Unitarized Chiral Theory of Two Hadrons in a Finite Volume

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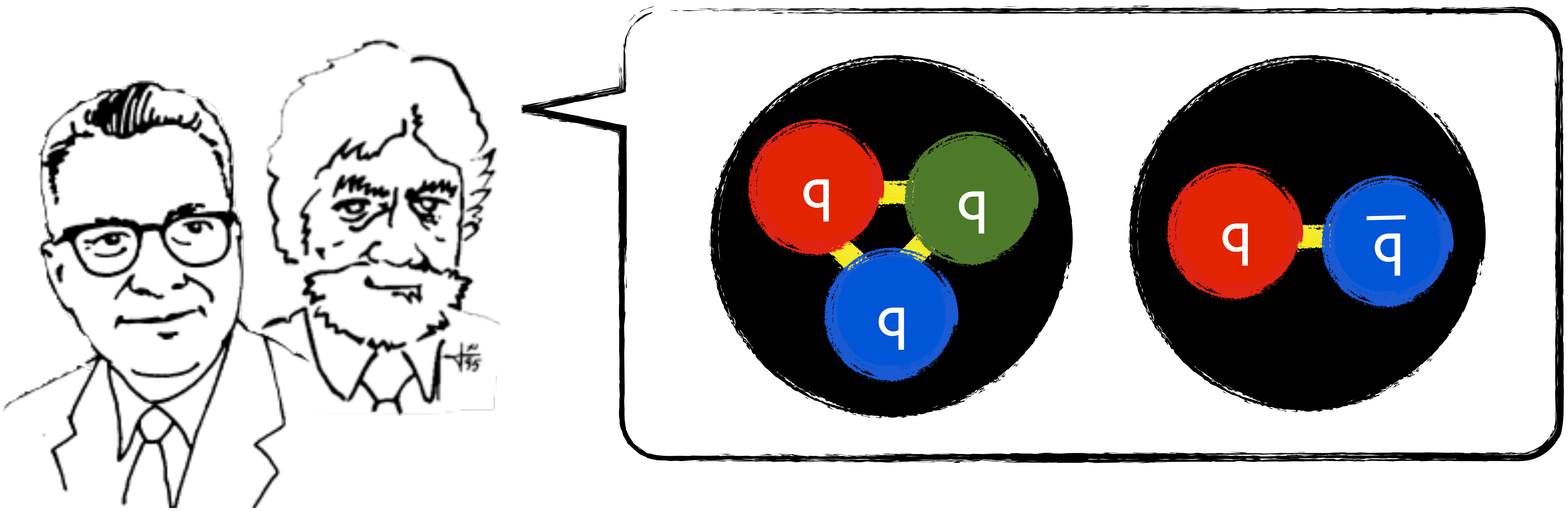
INTRODUCTION

- Quark model of Gell-Mann and Zweig:



- Resonances: excitations of these quarks from the ground state to different high energy levels.

- This picture is too simple to describe the properties of all the hadrons found in Nature

N*(1440) → Lowest excited state of the Nucleon observed

N*(1535) → Lowest excited state of the Nucleon based on a three quark model

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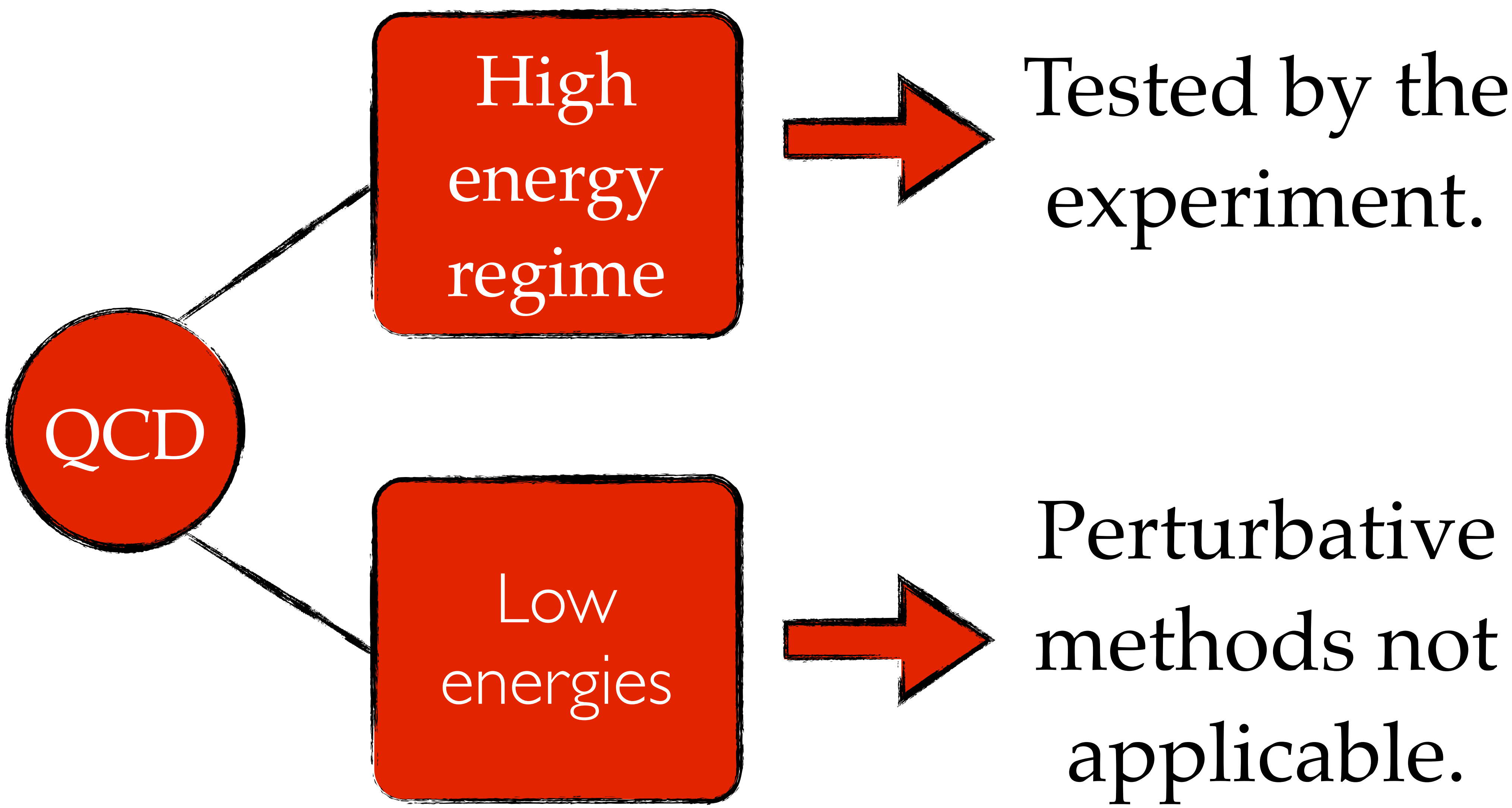
N*(1535) $\rightarrow$ **It requires 600 MeV of energy**

- There is enough energy to create one or two pions or an eta meson.

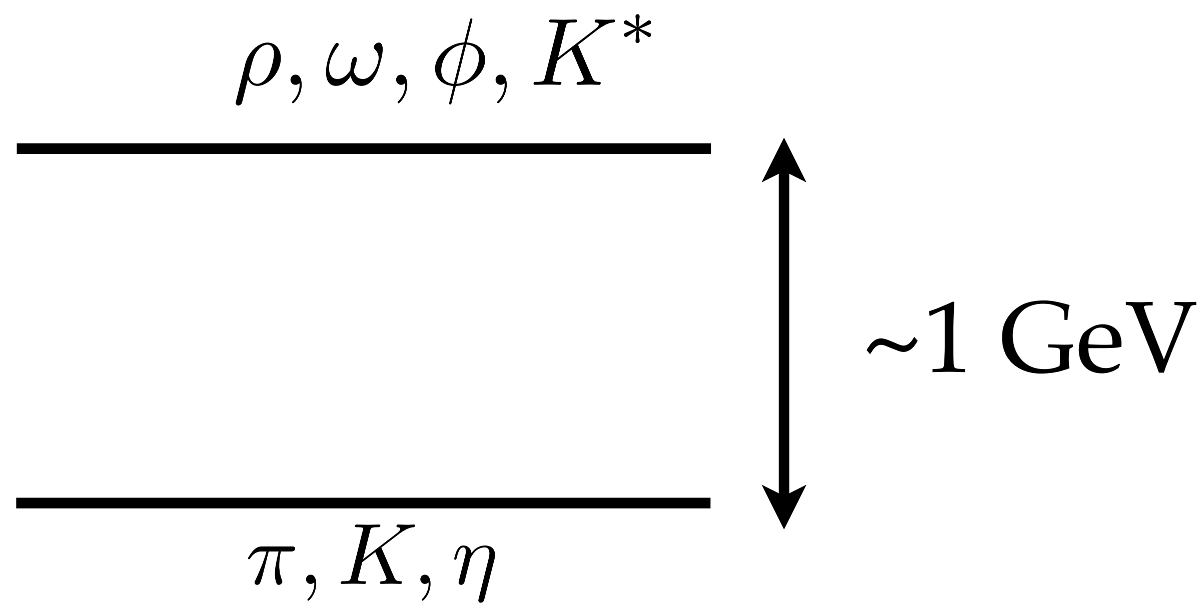
$$|N^*(1535) >= \alpha|qqq > + \beta|\pi N > + \cdots$$

- QCD is the theory for the strong interaction.



