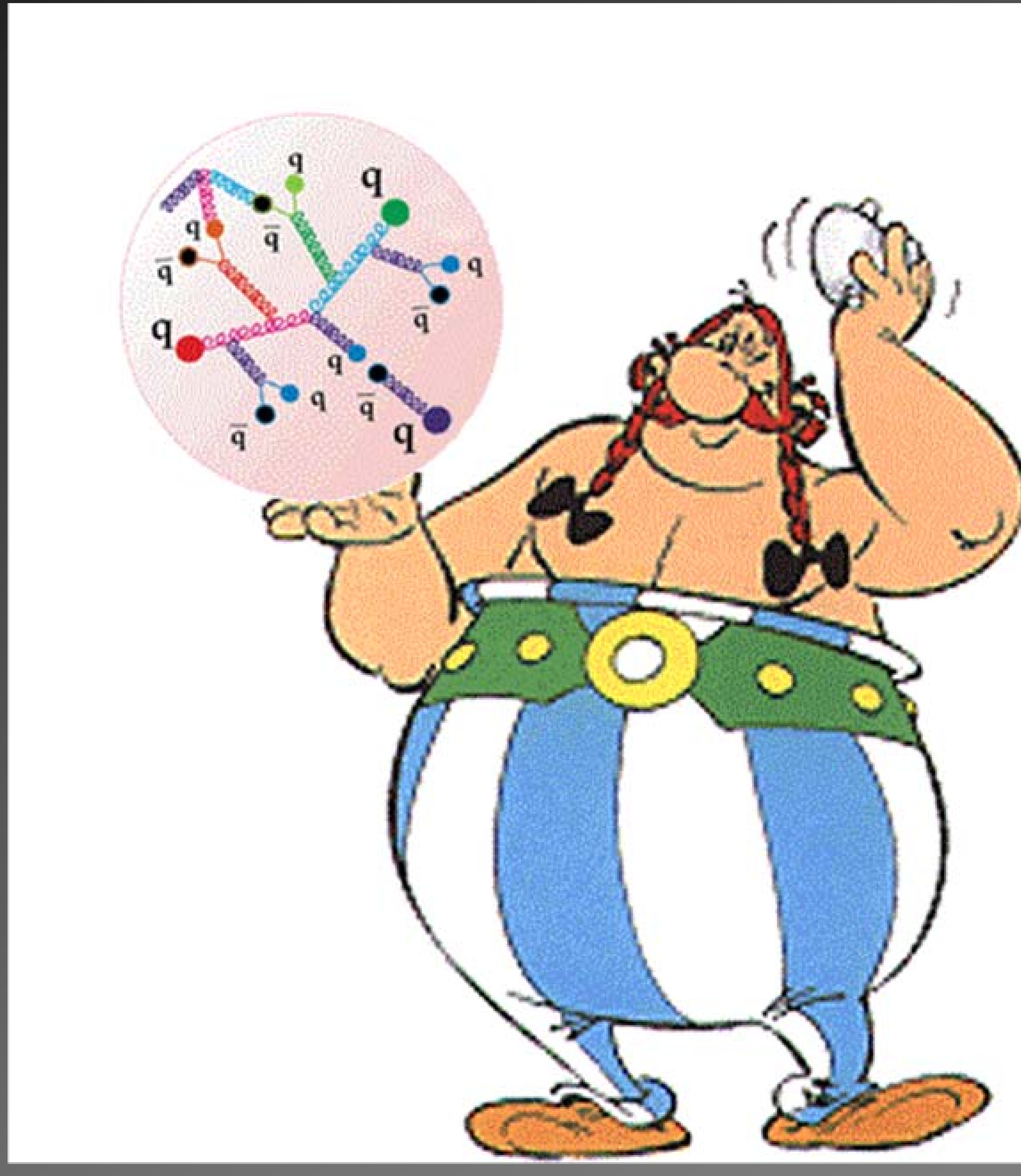
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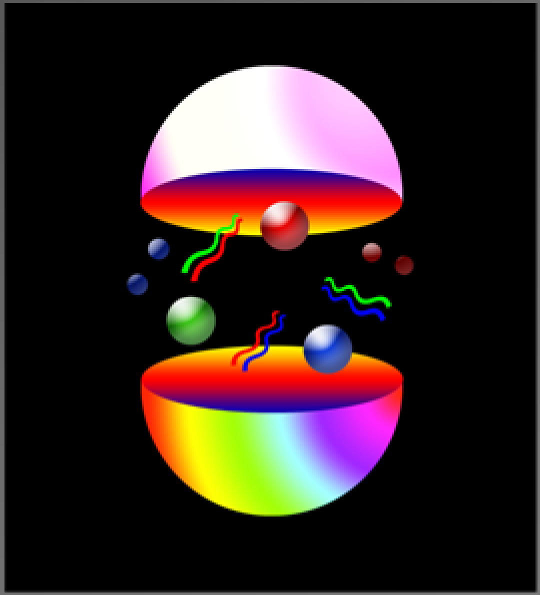
Symmetries in Nuclear and Particle Physics

- 1. Symmetries in Physics
- 2. Interacting Boson Model
- 3. Nuclear Supersymmetry
- 4. Quark Model
- 5. Unquenched Quark Model



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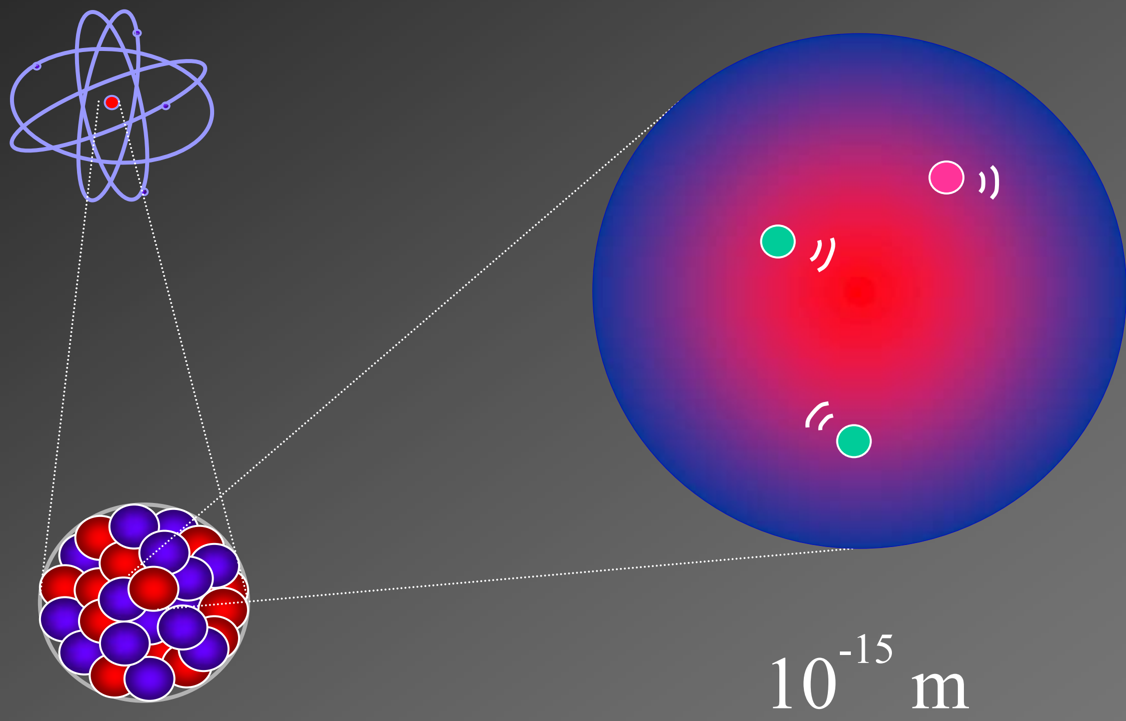
Outline

- Introduction
- Constituent quark model
- Mesons and baryons
- Stringlike collective model
- Masses, magnetic moments and electromagnetic couplings

