Two nucleon transfer reactions in Nuclear Supersymmetry

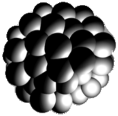
Ruslan Magaña Vsevolodovna[1,2] Roelof Bijker[2]

[1]Universidad Juárez Autónoma de Tabasco, DACB, México
[2]Universidad Nacional Autónoma de México, ICN, México

Andean School on Nuclear Physics in the 21st century, 2012

Outline

1. Introduction
   - Interacting Boson Model
   - Nuclear Supersymmetry
2. Theory of Generalizad F-spin
   - Wave Function in GF-Spin
   - Operators in GF-spin
   - Nuclear Reactions in GF-spin
   - Tensorial Transfer Operators
3. Nuclear Transfer Reactions
   - One nucleon transfer
   - Two nucleon transfer
4. Conclusions



Analytical Solutions

Phenomenological Collective Models

- Interacting Boson Model and extensions

"Group Theory"

Numerical Solutions

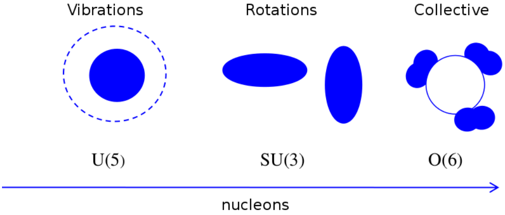
Ab-initio Models

- Green Montecalo Functions
- Effective Fields Models
- ...

Algebraic Nuclear Models

In Nuclear physics we have different symmetries

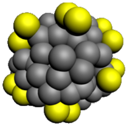
- $U(5)$ for vibrational symmetry
- $SU(3)$ for rotational symmetry
- $O(6)$ for collective symmetry (γ -unstable)



The collective behavior of the nuclei arise when the numbers of nuclei is near to 78 protons and 116 neutrons. The most external cores (valence nuclei) acquiere the property to be coupled in pairs

Interacting Boson Model (IBM)

The theory to describe this exploit of the nature of the nucleus is the IBM



which describe the collective exitations to even-even nuclei with angular momentum $L = 0$ and $L = 2$ (bosons s and d)

Creation and anihilation operators for bosons

$$b_i^\dagger, b_i \quad i = l, m \quad (l = 0, 2 \quad -l \leq l)$$

$$[b_i, b_j^\dagger] = \delta_{ij}, \quad [b_i^\dagger, b_j^\dagger] = [b_i, b_j] = 0$$

Generators of $U(6)$

$$G_i^j = b_i^\dagger b_j \quad i, j = 1, ...6$$

there are 36 bilineal products

$$[G_i^j, G_l^k] = G_i^j \delta_{j,k} - G_l^k \delta_{i,l}$$

where $i, j, j, l = 1, .., 6$

A. Arima y F. Iachello Phys. Rev. Lett. 35 1069 **(1975)**

Generating Spectrum Algebra (GSA)

When we describe one hamiltonian by operators in one symmetry we call that **dynamical symmetry**. Lets represent the Hamiltonian by elements of the algebra A

$$
\begin{aligned}
H &= f(G_k), \quad G_k \in A \\
H_B &= E_0 + \sum_{\alpha\beta} \varepsilon_{\alpha\beta} G^B + \frac{1}{2} \sum_{\alpha\beta\gamma\delta} G^B_{\alpha\beta} G^B_{\gamma\delta} + \ldots
\end{aligned}
$$

we can build the Hamiltionian in terms of invariant Cassimir operators

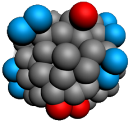
$$[C_k, G_k^k] = 0$$

from the chains of the 'algebra A $\subset$ A' $\subset$ A" $\subset$ ... we obtain **generating polynomial spectrum**

$$H = f(C_k)$$

F. Iachello and A. Arima, The interacting boson model, Cambridge University Press **(1987).**

Now if we include the grade of freedom for one particle (neutron or proton)



(fermion with angular momentum $j = j_1, j_2, ...$) Whereby we introduce one fermion operator which satisfies

Anhilation and creation operators for fermions

$$\{a_\mu, a_\nu^\dagger\} = \delta_{\mu\nu}, \;\; \{a_\mu^\dagger, a_\nu^\dagger\} = \{a_\mu, a_\nu\} = 0$$

F. Iachello and O. Scholten, Phys. Rev. Lett, **43**, 679 **(1979)**.

F.Iachello, Phys Rev. Lett **44** 772 **(1980)**.