- In the low energy region there is an interesting fact: isospin triplet with a mass much smaller than the rest of the QCD states.

- Extension to SU(3): lowest octect of pseudoscalar states (π, K, η).



Presence of a chiral symmetry in the light quark sector (u, d, s) which breaks down spontaneously

- QCD Lagrangian:

$$\mathcal{L}_{\rm QCD} = \bar{q}\left(i\gamma^\mu D_\mu - m_q\right)q - \frac{1}{2}\mathrm{Tr}_c\left(G^{\mu\nu}G_{\mu\nu}\right)$$

$$q^T = (u, d, s, c, b, t)$$

$$D_\mu = \partial_\mu - igG_\mu$$

$$G_{\mu\nu} = \partial_\mu G_\nu - \partial_\nu G_\mu - ig\left[G_\mu, G_\nu\right]$$

Low energies ➡ c, b,t infinitely heavy

Massless quarks ➡ chiral limit

$$\mathcal{L}_{\rm QCD} = \sum_{q=u,d,s}\left(\bar{q}_L i\gamma_\mu D^\mu q_L + \bar{q}_R i\gamma_\mu D^\mu q_R\right)$$

$$q_{R/L} = \frac{1}{2}(1 \pm \gamma_5)\,q$$

- Invariant under:

$$q_R \longrightarrow R\, q_R \qquad\qquad q_L \longrightarrow L\, q_L$$

$$R = e^{\theta_R^a \lambda^a} \qquad\qquad L = e^{\theta_L^a \lambda^a}$$

$$R \in \mathrm{SU}(3)_R \qquad\qquad L \in \mathrm{SU}(3)_L$$

$\mathrm{SU}(3)_L \times \mathrm{SU}(3)_R$ chiral symmetry

$$L_\mu^a = \sum_{q=u,d,s} \bar{q}_L \gamma_\mu \frac{\lambda^a}{2} q_L \qquad\qquad R_\mu^a = \sum_{q=u,d,s} \bar{q}_R \gamma_\mu \frac{\lambda^a}{2} q_R$$

- Associated charges: $Q_V^a = Q_R^a + Q_L^a \qquad Q_A^a = Q_R^a - Q_L^a$

$$Q_V^a \to Q_V^a \qquad Q_A^a \to -Q_A^a$$

$$\frac{dQ_{V,A}^a}{dt} = i[H_{QCD}, Q_{V,A}^a] = 0$$

$$\Downarrow$$

$$H_{\rm QCD}|\psi\rangle = E|\psi\rangle$$

$$\underbrace{Q_V^a|\psi\rangle \text{ and } Q_A^a|\psi\rangle \Rightarrow \text{Same energy, opposite parity}}_{\text{Nambu-Goldstone realization}}$$

$$Q_V^a|0\rangle = 0 \qquad Q_A^a|0\rangle \neq 0$$

- Weinberg (1979): the most general Lagrangian containning all terms allowed by the assumed symmetries gives rise to the most general S-matrix consistent with analyticity, unitarity and the assumed symmetries.

- Lagangian PP --> PP:

$$\Phi = \frac{\vec{\lambda}}{\sqrt{2}}\vec{\phi} = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta_8 \end{pmatrix}$$

$$U(\Phi) = e^{i\sqrt{2}\Phi/f}$$

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm eff}(U, \partial U, \partial^2 U, \ldots)$$

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm eff}^{(0)} + \mathcal{L}_{\rm eff}^{(2)} + \mathcal{L}_{\rm eff}^{(4)} + \ldots$$

$$\mathcal{L}_2 = \frac{f^2}{4} < D_\mu U^\dagger D^\mu U + U^\dagger \mathcal{M} + \mathcal{M}^\dagger U >$$

$$M = \begin{pmatrix} m_\pi^2 & 0 & 0 \\ 0 & m_\pi^2 & 0 \\ 0 & 0 & 2m_K^2 - m_\pi^2 \end{pmatrix}$$

- Chiral perturbation Theory (χPT): series of Lagrangians in a power momentum expansion.

- Validity of the series: p << 1 GeV.

- Convergence limited to a narrow interval $\left\{\begin{array}{l}\text{500 MeV, meson-meson scattering.} \\ \sim \text{threshold, meson-baryon scattering.}\end{array}\right.$

- Consequence: we can not study resonances.

- Unitarization

χPT + Unitarity

UχPT

- Unitarization:

$$SS^\dagger = 1 \qquad S = 1 - iT$$

$$T - T^\dagger = -iTT^\dagger$$

$$< f|T|i > - < f|T^\dagger|i >= -i\sum_a \int d\mathcal{Q}_a < f|T|a >< a|T^\dagger|i >$$

$$< f|T|i >= T_{fi}(2\pi)^4\delta^4(\sum_f p_f - \sum_i p_i)$$

$$Im\{T^{-1}_{fi}\} = -\rho_{fi} = \frac{|\vec{p}_i|}{8\pi E}\delta_{fi}$$

- Dispersion relation:

$$T_{fi}^{-1} = V_{fi}^{-1} - \delta_{fi}\left[a_i(s_0) + \frac{s-s_0}{\pi}\int_{s_i}^{\infty} ds' \frac{\rho_{fi}(s')}{(s-s')(s'-s_0)}\right]$$

$$\equiv V_{fi}^{-1} - G_i(s)\delta_{fi}$$

$$T = (1-VG)^{-1}V$$

$$T - VGT = V$$

$$T = V + VGT$$

Bethe-Salpeter equation

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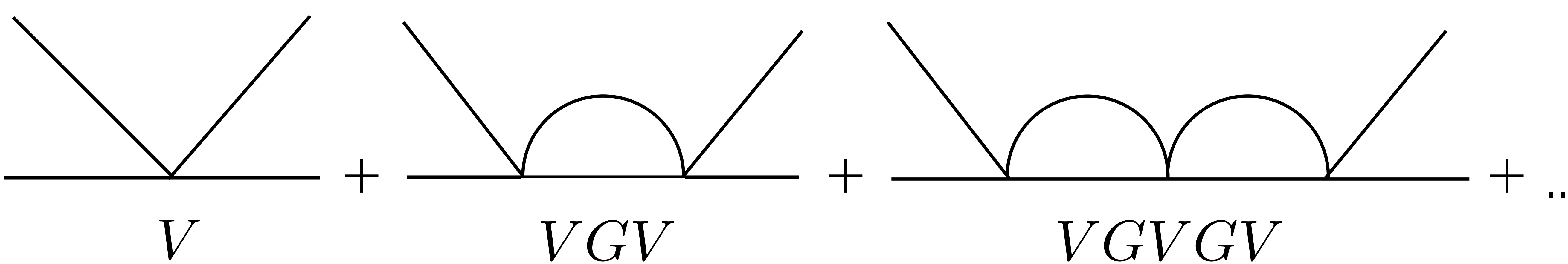


FORMALISM

- Lowest order chiral Lagrangian:

$$\mathcal{L}_{PB} = \frac{1}{4f^2}\langle \bar{B} i\gamma^{\mu}[(\partial_{\mu}\Phi\Phi - \Phi\partial_{\mu}\Phi))B - B(\partial_{\mu}\Phi\Phi - \Phi\partial_{\mu}\Phi))\rangle$$

$$\mathcal{L}_{PP} = \frac{1}{12f^2}\langle (\partial_{\mu}\Phi\Phi - \Phi\partial_{\mu}\Phi)^2 + M\Phi^4\rangle$$

$$\Phi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix} \quad B = \begin{pmatrix} \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda \end{pmatrix}$$

- We determine the lowest order amplitude:

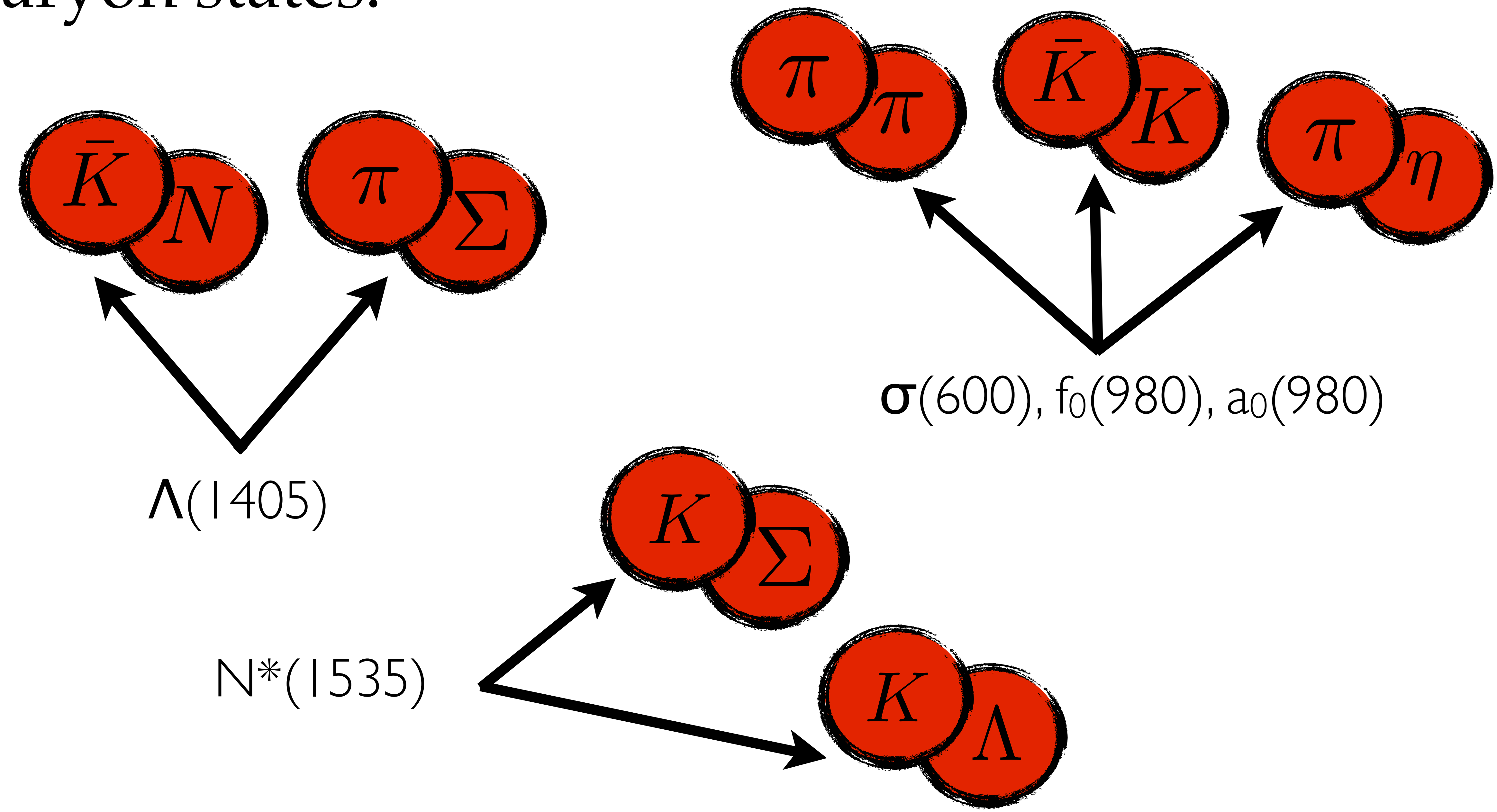
$$\mathcal{V}_{ij}(E) = -C_{ij}\frac{1}{4f_i f_j}(2E - M_i - M_j)\sqrt{\frac{M_i + E_i(E)}{2M_i}}\sqrt{\frac{M_j + E_j(E)}{2M_j}}$$

- We solve the Bethe-Salpeter equation:

$$T = [1 - \mathcal{V}G]^{-1}\mathcal{V}$$

$$G_i(E) = N_i \int \frac{d^4q}{(2\pi)^4}\frac{1}{q^2 - m^2}\frac{1}{(P-q)^2 - M^2} \qquad N_i = \begin{cases} 1 & \text{meson-meson} \\ 2M & \text{meson-baryon} \end{cases}$$

- The use of effective field theories based on UχPT has shed ligth on the nature of several meson and baryon states.



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- J. A. Oller, E. Oset, Nucl. Phys. A 620 (1997) 438 ; J. A. Oller, E. Oset, J. R. Peláez, Phys. Rev. D 59 074001 (1999).
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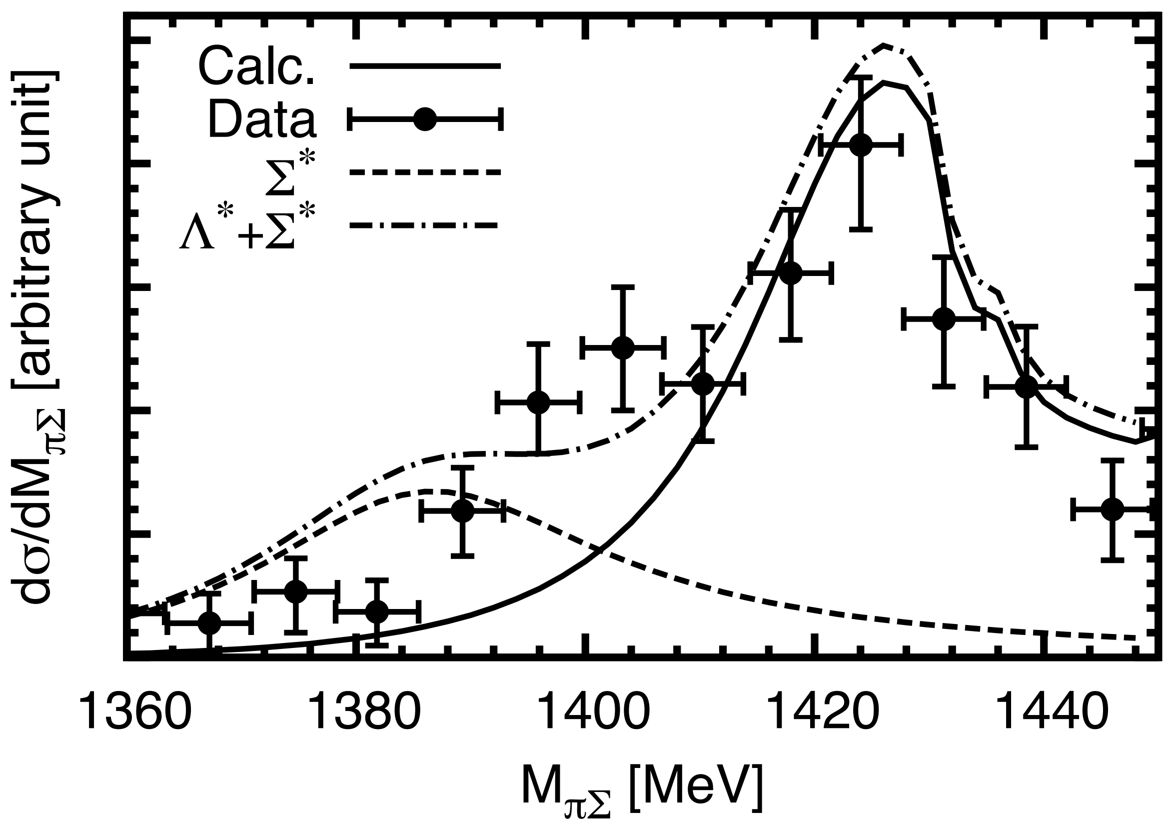
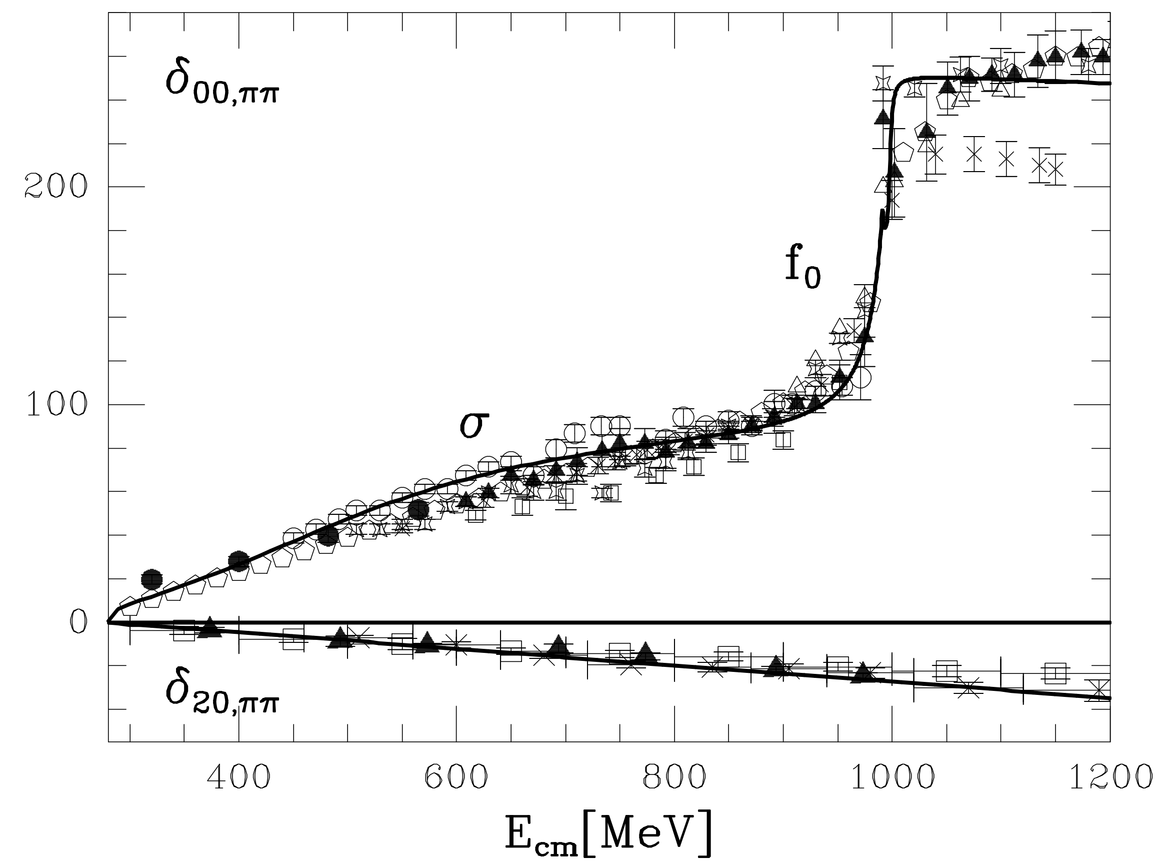
$$K^- d \to \pi^+ \Sigma^- n$$