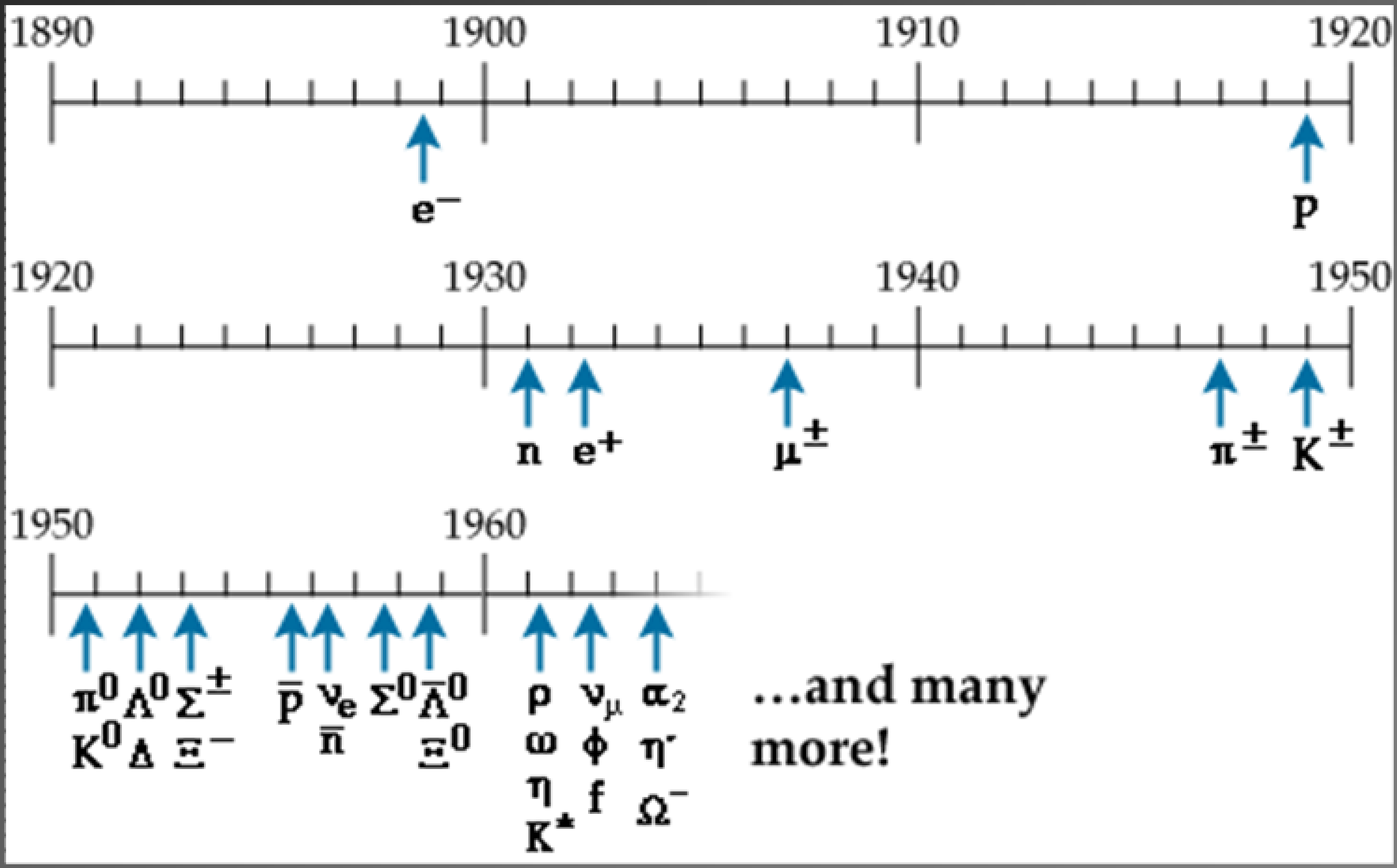
Where the hell ...?

The Structure of Nucleons



Neutrons and protons contain quarks

The Particle Zoo



"Quarks. Neutrinos. Mesons. All those damn particles you can't see. That's what drove me to drink. But now I can see them!"

"Particles, particles, particles."

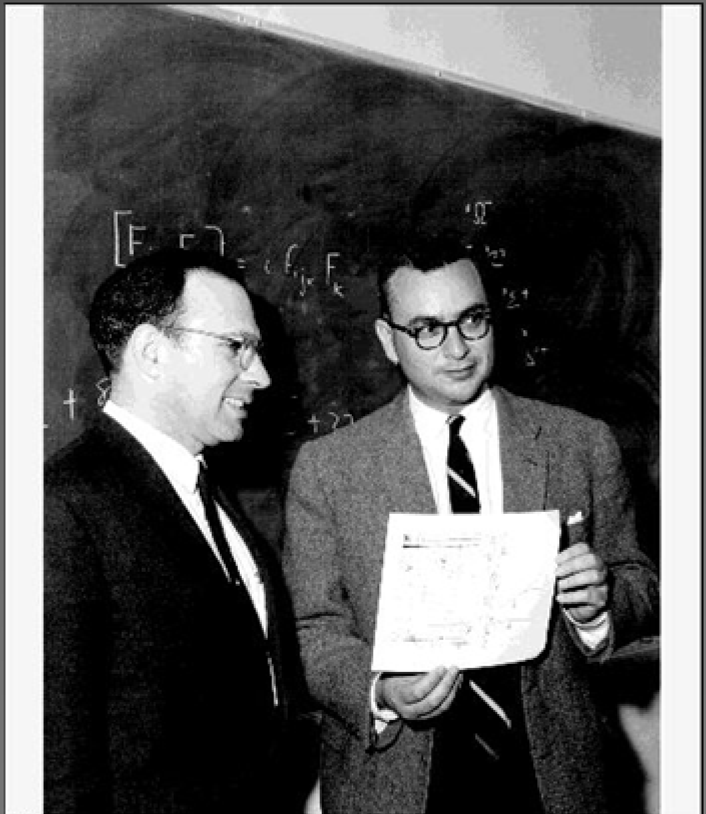
The Eightfold Way

In 1961, Murray Gell-Mann (1929) and Yuval Ne'eman (1925-2006) proposed independently to classify the strongly interacting particles (hadrons) according to their electric charge, isospin and strangeness into multiplets of SU(3) flavor:

Mesons: octet and singlet

Baryons: octet and decuplet

Periodic table of elementary particles a la Mendeleev!



SU(3) Flavor Symmetry

- Extend SU(2) isospin symmetry to SU(3) flavor symmetry

Algebras :	$SU(3)$	$\supset$	$[SU(2)$	$\supset$	$SO(2)]$	$\otimes$	$U(1)$
Generators :	$\hat{I}_\pm, \hat{I}_3, \hat{Y}, \hat{U}_\pm, \hat{V}_\pm$		$\hat{I}_\pm, \hat{I}_3$		$\hat{I}_3$		$\hat{Y}$
Labels :	$\mid (\lambda, \mu)$	,	I	,	I_3	,	$Y\rangle$

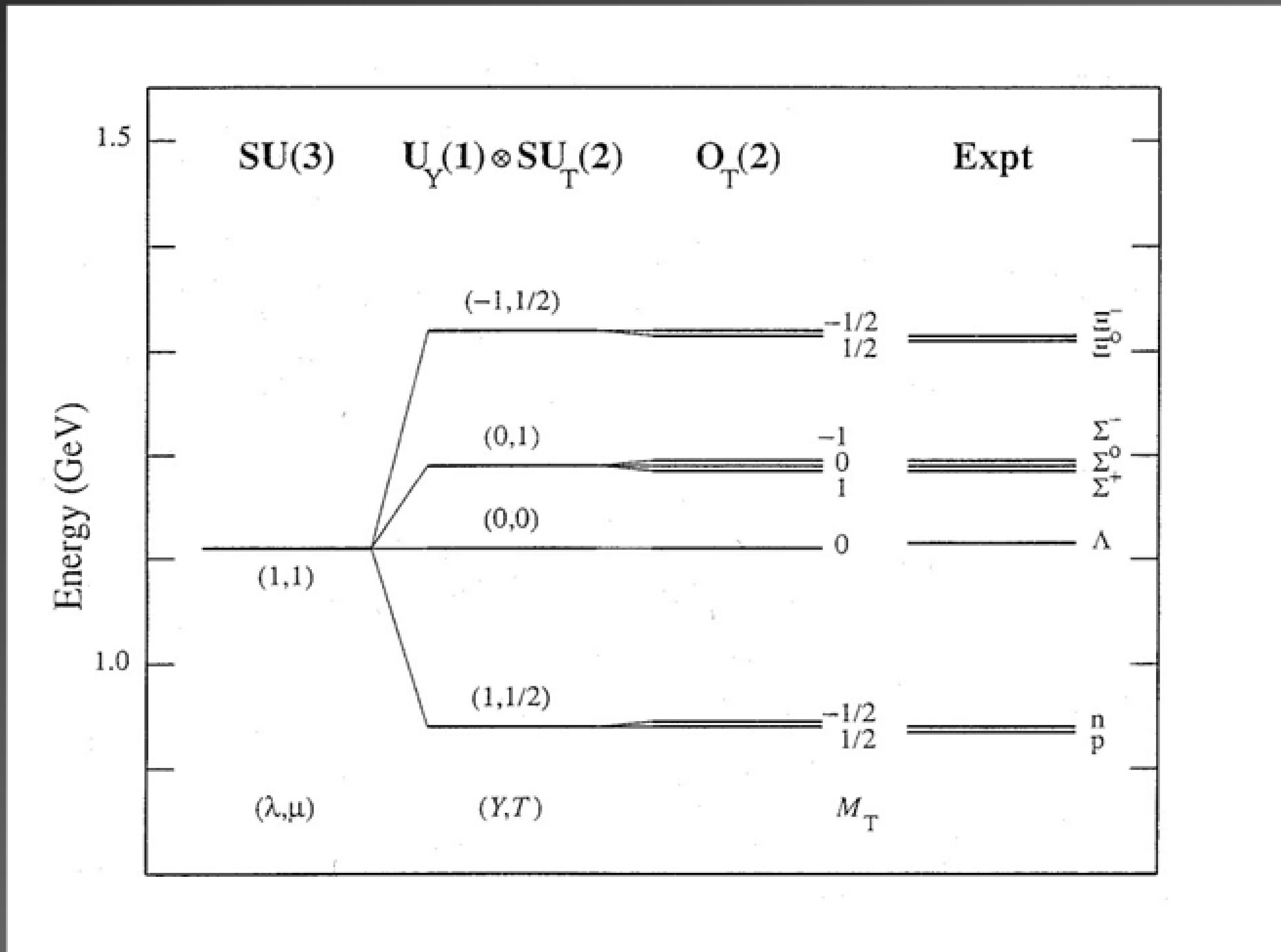
- Mass operator *M* with SU(3) dynamical symmetry: Gell-Mann-Okubo mass formula