Model	Year	Generator	Invariant	Symmetry
IBM	1975	$b_i^\dagger b_j$	N	$U(6)$
IBFM	1979	$b_i^\dagger b_j \;,\; a_k^\dagger a_l$	$N,\ M$	$U(6) \otimes U(m)$
SUSY	1980	$b_i^\dagger b_j \;,\; a_k^\dagger a_l \;,\; b_i^\dagger a_k \;,\; a_k^\dagger b_i$	$\mathcal{N} = N + M$	$U(6/m)$

$$\begin{array}{ll} N = \sum_i b_i^\dagger b_i & \text{bosons} \\ M = \sum_\mu a_\mu^\dagger a_\mu & \text{fermions} \\ \mathcal{N} = \mathcal{N} + \mathcal{M} & \text{bosons and fermions} \end{array}$$

where **bosons** have angular momemtum $l = 0, 2$ y **fermions** $j = j_1, j_2, \ldots$
whence there are four types of nuclei:

$$
\begin{aligned}
|\Psi_{\mathrm{ee}}\rangle &= |[\mathcal{N}_\nu],[\mathcal{N}_\pi];[\mathcal{N}_\nu+\mathcal{N}_\pi-i,i]; \\
&\quad (\Sigma_1,\Sigma_2,\Sigma_3);(\tau_1,\tau_2);L\rangle \\
|\Psi_{\mathrm{op}}\rangle &= |[\mathcal{N}_\nu],[\mathcal{N}_\pi-1];[\mathcal{N}_\nu+\mathcal{N}_\pi-1-i,i]; \\
&\quad (\Sigma_1,\Sigma_2,\Sigma_3),(\tfrac{1}{2}\tfrac{1}{2}\tfrac{1}{2})_\pi;(\sigma_1,\sigma_2,\sigma_3);(\tau_1,\tau_2);J\rangle \\
|\Psi_{\mathrm{on}}\rangle &= |[\mathcal{N}_\nu-1],[\mathcal{N}_\pi];[\mathcal{N}_\nu+\mathcal{N}_\pi-1-i,i][1]_\nu; \\
&\quad [\mathcal{N}_\nu+\mathcal{N}_\pi-j,j-k,k];(\Sigma_1,\Sigma_2,\Sigma_3);,(\tau_1,\tau_2);J,(\tfrac{1}{2});L\rangle \\
|\Psi_{\mathrm{oo}}\rangle &= |[\mathcal{N}_\nu-1],[\mathcal{N}_\pi-1];[\mathcal{N}_\nu+\mathcal{N}_\pi-2-i,i][1]_\nu; \\
&\quad [\mathcal{N}_\nu+\mathcal{N}_\pi-1-j,j-k,k];(\Sigma_1,\Sigma_2,\Sigma_3),(\tfrac{1}{2}\tfrac{1}{2}\tfrac{1}{2})_\pi; \\
&\quad (\sigma_1,\sigma_2,\sigma_3);(\tau_1,\tau_2);J,\tfrac{1}{2};L\rangle
\end{aligned}
$$

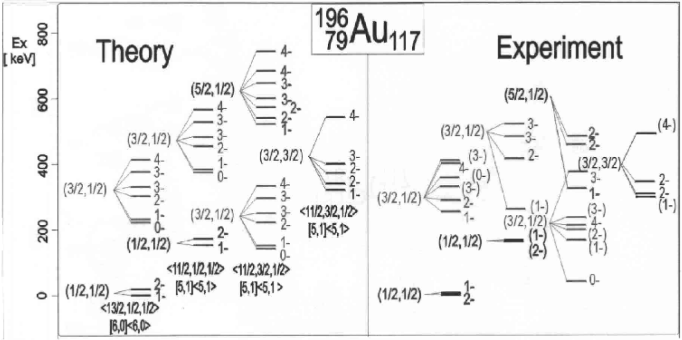
Where the Hamiltonian is

$$\begin{aligned} H &= aC_{2U_{BF_\nu}(6)} + bC_{2SO_{BF_\nu}(6)} + cC_{2Spin(6)} \\ &+dC_{2Spin(5)} + eC_{2Spin(3)} + fC_{2SU(2)} \end{aligned}$$

with energies

$$\begin{aligned} E &= a[N_1(N_1+5) + N_2(N_2+3) + N_3(N_3+1)] \\ &+b[\Sigma_1(\Sigma_1+4) + \Sigma_2(\Sigma_2+2) + \Sigma_3^2] \\ &+c[\sigma_1(\sigma_1+4) + \sigma_2(\sigma_2+2) + \sigma_3^2] \\ &+d[\tau_1(\tau_1+3) + \tau_2(\tau_2+1)] \\ &+eJ(J+1) \\ &+fL(L+1) \end{aligned}$$

even-even	s, d
odd proton	$j_\pi = 2d_{3/2}$
odd neutron	$j_\nu = 3p_{1/2}, 3p_{3/2}, 2f_{5/2}$
odd-odd	$j_\pi = 2d_{3/2}$
	$j_\nu = 3p_{1/2}; 3p_{3/2}; 2f_{5/2}$



A.Metz, J.Jolie, G. Graw, R. Hertenberger, J. Groger, C. Gunther, N. Warr, and Y. Eisermann Phys. Rev. Lett, 83, 1542 **(1999)**.