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FINITE VOLUME

- Challenge of lattice QCD: determination of the spectra of mesons and baryons (lattice: discretized, finite space-time volume)

$$S[\zeta] = \int d^4x \mathcal{L}(\zeta(x), \partial^\mu \zeta(x))$$

$$\mathcal{G}^n(x_1, \cdots, x_n) = \frac{\int [d\zeta]\zeta(x_1)\cdots\zeta(x_n)e^{iS[\zeta]}}{\int [d\zeta]e^{iS[\zeta]}}$$

- Resonances do not correspond to isolated energy levels in the spectrum of the QCD Hamiltonian on the lattice.

- One channel problem $\Rightarrow$ Luescher framework

Relates the measured discrete value of the energy in a finite volume to the scattering phase shift at the same energy, for the same system in the infinite volume.

- M. Luescher, Commun. Math. Phys. 105, 153 (1986).
- M. Luescher, Nucl. Phys. B 354, 531 (1991).

- Consider a cubic box of side length L.

- Using periodic boundary conditions, the finite volume allows only discrete momenta:

$$\int \frac{d^3\vec{q}}{(2\pi)^3} I(|\vec{q}|) \to \frac{1}{L^3} \sum_{\vec{q}} I(|\vec{q}|)$$

$$\vec{q} = \frac{2\pi}{L}\,\vec{n}, \quad \vec{n} \in \mathbb{Z}^3$$

- To use UχPT in a finite volume we replace the loop function G by $\tilde{G}$.

- Dimensional regularization:

$$\tilde{G}(E) = G^D(E) + \lim_{q_{\max}\to\infty}\left[\frac{1}{L^3}\sum_{q_i}^{q_{\max}} I(q_i) - \int_{q<q_{\max}} \frac{d^3q}{(2\pi)^3} I(q)\right]$$

$$I(q) = \frac{1}{2\omega_1(\vec{q})\,\omega_2(\vec{q})}\,\frac{\omega_1(\vec{q}) + \omega_2(\vec{q})}{E^2 - (\omega_1(\vec{q}) + \omega_2(\vec{q}))^2 + i\epsilon}$$

- One channel case:

$$T(E) = \left[V^{-1}(E) - G^D(E)\right]^{-1} \xrightarrow{\text{Finite Volume}} \tilde{T}(E) = \left[V^{-1}(E) - \tilde{G}(E)\right]^{-1}$$

- We search for the poles of $\tilde{T}$: $V^{-1}(E) = \tilde{G}(E)$

$\Big\downarrow$ Infinite Volume

$$T(E) = \left[V^{-1}(E) - G^D(E)\right]^{-1} = \left[\tilde{G}(E) - G^D(E)\right]^{-1}$$

$$T(E)^{-1} = \lim_{q_{max}\to\infty}\left[\frac{1}{L^3}\sum_{q_i}^{q_{max}} I(q_i) - \int_{q<q_{max}} \frac{d^3q}{(2\pi)^3} I(q)\right]$$

Regularization scale independent !

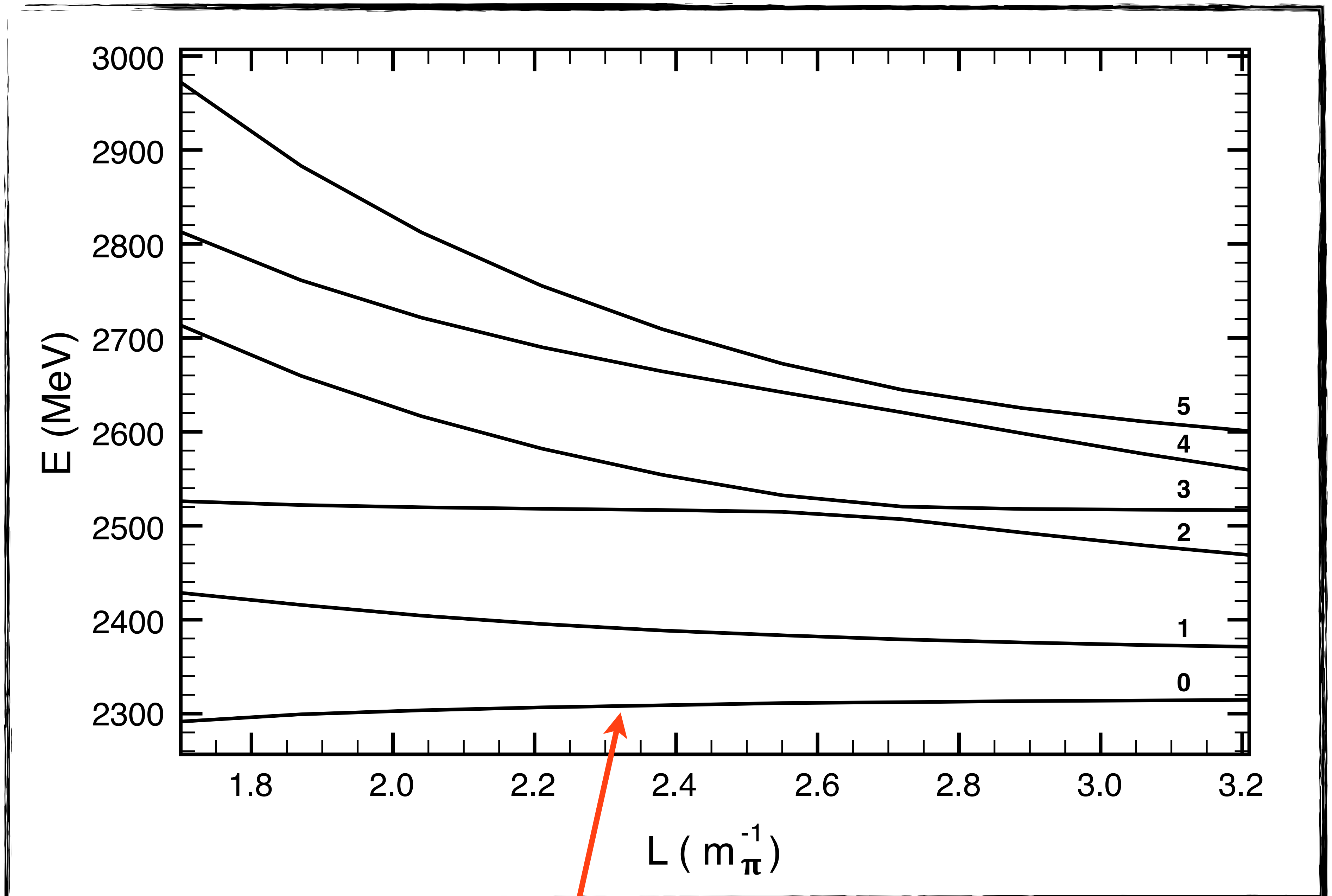
- Equivalent to Luescher formula but keeps all the terms of the relativistic two-body propagator (M. Doering, U.-G. Meißner, E. Oset, and A. Rusetsky, arxiv11073.3988 [hep-lat]).