$$\hat{M}|(\lambda,\mu)II_3Y\rangle = M|(\lambda,\mu)II_3Y\rangle$$

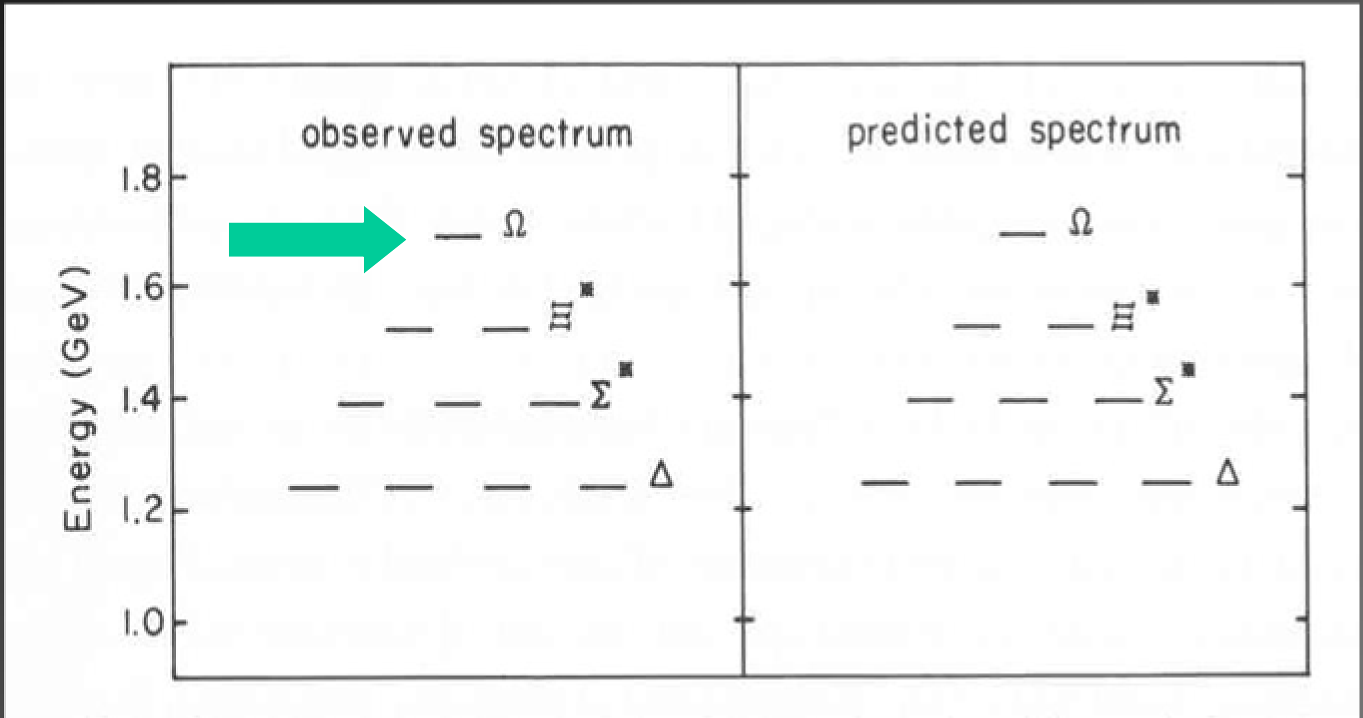
$$M = \kappa C(\lambda,\mu) + aY + b\left[I(I+1) - \frac{Y^2}{4}\right] + cI_3 + dI_3^2$$

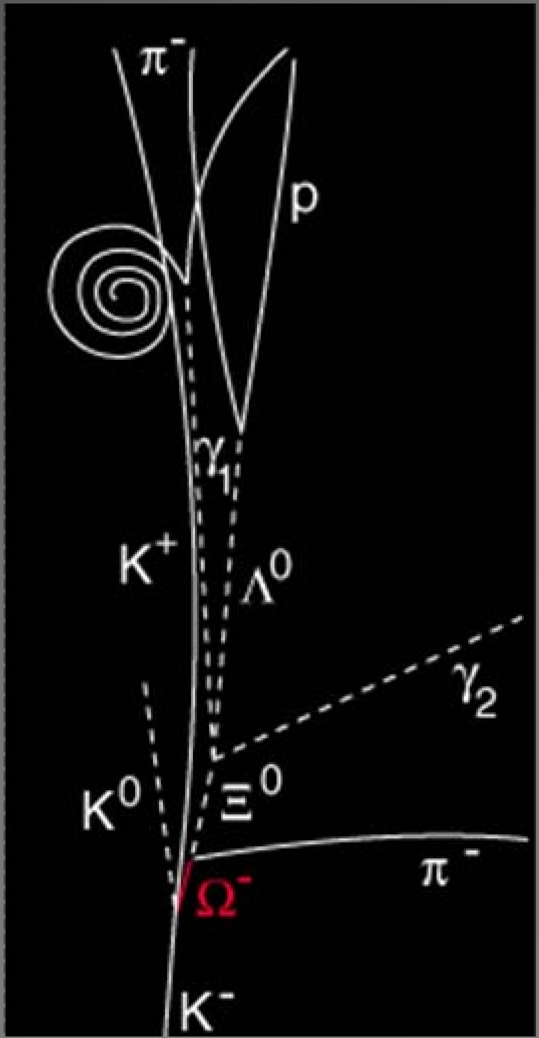
M. Gell-Mann, Phys. Rev. 125, 1067 (1962)
S. Okubo, Prog. Theor. Phys. 27, 949 (1962)

Baryon Octet

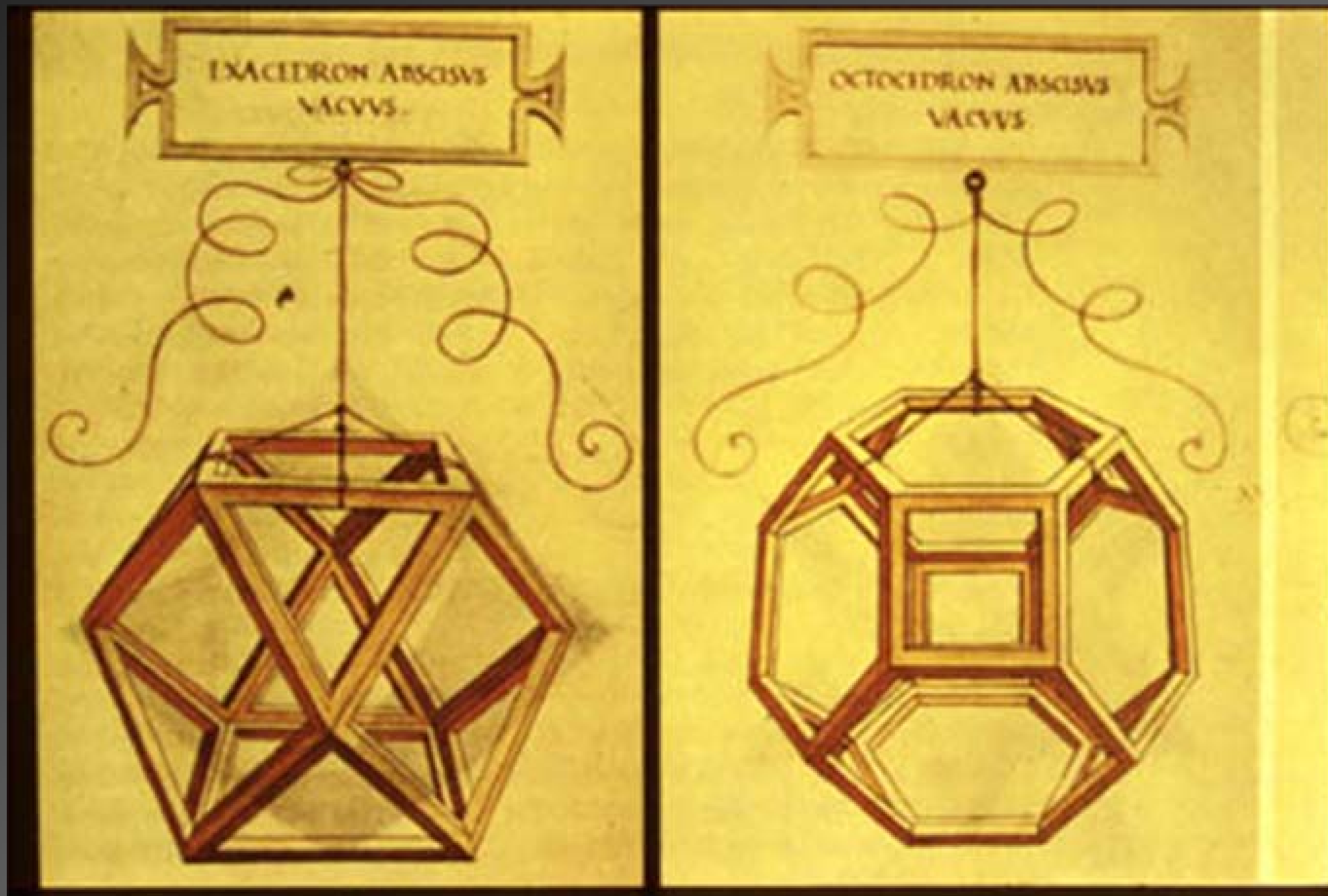


Baryon Decuplet





In 1964, Barnes et al. discovered the Ω^- at Brookhaven precisely as Gell-Mann and Ne'eman had predicted!