Theory of Generalizad F-spin

The lowest energy levels of even-even and odd-neutron nuclei in Supersymmetry theory share the same spectrum

Even-even nuclei		*Odd-neutron nuclei*	
$U_B(6)$ $[\mathcal{N}-i,i]$	$SO_B(6)$ $(\Sigma_1,\Sigma_2,\Sigma_3)$	$U_{BF_\nu}(6)$ $[\mathcal{N}-j,j-k,k]$	$SO_{BF_\nu}(6)$ $(\Sigma_1,\Sigma_2,\Sigma_3)$
$[\mathcal{N},0]$	$(\mathcal{N},0,0)$ $(\mathcal{N}-2,0,0)$ $\vdots$	$[\mathcal{N},0,0]$	$(\mathcal{N},0,0)$ $(\mathcal{N}-2,0,0)$ $\vdots$
$[\mathcal{N}-1,1]$	$(\mathcal{N}-1,1,0)$ $(\mathcal{N}-2,0,0)$ $\vdots$	$[\mathcal{N}-1,1,0]$	$(\mathcal{N}-1,1,0)$ $(\mathcal{N}-2,0,0)$ $\vdots$

where $\mathcal{N}=\mathcal{N}_\nu+\mathcal{N}_\pi$. So we have correlation between different nuclei.

We see that the wavefunctions in nuclear supersymmetry are made by three different repesentations: bosons of proton, bosons of neutron and orbital part of neutron.

$$\begin{array}{ccccc} U(18) & \supset & U(6) & \otimes & U(3) \\ \downarrow & & \downarrow & & \downarrow \\ [N] & & [N_1, N_2, N_3] & & [N_1, N_2, N_3] \end{array}$$

whereby we have couplings and we get our GF-spin space

$$\begin{array}{ccccccccc} U(3) & \supset & SU(3) & \supset & (SU(2) & \supset & SO(2)) & \otimes & U(1) \\ \downarrow & & \downarrow & & \downarrow & & \downarrow & & \downarrow \\ [N_1, N_2, N_3] & & (\lambda, \mu) & & F & & F_z & & Y \end{array}$$

We take the analogy of the quarks model (Gell-Mann 1961,1964) of tree flavors (u,d, s) with the symmetry

$$SU(3) \supset SU(2) \otimes U(1)$$

GF-spin Wave Function

$$|\Psi\rangle_{N-SUSY} = |[N_\nu],[N_\pi];[N_\nu + N_\pi - i, i],[N_\rho];[N-j, j-k, k], \alpha\rangle$$

we transform into

$$|\Psi\rangle_{\text{F-spin}} = |[N];(\lambda,\mu),F,F_z,Y,\alpha\rangle$$

with the following rules of transformation

$$\begin{aligned}
&N = N_\nu + N_\pi + N_\rho \\
&(\lambda,\mu) = (N_\nu + N_\pi - 2j + k, j - 2k) \\
&F = \tfrac{1}{2}(N_\pi + N_\nu - 2i) \\
&F_z = \tfrac{1}{2}(N_\pi - N_\nu) \\
&Y = \tfrac{1}{3}(N_\pi + N_\nu - 2N_\rho).
\end{aligned}$$

Ruslan Magaña Tesis Paper U.N.A.M. **(2010)**.

Roelof Bijker Journal of Physics Conferences Series **284** 012013 **(2011)**.