- Multichannel case:

$$\tilde{T}(E) = \left[1 - V\tilde{G}(E)\right]^{-1} V$$

$$\det[1 - V\tilde{G}] = 0$$

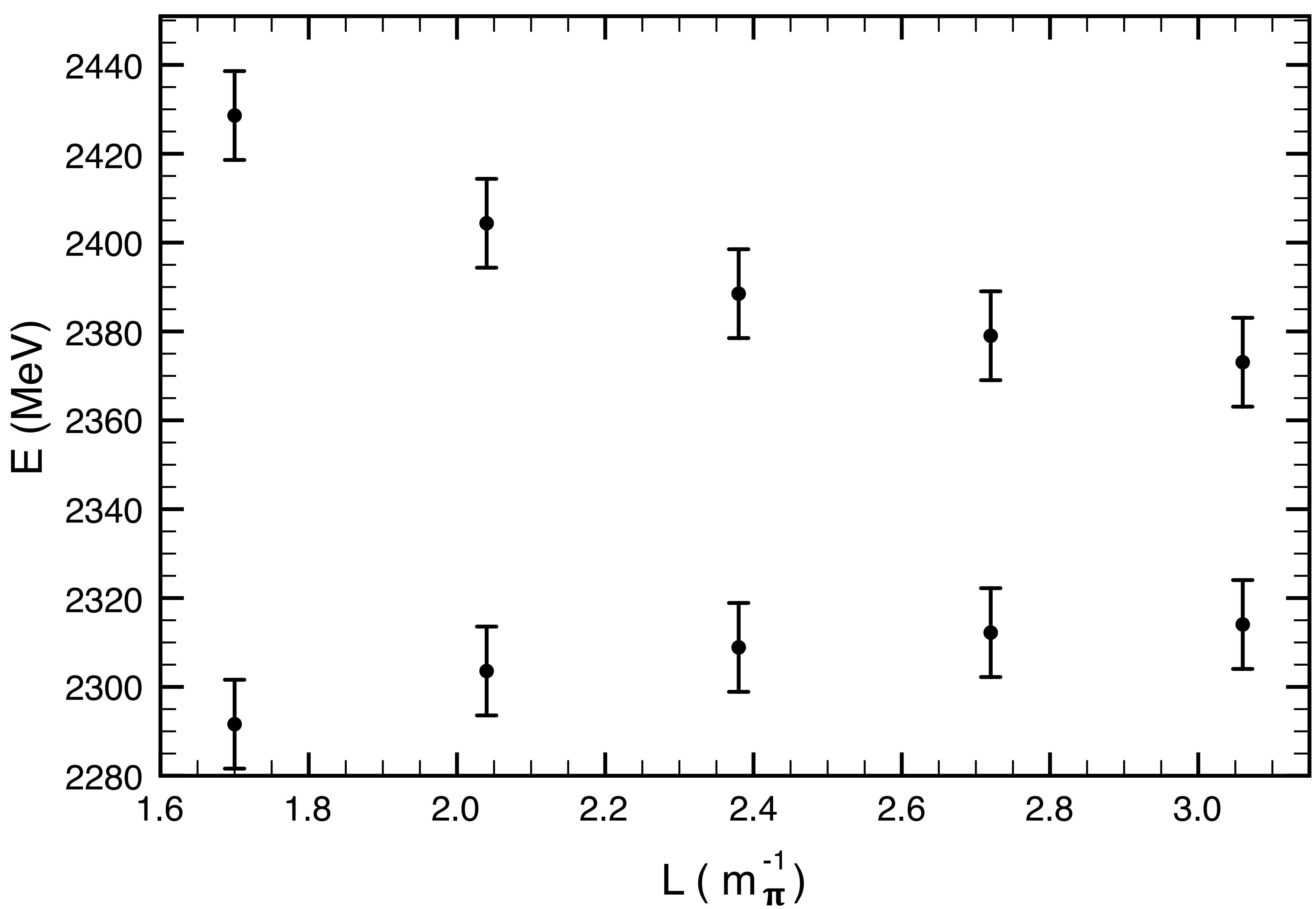
- We have applied it to the case of the $D_{s^*0}(2317)$: Dynamically generated in the $KD, \eta D_s$ system (D. Gamermann, E. Oset, D. Strottman, M. J. Vicente Vacas, Phys. Rev. D76, 074016 (2007)).



It converges to the energy of the bound state of the infinite volume for $L = 3\, m_\pi^{-1}$.

- We use these levels to face the problem of getting bound states and phase shifts in the infinite volume.

- We consider the energy levels obtained as “synthetic” lattice data.



- Inverse problem:

$$V_{ij} = a_{ij} + b_{ij}[s - (m_K + M_D)^2]$$

- We make a fit to the data to determine the parameters minimazing the χ^2.

- We generate random numbers such that $\chi^2 < \chi^2_{min} + 1$

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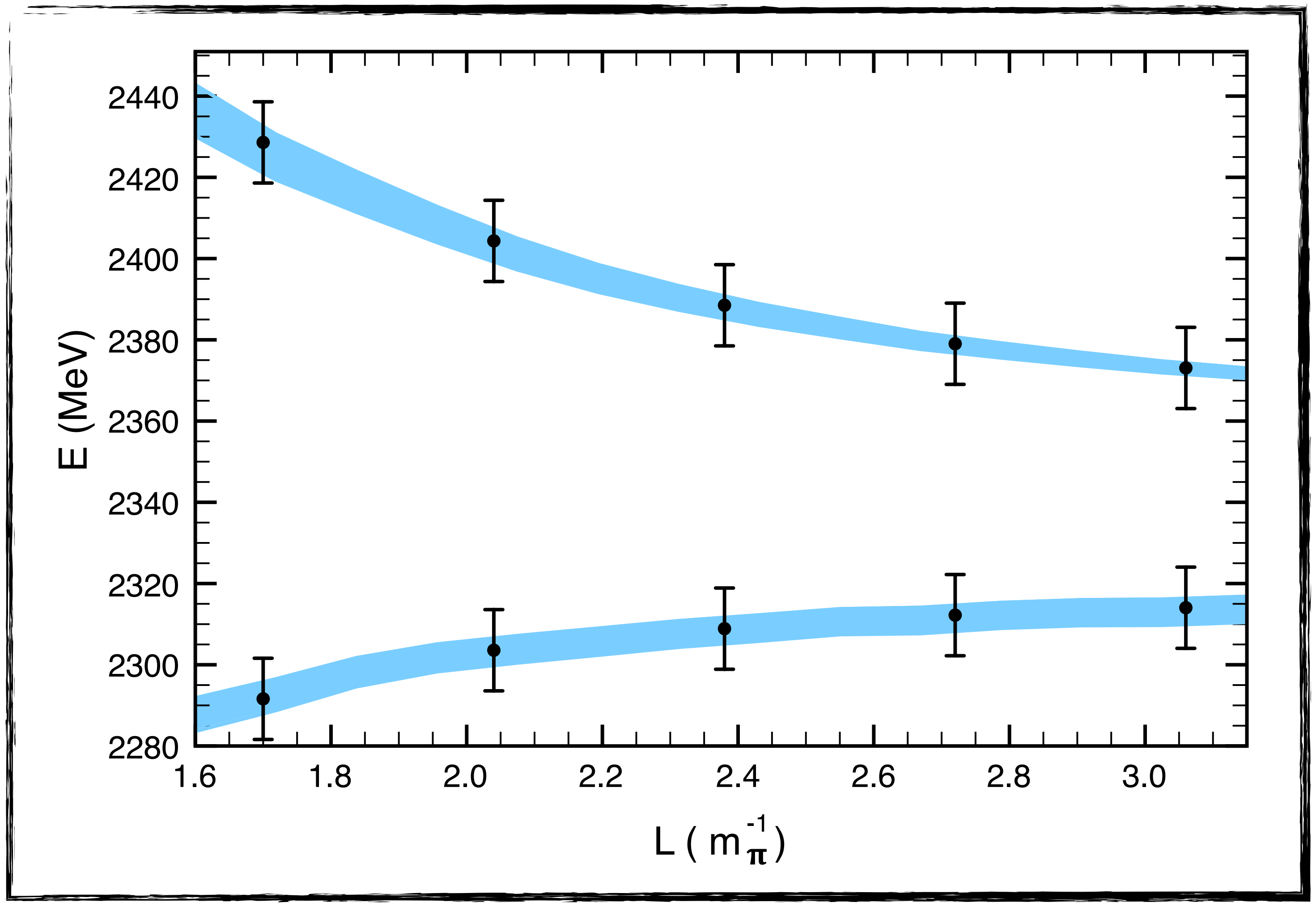