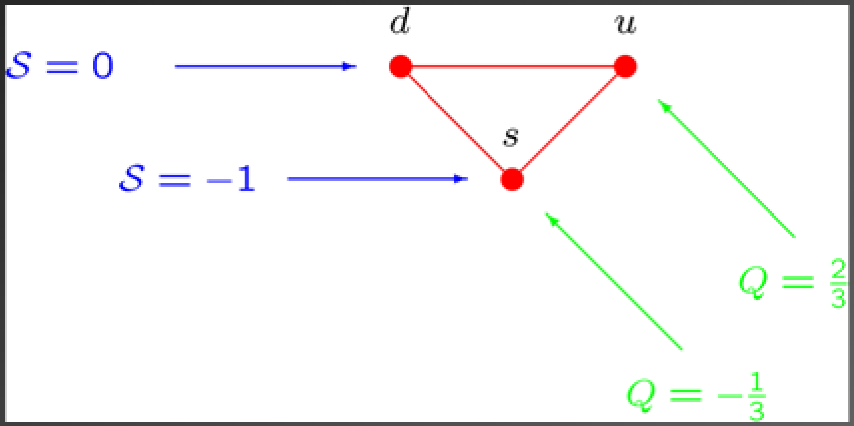

EXACEDRON ABSCISVS VACVVS.
OCTOCEDRON ABSCISVS VACVVS

Quarks

In 1964, Murray Gell-Mann (1929) and George Zweig (1937) independently introduced the idea of quarks and suggested that baryons consists out of three quarks and mesons out of a quark and an antiquark.

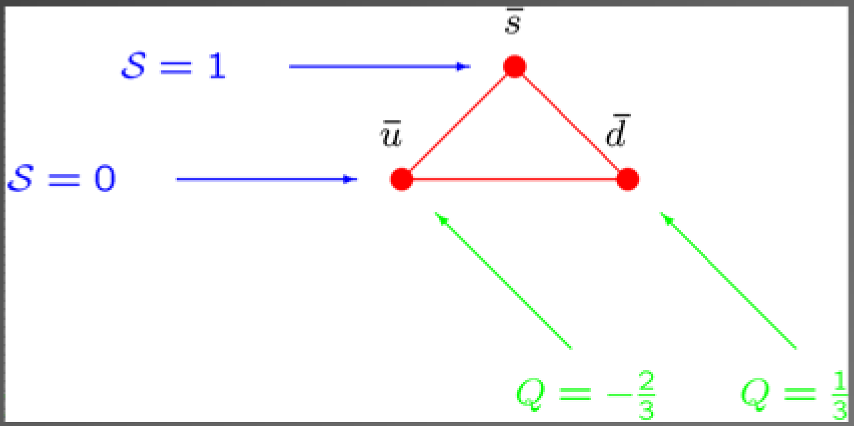
James Joyce – Finnegan's Wake
"Three quarks for Muster Mark"

Quarks and Antiquarks



Quark triplet $\mathbf{3}$

up
down
strange



Antiquark antitriplet $\bar{\mathbf{3}}$

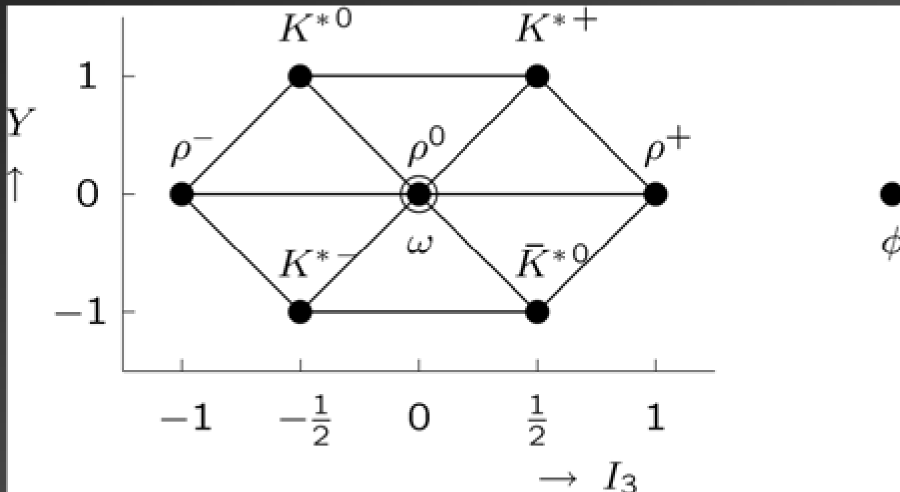
anti-up
anti-down
anti-strange

Quantum Numbers

B	baryon number
J^P	spin and parity
$I,\ I_3$	isospin
$\mathcal{S}$	strangeness
$Y = B + \mathcal{S}$	hypercharge
$Q = I_3 + \frac{Y}{2}$	electric charge

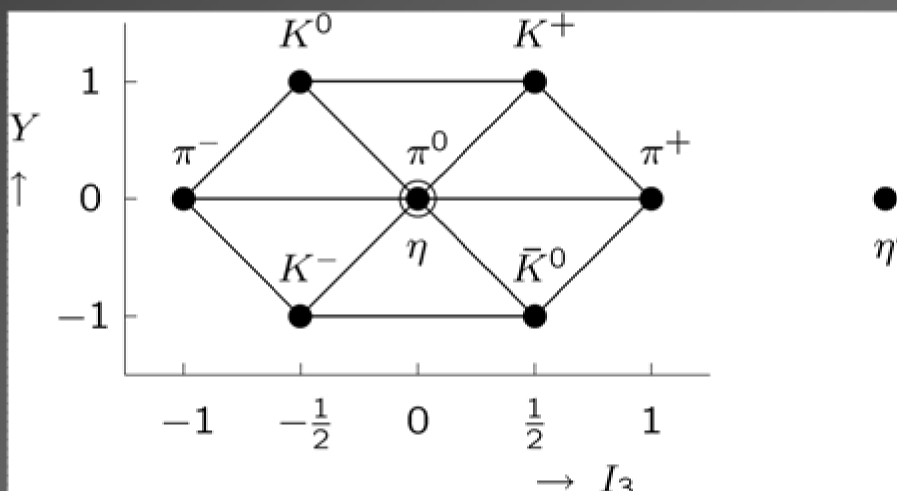
	B	J^P	I	I_3	$\mathcal{S}$	Y	Q
u	$\frac{1}{3}$	$\frac{1}{2}^+$	$\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{1}{3}$	$\frac{2}{3}$
d	$\frac{1}{3}$	$\frac{1}{2}^+$	$\frac{1}{2}$	$-\frac{1}{2}$	0	$\frac{1}{3}$	$-\frac{1}{3}$
s	$\frac{1}{3}$	$\frac{1}{2}^+$	0	0	-1	$-\frac{2}{3}$	$-\frac{1}{3}$
$\bar{u}$	$-\frac{1}{3}$	$\frac{1}{2}^-$	$\frac{1}{2}$	$-\frac{1}{2}$	0	$-\frac{1}{3}$	$-\frac{2}{3}$
$\bar{d}$	$-\frac{1}{3}$	$\frac{1}{2}^-$	$\frac{1}{2}$	$\frac{1}{2}$	0	$-\frac{1}{3}$	$\frac{1}{3}$
$\bar{s}$	$-\frac{1}{3}$	$\frac{1}{2}^-$	0	0	1	$\frac{2}{3}$	$\frac{1}{3}$

Mesons



$$q\bar{q} \equiv \mathbf{3} \otimes \bar{\mathbf{3}} = \mathbf{8} \oplus \mathbf{1}$$

Vector meson
octet and singlet
$J^P = 1^-$



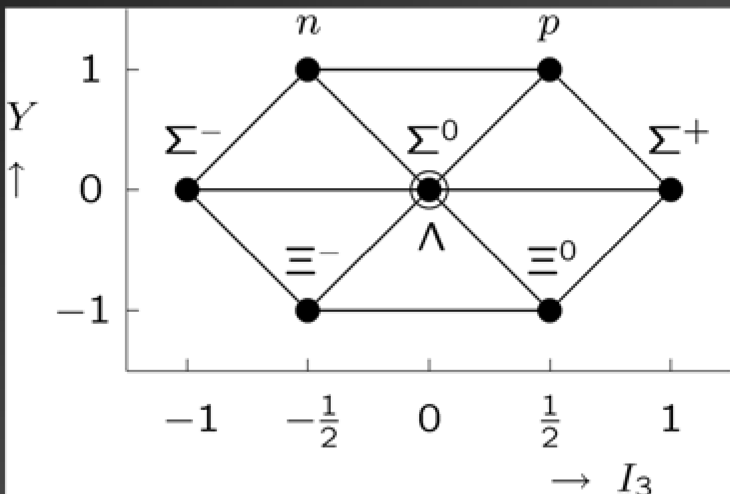
Pseudoscalar meson
octet and singlet
$J^P = 0^-$