GF-spin Operators

F-spin in IBM-2 (1987)

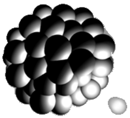
	F	F_z
$b_\pi^\dagger$	$\frac{1}{2}$	$\frac{1}{2}$
$b_\nu^\dagger$	$\frac{1}{2}$	$-\frac{1}{2}$
$-b_\pi$	$\frac{1}{2}$	$-\frac{1}{2}$
b_ν	$\frac{1}{2}$	$\frac{1}{2}$

Generalized F-spin in N-SUSY(2010)

	F	F_z	Y
$b_\pi^\dagger$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{3}$
$b_\nu^\dagger$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{3}$
$a_\nu^\dagger$	0	0	$-\frac{2}{3}$
$-b_\pi$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{3}$
b_ν	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{3}$
a_ν	0	0	$\frac{2}{3}$

F. Iachello and A. Arima, IBM , Cambridge University Press **(1987)**

Ruslan Magaña - Tesis Paper UNAM **(2010)**



In order to reproduce nuclear reactions we must build the appropiate operators.
Our theory of F-spin in the algebra $SU(3)$ has two main irreducible representations:

Irreps for Transfer Operators

- Irrep $(1,0)$ for creation operators $t^\dagger$
- Irrep $(0,1)$ for anihilation operators $t^\dagger$

We will build our nuclear reactions operators by the **direct product of irreps** or **tensor products**.
For example:

$$(1,0) \otimes (0,1) \quad = \quad \underbrace{(1,1)}_{\text{one nucleon transfer}} \oplus (0,0)$$

So the general product of two irreps is given by:

Tensorial product

$$T^{(\lambda,\mu)}_{\mathbf{FF_zY}} = \sum_{\substack{F,F_z,Y\\F',F'_z,Y'}} \left\langle \begin{matrix}(1,0)\\ F,F_z,Y\end{matrix} \quad \begin{matrix}(0,1)\\ F',F'_z,Y'\end{matrix} \;\middle|\; \begin{matrix}(\lambda,\mu)\\ \mathbf{F},\mathbf{F_z},\mathbf{Y}\end{matrix} \right\rangle T^{(1,0)}_{F,F,F_z,Y}\, T^{(0,1)}_{F',F',-F'_z,-Y'}$$

where we can use the Wigner Eckart theorem to decomposed into the product of two direct sum of irreps

Wigner Eckart Theorem

$$\left\langle \begin{matrix}(\lambda_1,\mu_2)\\ F_1,F_{z_1}Y_1\end{matrix} \quad \begin{matrix}(\lambda_2,\mu_2)\\ F_1,F_{z_2},Y_2\end{matrix} \;\middle|\; \begin{matrix}(\lambda,\mu)\\ F,F_z,Y\end{matrix} \right\rangle = \underbrace{\left\langle \begin{matrix}(\lambda_1,\mu_1)\\ F_1,Y_1\end{matrix} \quad \begin{matrix}(\lambda_2,\mu_2)\\ F_2,Y_2\end{matrix} \;\middle|\; \begin{matrix}(\lambda,\mu)\\ F,Y\end{matrix} \right\rangle}_{SU(3)\supset SU(2)\otimes U_Y(1)} \overbrace{\langle F_1,F_{z_1};F_2,F_{z_2}|F,F'_z\rangle}^{SU_F(2)\supset SO_{F_z}(2)}$$

The irreducible labels for the transfer of one neutron are

$$(\lambda,\mu)=(1,1) \quad F=\tfrac{1}{2} \quad F_z=-\tfrac{1}{2} \quad Y=-1$$

so the series decomposition is given by

$$\begin{aligned}
T^{(\mathbf{1},\mathbf{1})}_{\frac{1}{2},\frac{1}{2},-1} &= \sum_{F,F_z,Y}\sum_{F',F'_z,Y'} \left\langle \begin{matrix}(1,0) & (0,1)\\ F,F_z,Y & F',F'_z,Y'\end{matrix} \middle| \begin{matrix}(\mathbf{1},\mathbf{1})\\ \frac{1}{2},\frac{1}{2},-\mathbf{1}\end{matrix}\right\rangle T^{(1,0)}_{F,F,F_z,Y}\,T^{(0,1)}_{F',F',F'_z,Y'}\\
&= \sum_{F,Y}\sum_{F',Y'} \left\langle \begin{matrix}(1,0) & (0,1)\\ F,Y & F',Y'\end{matrix} \middle| \begin{matrix}(\mathbf{1},\mathbf{1})\\ \frac{1}{2},-\mathbf{1}\end{matrix}\right\rangle \sum_{F_z}\langle F,F_z,F',F'_z|\frac{\mathbf{1}}{\mathbf{2}},\frac{\mathbf{1}}{\mathbf{2}}\rangle \mathbf{T}^{(1,0)}_{\mathbf{F},\mathbf{F_z},\mathbf{Y}}\mathbf{T}^{(0,1)}_{\mathbf{F'},\mathbf{F'_z},\mathbf{Y'}}\\
&= \underbrace{\left\langle \begin{matrix}(1,0) & (0,1)\\ 0,-\frac{2}{3} & \frac{1}{2},-\frac{1}{3}\end{matrix} \middle| \begin{matrix}(\mathbf{1},\mathbf{1})\\ \frac{1}{2},-\mathbf{1}\end{matrix}\right\rangle}_{\text{Isoscalar factor}} \underbrace{\langle 0,0,\frac{1}{2},\frac{1}{2}|\frac{\mathbf{1}}{\mathbf{2}},\frac{\mathbf{1}}{\mathbf{2}}\rangle}_{\text{Clebsch-Gordan coefficient}} T^{(1,0)}_{0,0,-\frac{2}{3}}\,T^{(0,1)}_{\frac{1}{2},\frac{1}{2},-\frac{1}{3}}
\end{aligned}$$