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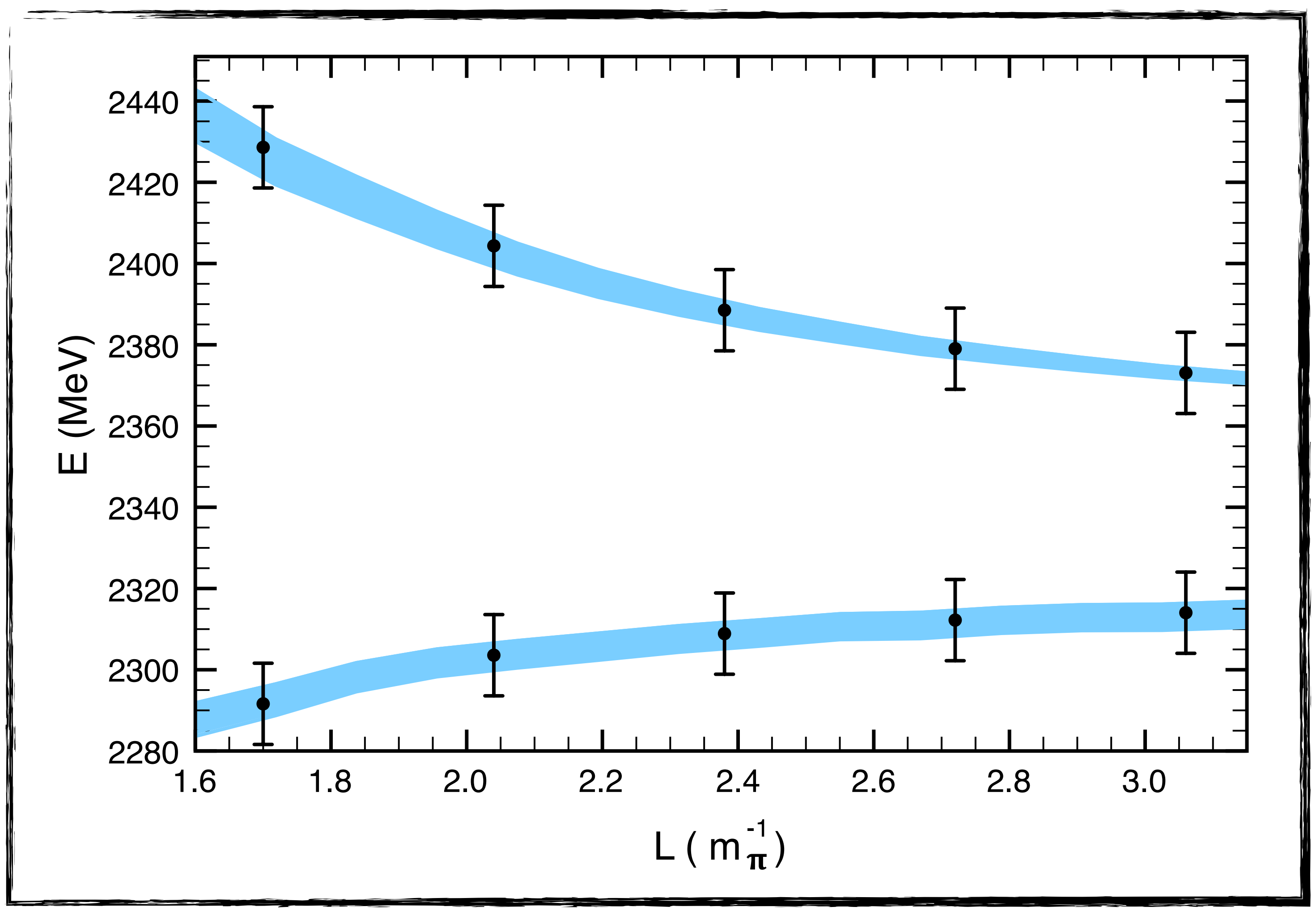
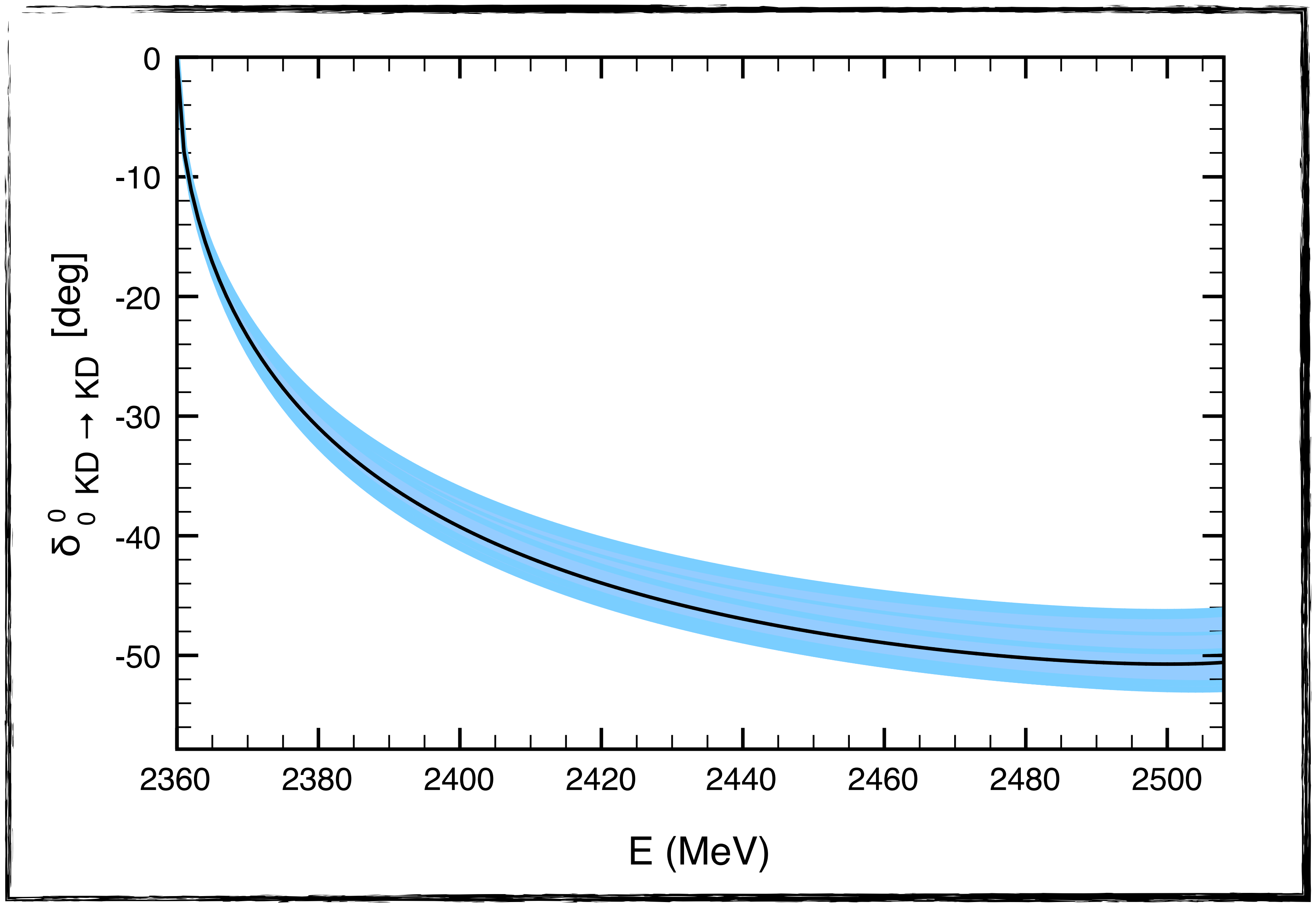


Arbitrary subtraction constant

$$a \in [-1, -2]$$

- We determine the KD phase shift in the infinite volume.