Meson Quark Content

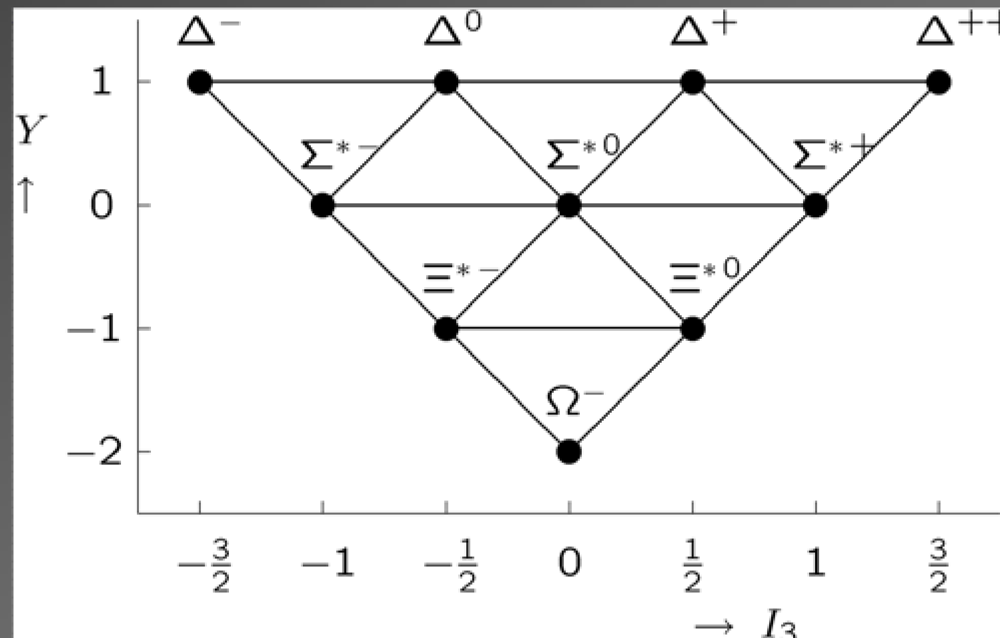
	pseudoscalar	vector
$-d\bar{s}$	K^0	K^{*0}
$-u\bar{s}$	K^+	K^{*+}
$d\bar{u}$	π^-	ρ^-
$\frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$	π^0	ρ^0
$-u\bar{d}$	π^+	ρ^+
$s\bar{u}$	K^-	K^{*-}
$-s\bar{d}$	$\bar{K}^0$	$\bar{K}^{*0}$
$\frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s})$	η_8	ω_8
$\frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s})$	η_1	ω_1

Baryons

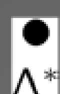
$$qqq \equiv \mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} = \mathbf{10} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{1}$$



Baryon octet
$J^P = 1/2^+$



Baryon decuplet
$J^P = 3/2^+$



Baryon singlet
$J^P = 1/2^-$

Baryon Quark Content

	octet	decuplet	singlet
uuu		Δ^{++}	
uud	p	Δ^{+}	
udd	n	Δ^{0}	
ddd		Δ^{-}	
uus	Σ^{+}	Σ^{*+}	
uds	Σ^{0}, Λ	Σ^{*0}	Λ^{*}
dds	Σ^{-}	Σ^{*-}	
uss	Ξ^{0}	Ξ^{*0}	
dss	Ξ^{-}	Ξ^{*-}	
sss		Ω^{-}	

qqq Baryons

Internal degrees of freedom

spin	$\uparrow, \downarrow$
flavor	u, d, s
color	r, g, b

Spatial degrees of freedom: Jacobi vectors

$$\vec{\rho} = \frac{1}{\sqrt{2}}(\vec{r}_1 - \vec{r}_2)$$

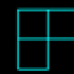
$$\vec{\lambda} = \frac{1}{\sqrt{6}}(\vec{r}_1 + \vec{r}_2 - 2\vec{r}_3)$$

Baryon wave function

Ψ = Ψ(space) χ(spin) ϕ(flavor) ψ(color)

Permutation Symmetry

Three identical objects

Young tableau	S_3	D_3
	S	A_1
	M	E
	A	A_2

color	singlet	ψ_{A_2}
flavor	decuplet	ϕ_{A_1}
	octet	ϕ_E
	singlet	ϕ_{A_2}
spin	$S=\frac{3}{2}$	χ_{A_1}
	$S=\frac{1}{2}$	χ_E
space		$\Psi_{A_1}, \Psi_E, \Psi_{A_2}$

Flavor Wave Functions

$$qqq \equiv \mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} = \mathbf{10} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{1}$$

decuplet : $\phi_{A_1}(\Delta^{++}) = |uuu\rangle$

decuplet : $\phi_{A_1}(\Delta^{+}) = \frac{1}{\sqrt{3}}\left[|uud\rangle + |udu\rangle + |duu\rangle\right]$

octet : $\phi_{E_\rho}(p) = \frac{1}{\sqrt{2}}\left[|udu\rangle - |duu\rangle\right]$

octet : $\phi_{E_\lambda}(p) = \frac{1}{\sqrt{6}}\left[2\,|uud\rangle - |udu\rangle - |duu\rangle\right]$

Spin Wave Functions

$$\frac{1}{2} \otimes \frac{1}{2} \otimes \frac{1}{2} = \frac{1}{2} \oplus \frac{1}{2} \oplus \frac{3}{2}$$

$$|S, M_S\rangle = \left|\frac{3}{2}, \frac{3}{2}\right\rangle : \quad \chi_{A_1}(q^3) \;=\; |\uparrow\uparrow\uparrow\rangle$$

$$|S, M_S\rangle = \left|\frac{3}{2}, \frac{1}{2}\right\rangle : \quad \chi_{A_1}(q^3) \;=\; \frac{1}{\sqrt{3}}\Big[|\uparrow\uparrow\downarrow\rangle + |\uparrow\downarrow\uparrow\rangle + |\downarrow\uparrow\uparrow\rangle\Big]$$

$$\left|\frac{1}{2}, \frac{1}{2}\right\rangle : \quad \chi_{E_\rho}(q^3) \;=\; \frac{1}{\sqrt{2}}\Big[|\uparrow\downarrow\uparrow\rangle - |\downarrow\uparrow\uparrow\rangle\Big]$$

$$\left|\frac{1}{2}, \frac{1}{2}\right\rangle : \quad \chi_{E_\lambda}(q^3) \;=\; \frac{1}{\sqrt{6}}\Big[2\,|\uparrow\uparrow\downarrow\rangle - |\uparrow\downarrow\uparrow\rangle - |\downarrow\uparrow\uparrow\rangle\Big]$$

Baryon Wave Functions

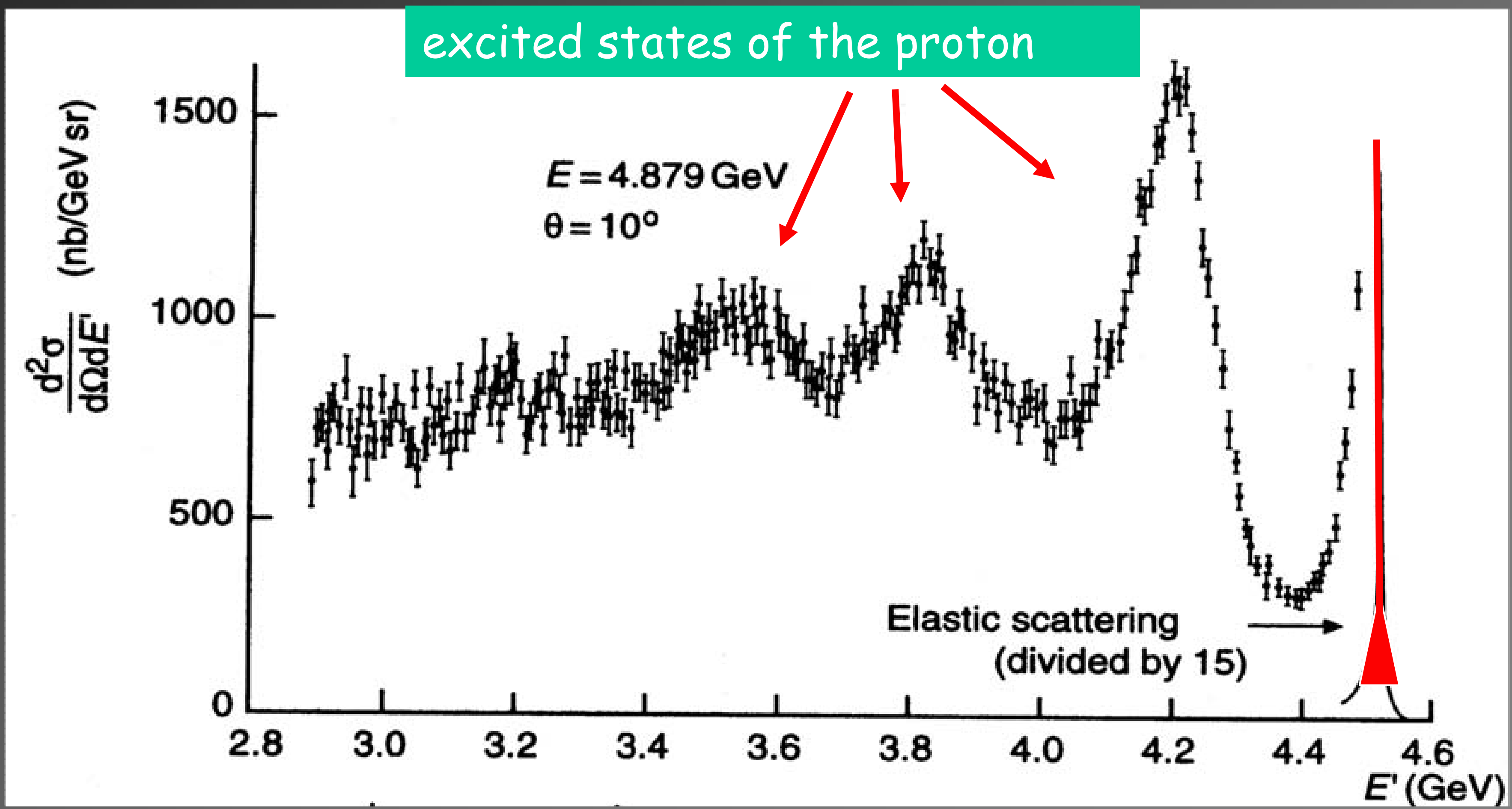
- Internal and spatial degrees of freedom
- Physical states are color singlets: ψ_{A_2}(color)
- Pauli principle: baryon wave function is antisymmetric

$$\psi = \underbrace{\psi(\text{space})\phi(\text{flavor})\chi(\text{spin})}_{A_1}\underbrace{\psi(\text{color})}_{A_2}$$

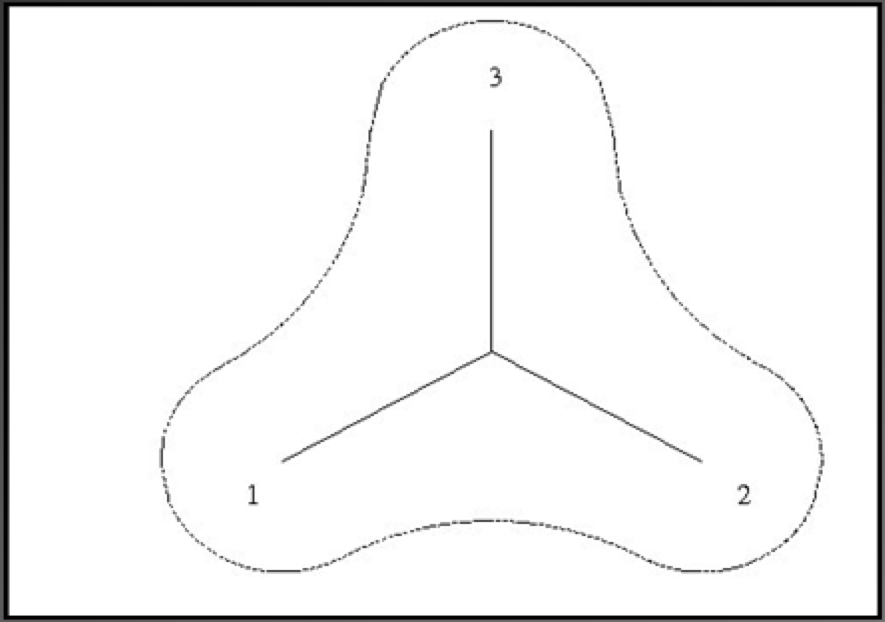
- Baryon wave function is symmetric in spin-flavor-spatial coordinates

$$\begin{aligned} \psi_{\text{oct}} &= \frac{1}{\sqrt{2}}\psi_{A_1}\left(\phi_{E_\rho}\chi_{E_\rho} + \phi_{E_\lambda}\chi_{E_\lambda}\right) \\ \psi_{\text{dec}} &= \psi_{A_1}\phi_{A_1}\chi_{A_1} \end{aligned}$$

Excited Nucleons



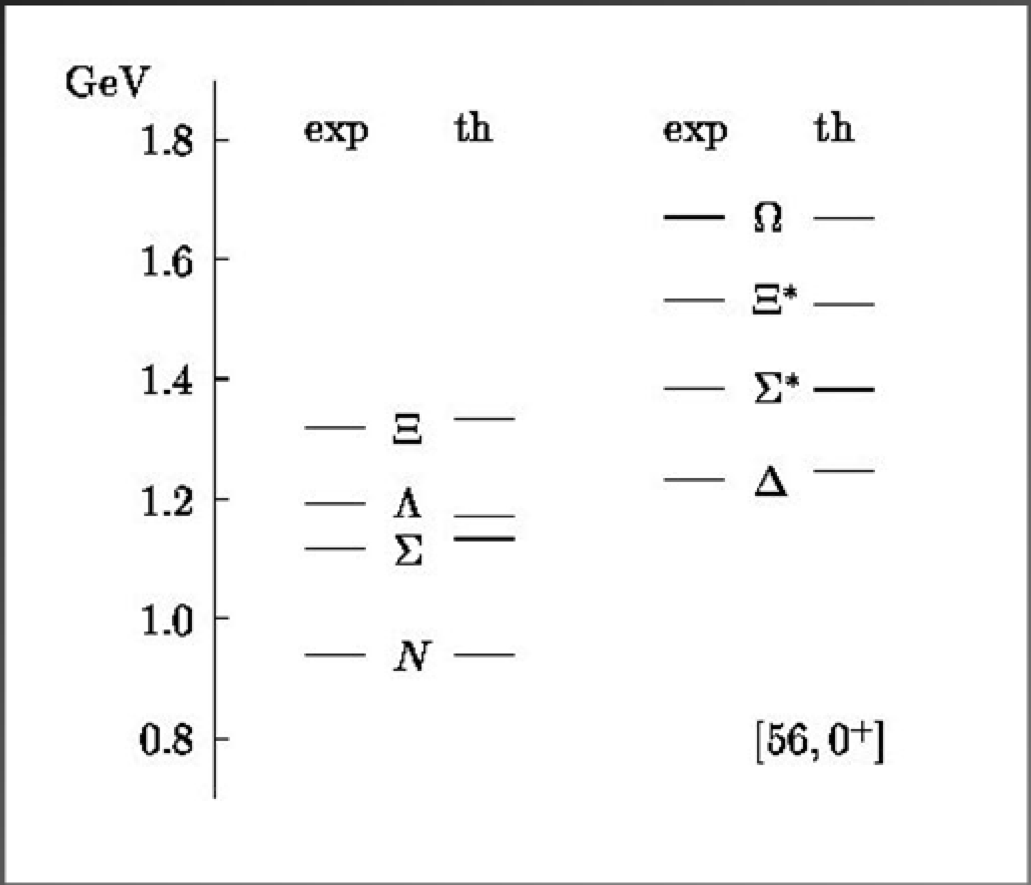
Stringlike Model of Baryons



Mass formula (dynamical symmetry)

$$M^2 = M_0^2 + M^2(\text{space}) + M^2(\text{spin} - \text{flavor})$$

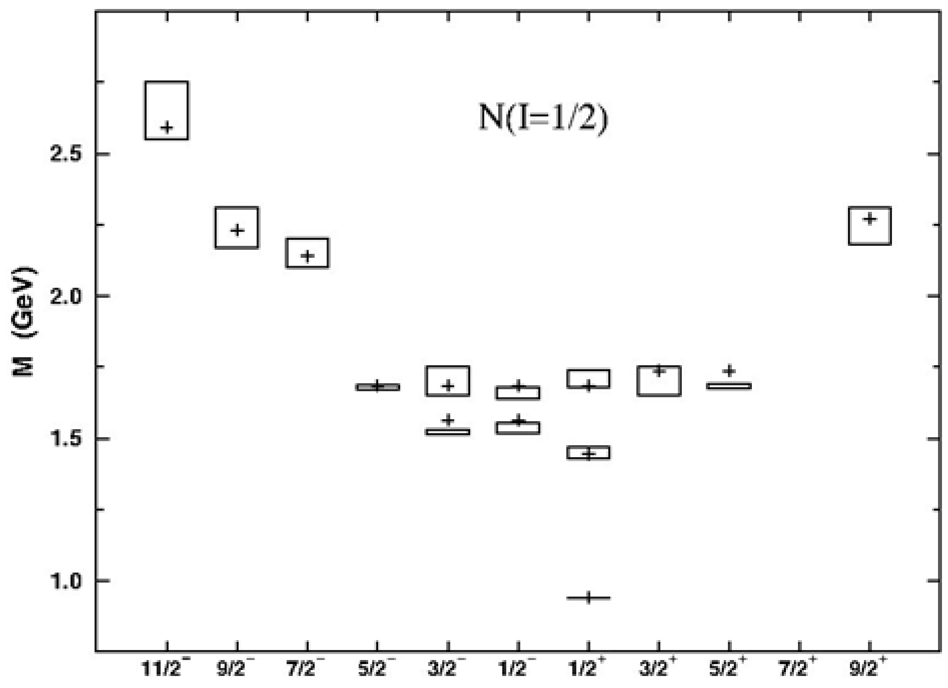
Ground State Baryons



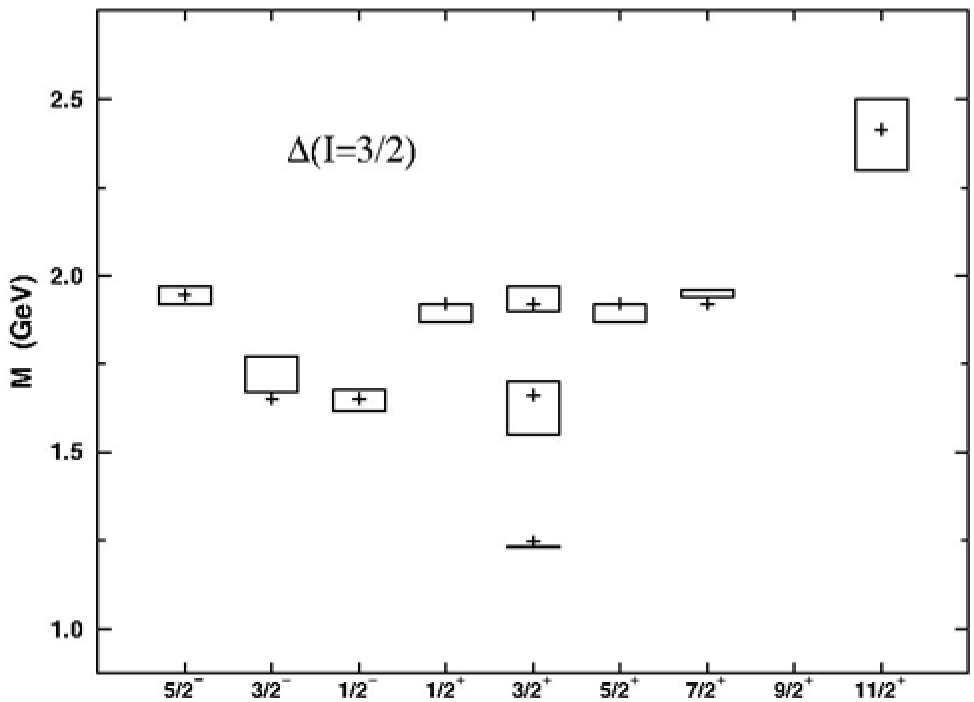
octet decuplet

Bijker, Iachello, Leviatan
Ann. Phys. 236, 69 (1994)
Ann. Phys. 284, 89 (2000)