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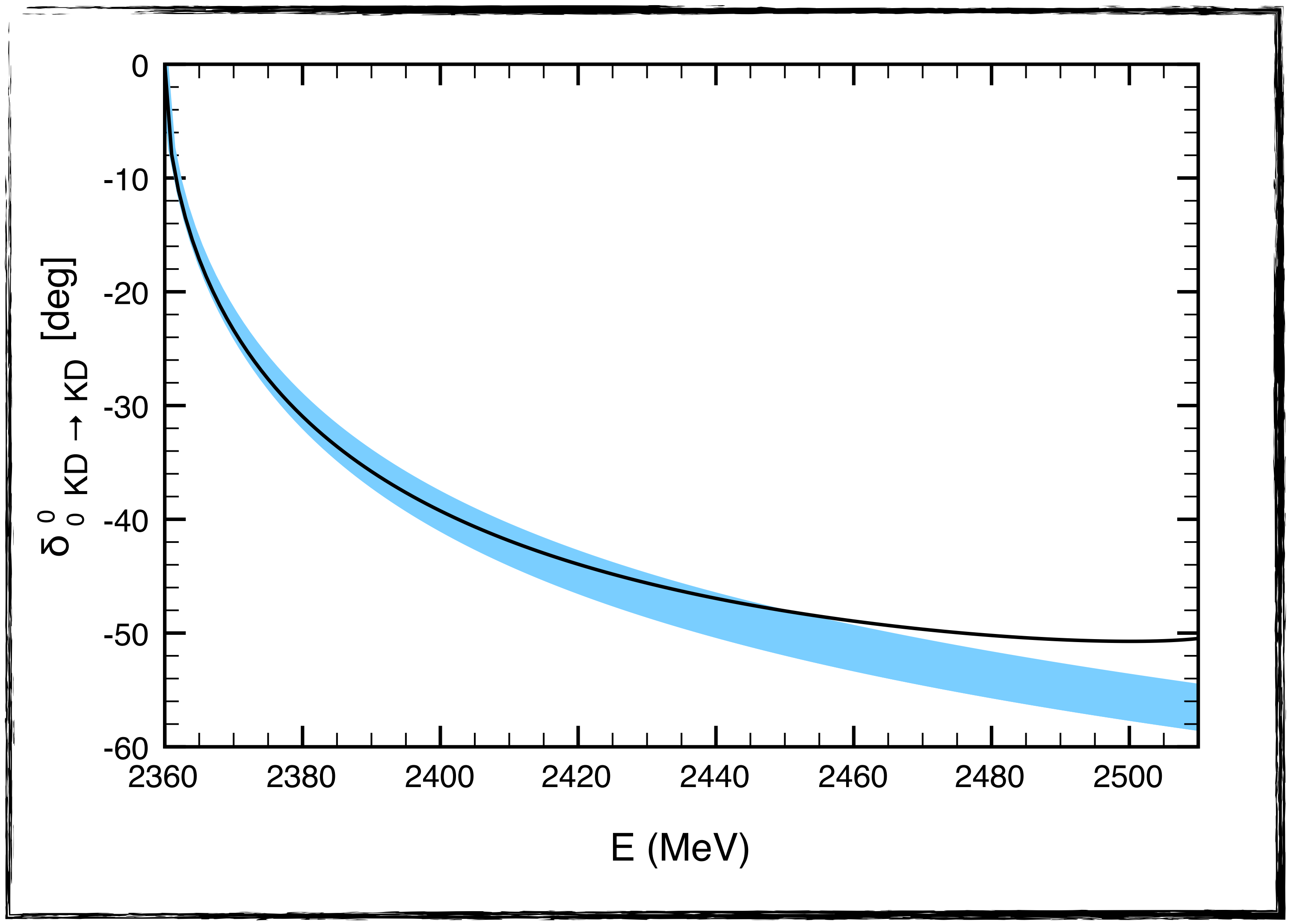
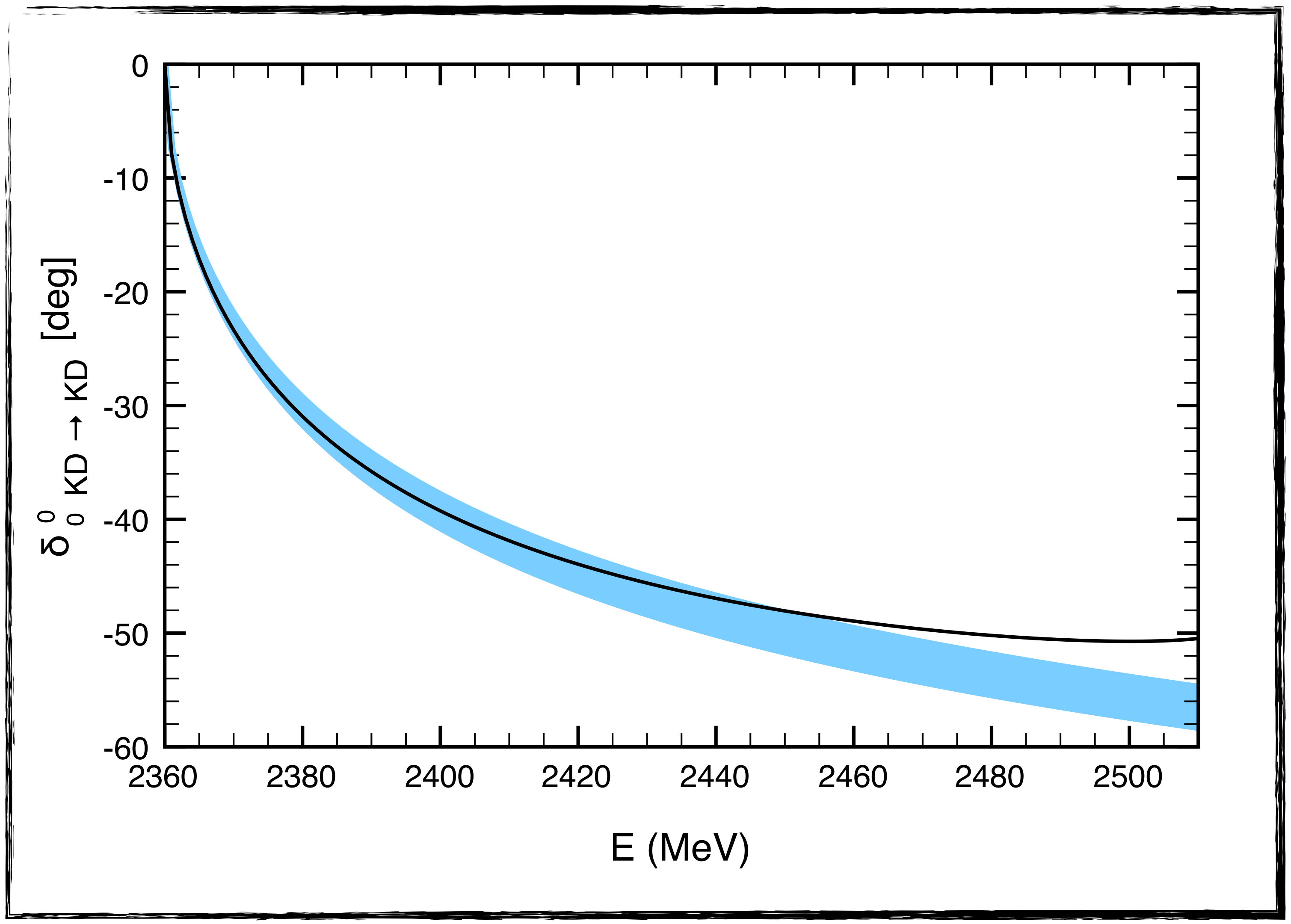


- Is it possible to fit the lattice data with only the *KD* channel?



- In both cases, i.e., two and one channel, we get a pole at 2317 ± 5 MeV.

- Can we get information about the nature of the $D_{s^*0}(2317)$ from the lattice data?

$$|R> = A|HH> + B|H'H'> + \cdots$$

$$\sum_i g_i^2 \left.\frac{dG_{ii}}{ds}\right|_{E=E_\alpha} = -1$$

- D. Gamermann, J. Nieves, E. Oset, E. Ruiz Arriola, Phys. Rev. D81, 014029 (2010).

- Can we get information about the nature of the $D_{s^*0}(2317)$ from the lattice data?

$$|R >= A|HH > +B|H'H' > + \cdots$$

$$\sum_i g_i^2 \left. \frac{dG_{ii}}{ds} \right|_{E=E_\alpha} = -1 + Z$$

overlap with a genuine particle

- D. Gamermann, J. Nieves, E. Oset, E. Ruiz Arriola, Phys. Rev. D81, 014029 (2010).