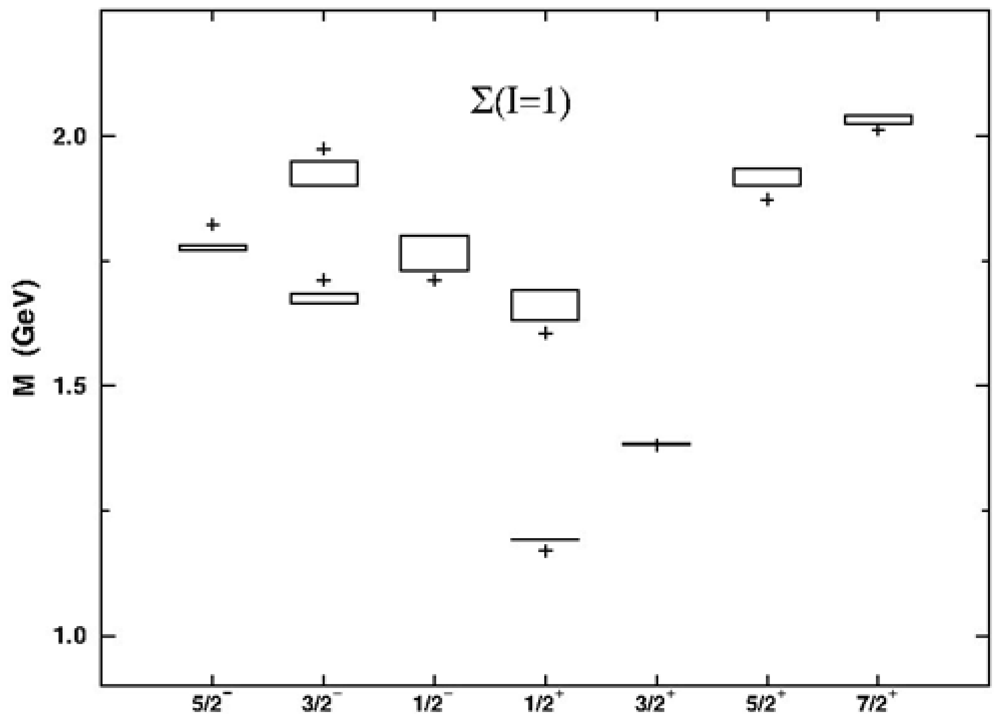
Nucleon Resonances



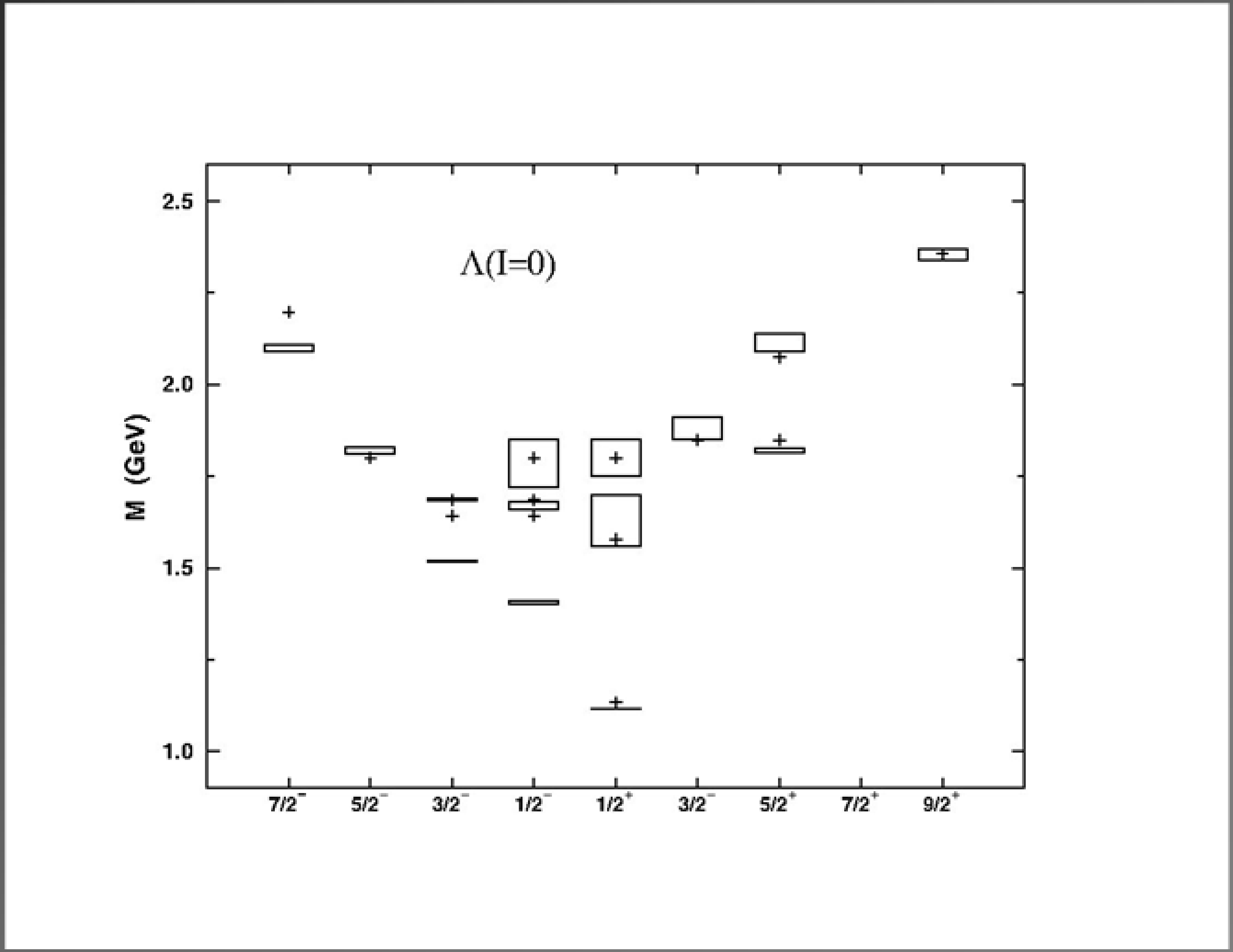
Delta Resonances



Sigma Resonances



Lambda Resonances



Magnetic Moments

$$\vec{\mu} = \vec{\mu}_{\text{spin}} + \vec{\mu}_{\text{orb}}$$
$$= \sum_i \mu_i (2\vec{s}_i + \vec{\ell}_i)$$
$$\mu_i = \frac{e_i}{2m_i}$$

Ground state baryons:
Ψ_{A_1}(space,$L^P=0^+$)
→ no contribution
from orbital part

For example:

$$\mu_{\Delta^{++}} = \left\langle \Delta^{++} \middle| \mu_z \middle| \Delta^{++} \right\rangle$$
$$= 3 \left\langle \Delta^{++} \middle| 2\mu_3 s_{3,z} \middle| \Delta^{++} \right\rangle = 3\mu_u$$

with $\left|\Delta^{++}\right\rangle = \left|u\uparrow u\uparrow u\uparrow\right\rangle$

Nucleon Magnetic Moment

Proton $\mu_p = \langle p | \mu_z | p \rangle = (4\mu_u - \mu_d)/3$

with $|p\rangle = \frac{1}{\sqrt{2}} \left(\phi_{E_\rho}(p) \chi_{E_\rho} + \phi_{E_\lambda}(p) \chi_{E_\lambda} \right)$

Neutron $\mu_n = (4\mu_d - \mu_u)/3$

Isospin symmetry: $\mu_u = -2\mu_d$ $\frac{\mu_n}{\mu_p} = \frac{4\mu_d - \mu_u}{4\mu_u - \mu_d} = -\frac{2}{3}$

Experimental value: $\frac{\mu_n}{\mu_p} = -\frac{1.913}{2.793} = -0.685$

	μ_{th}	μ_{calc}	μ_{exp}
p	$(4\mu_u - \mu_d)/3$	2.793	2.793
n	$(4\mu_d - \mu_u)/3$	−1.913	−1.913
Λ	μ_s	−0.613	-0.613 ± 0.004
Σ^+	$(4\mu_u - \mu_s)/3$	2.674	2.458 ± 0.010
Σ^0	$(2\mu_u + 2\mu_d - \mu_s)/3$	0.791	
Σ^-	$(4\mu_d - \mu_s)/3$	−1.092	-1.160 ± 0.025
Ξ^0	$(4\mu_s - \mu_u)/3$	−1.435	-1.250 ± 0.014
Ξ^-	$(4\mu_s - \mu_d)/3$	−0.493	-0.651 ± 0.003

	μ_{th}	μ_{calc}	μ_{exp}
Δ^{++}	$3\mu_u$	5.556	5.6 ± 1.9
Δ^{+}	$2\mu_u + \mu_d$	2.732	
Δ^{0}	$\mu_u + 2\mu_d$	−0.092	
Δ^{-}	$3\mu_d$	−2.916	
$\Sigma^{*,+}$	$2\mu_u + \mu_s$	3.091	
$\Sigma^{*,0}$	$\mu_u + \mu_d + \mu_s$	0.267	
$\Sigma^{*,-}$	$2\mu_d + \mu_s$	−2.557	
$\Xi^{*,0}$	$\mu_u + 2\mu_s$	0.626	
$\Xi^{*,-}$	$\mu_d + 2\mu_s$	−2.198	
Ω^{-}	$3\mu_s$	−1.839	-2.02 ± 0.05

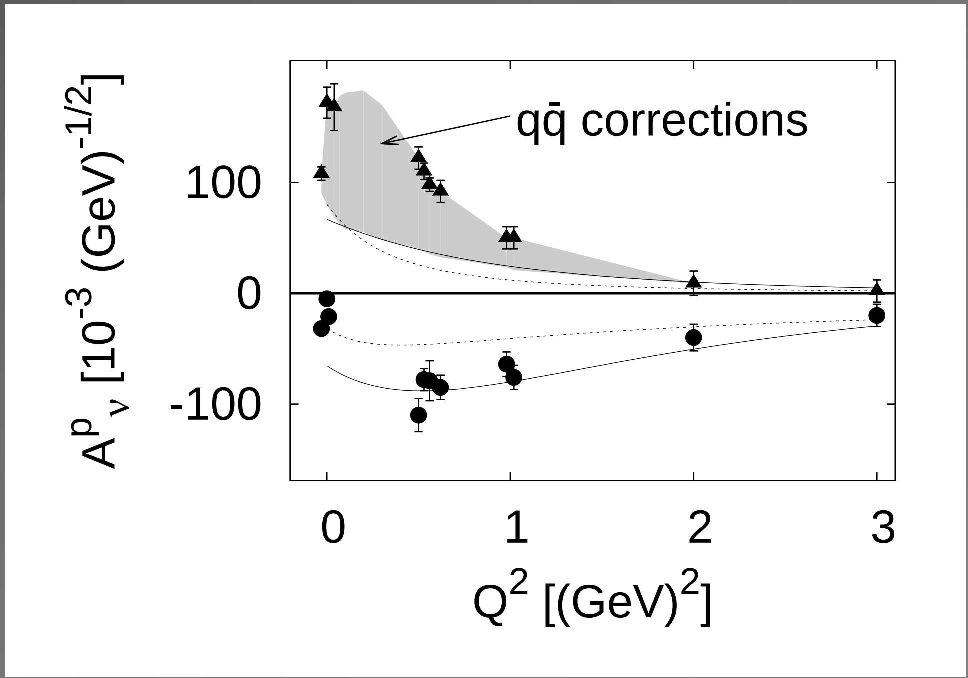
Electromagnetic Couplings

$$\mathcal{H}\mathrm{em} = -\sum_{j=1}^{3}\left[\frac{e_j}{2m_j}(\vec{p}_j\cdot\vec{A}_j+\vec{A}_j\cdot\vec{p}_j)+2\mu_j\vec{s}_j\cdot(\vec{\nabla}\times\vec{A}_j)\right]$$

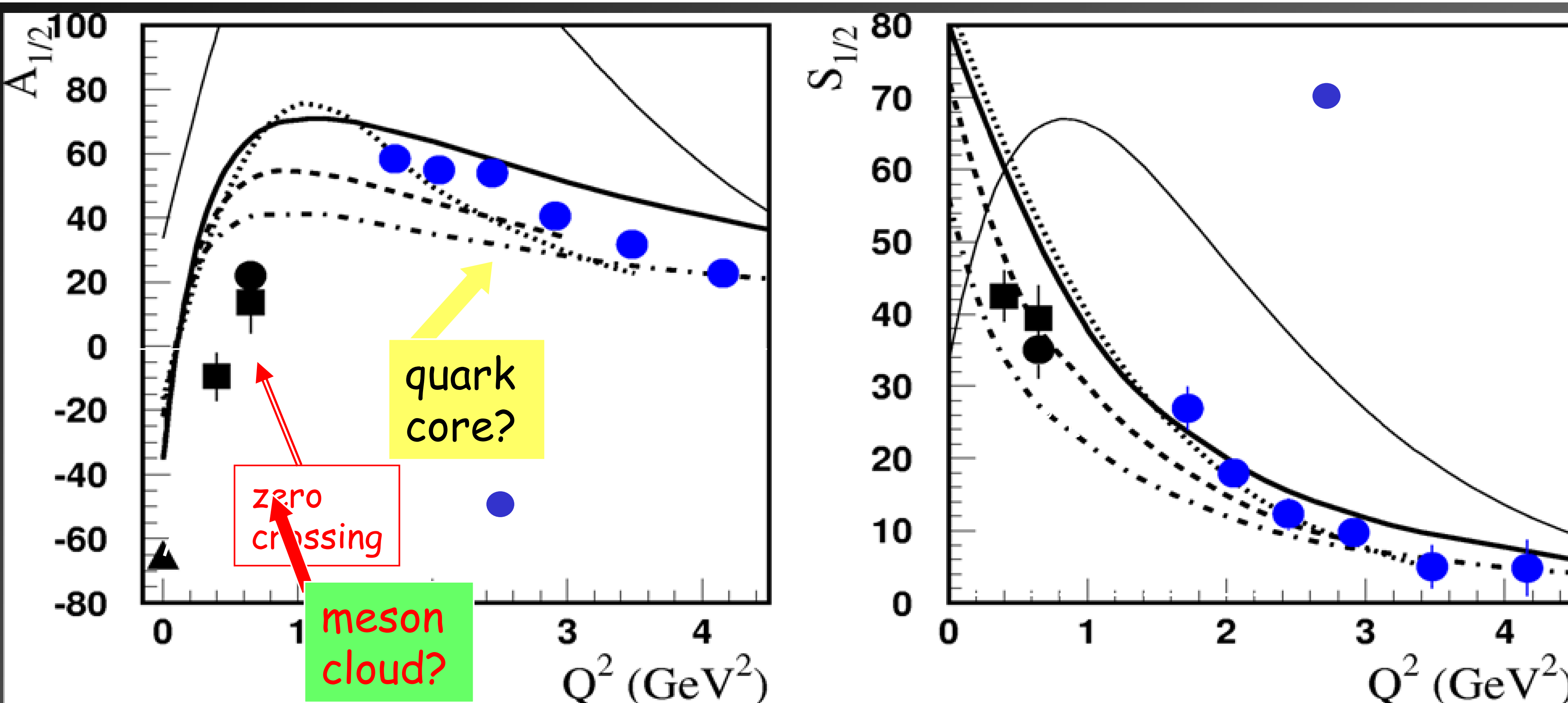
N(1520)D_{13}

$$A_{1/2} = 2i\sqrt{\frac{\pi}{k_0}}\mu\frac{1}{(1+k^2a^2)^2}\left[\frac{m_qk_0a}{g}-k^2a\right]$$

$$A_{3/2} = 2i\sqrt{3}\sqrt{\frac{\pi}{k_0}}\mu\frac{1}{(1+k^2a^2)^2}\frac{m_qk_0a}{g}$$

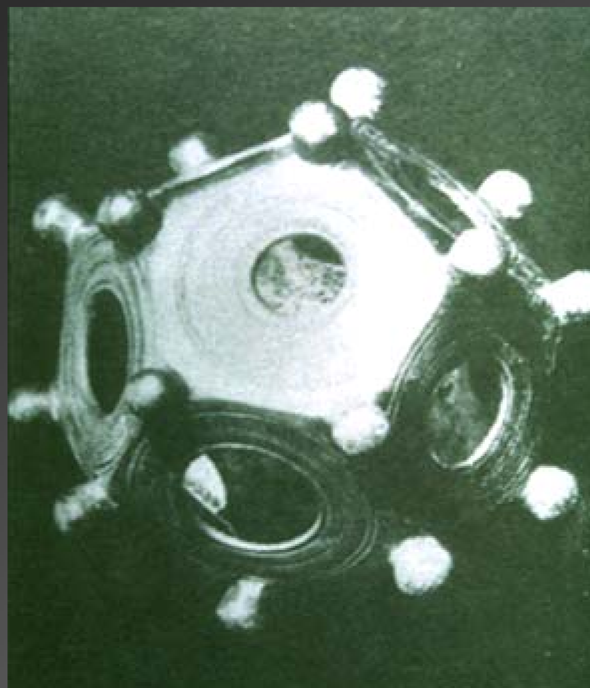


CLAS Nature of the Roper $N(1440)P_{11}$?



- Roper is not a gluonic excitation Q^3G
- At short distances consistent with Q^3- radial excitation
- At large distances meson couplings may be important

Ungaro – MENU 2007



Etruscan Dodecahedron
(Padova)

San Marco Cathedral
(Venice)

Summary and Conclusions

- Successes of the CQM
- Masses, magnetic moments, electromagnetic couplings
- But, also some systematic deviations
- Transition form factors at large distances, strong couplings
- Exotic (non-qqq) degrees of freedom

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