- Castillejo-Dalitz-Dyson (CDD) potential

$$V = V_M + \frac{g_{CDD}^2}{s - s_{CDD}}$$

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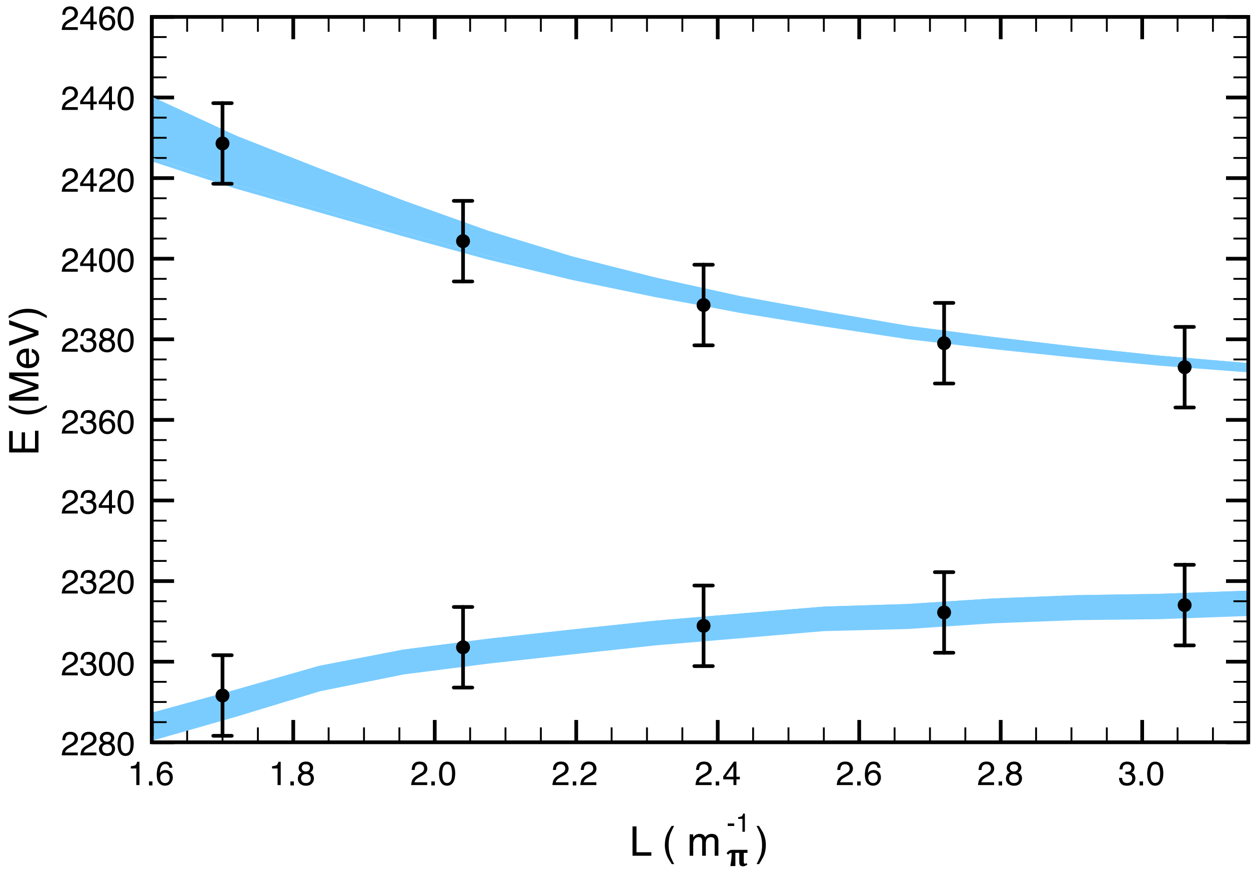
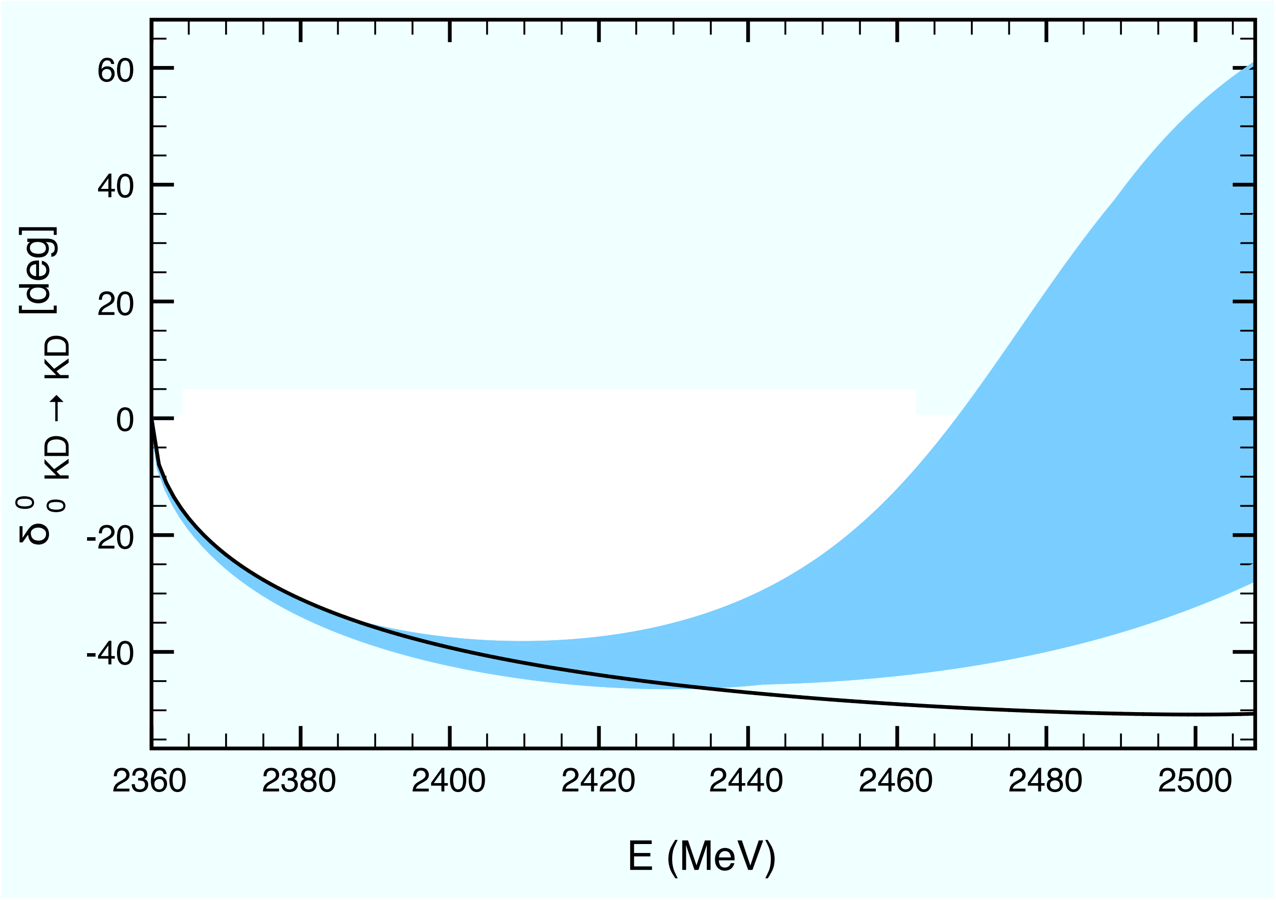
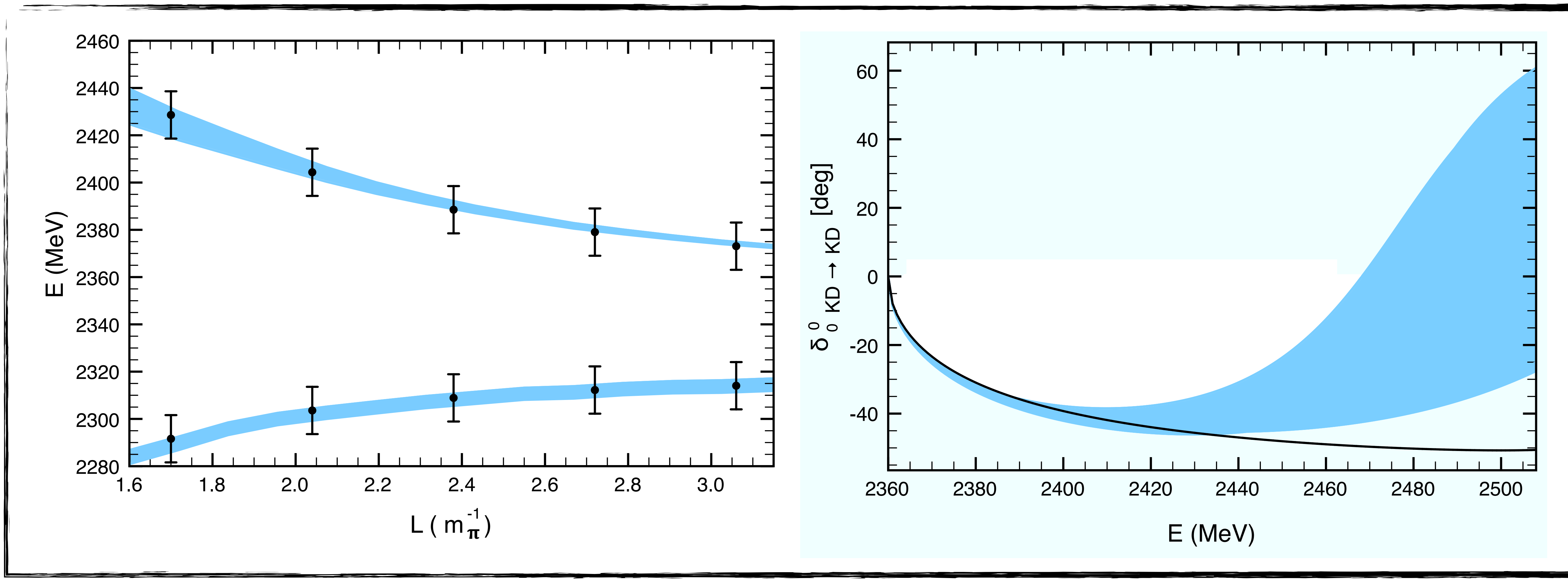




- Castillejo-Dalitz-Dyson (CDD) potential

$$V = V_M + \frac{g^2_{CDD}}{s - s_{CDD}}$$



CDD pole found ~ 2500 MeV
Z ~ 0.15

KD bound state
(a more precise determination
implies one more level)

CONCLUSIONS

- Two-body problem in finite Volume:

  - We have generate the energy levels in a cubic box for the *DK*, ηD_s system using UχPT.

  - Assuming our results as lattice data, we determine poles and phase shifts in the case of infinite volume.

  - The scheme does not depend on the regularization scale.

  - Low energy region of *KD*: one channel case quite good.

  - Information about the nature of the state: two levels and use of a sume rule.

$$
\begin{aligned}
\frac{1}{2\,\omega_1\,\omega_2}\,\frac{\omega_1+\omega_2}{E^2-(\omega_1+\omega_2)^2+i\epsilon} = {} & \frac{1}{2E}\,\frac{1}{p^2-\vec{q}^2+i\epsilon} - \frac{1}{2\omega_1\,\omega_2}\,\frac{1}{\omega_1+\omega_2+E} \\
& - \frac{1}{4\,\omega_1\,\omega_2}\,\frac{1}{\omega_1-\omega_2-E} - \frac{1}{4\,\omega_1\,\omega_2}\,\frac{1}{\omega_2-\omega_1-E}
\end{aligned}
$$