Neutron transfer operator

$$T^{(\mathbf{1},\mathbf{1})}_{\frac{1}{2},\frac{1}{2},-\mathbf{1}} = a^\dagger_\nu b_\nu$$

Tensorial Character

So we have the tensorial character in GF-spin of nucleon transfers

Transfer	*Operator*	(λ, μ)	F	F_z	Y
1) Proton	$a_\pi^\dagger \tilde{b}_\pi$	$(0,1)$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{3}$
	$b_\pi^\dagger \tilde{a}_\pi$	$(1,0)$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{3}$
2) Neutron	$a_\nu^\dagger \tilde{b}_\nu$	$(1,1)$	$\frac{1}{2}$	$\frac{1}{2}$	-1
	$b_\nu^\dagger \tilde{a}_\nu$	$(1,1)$	$\frac{1}{2}$	$-\frac{1}{2}$	1
3) Two protons	$b_\pi^\dagger$	$(1,0)$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{3}$
	b_π	$(0,1)$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{3}$
4) Two neutrons	$b_\nu^\dagger$	$(1,0)$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{3}$
	b_ν	$(0,1)$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{3}$
5)Neutron and proton	$a_\nu^\dagger b_\nu b_\pi$	$(1,2)$	1	0	$-\frac{4}{3}$
	$b_\nu^\dagger a_\nu b_\pi$	(1,2)	1	-1	$\frac{2}{3}$
	$a_\nu^\dagger b_\pi b_\pi$	(1,2)	1	-1	$-\frac{4}{3}$

Nuclear Transfer Reactions

Proton Transfer

$$b_\pi^\dagger \tilde{a}_\pi \quad \begin{array}{c} {}^{195}_{79}\mathrm{Au}_{116} \\ \downarrow \uparrow \\ {}^{194}_{78}\mathrm{Pt}_{116} \end{array} \quad a_\pi^\dagger \tilde{b}_\pi \qquad b_\pi^\dagger \tilde{a}_\pi \quad \begin{array}{c} {}^{196}_{79}\mathrm{Au}_{117} \\ \downarrow \uparrow \\ {}^{195}_{78}\mathrm{Pt}_{117} \end{array} \quad a_\pi^\dagger \tilde{b}_\pi$$

Neutron Transfer

$${}^{195}_{79}\mathrm{Au}_{116} \quad \overset{a_\nu^\dagger \tilde{b}_\nu}{\underset{b_\nu^\dagger \tilde{a}_\nu}{\rightleftarrows}} \quad {}^{196}_{79}\mathrm{Au}_{117}$$

$${}^{194}_{78}\mathrm{Pt}_{116} \quad \overset{a_\nu^\dagger \tilde{b}_\nu}{\underset{b_\nu^\dagger \tilde{a}_\nu}{\rightleftarrows}} \quad {}^{195}_{78}\mathrm{Pt}_{117}$$

Example: Let's assume that we want add one proton to even-even nucleus.

$$|^{194}\mathrm{Pt}\rangle_{ee} \quad \xrightarrow{a^\dagger_\pi \widetilde{b}_\pi} \quad |^{195}\mathrm{Au}\rangle_{op}$$

So in GF-spin we have

$$\begin{aligned}
C_1 &= \left\langle (\mathcal{N}-1-2i,i);\, \tfrac{\mathcal{N}-1-2i}{2}, \tfrac{\mathcal{N}_\pi-\mathcal{N}_\nu-1}{2}, \tfrac{\mathcal{N}-1}{3}; \alpha' \right| T^{(0,1)}_{\frac{1}{2},-\frac{1}{2},-\frac{1}{3}} \left| (\mathcal{N},0);\, \tfrac{\mathcal{N}}{2}, \tfrac{\mathcal{N}_\pi-\mathcal{N}_\nu}{2}, \tfrac{\mathcal{N}}{3}; \alpha \right\rangle \\
&= \left\langle \begin{matrix} (\mathcal{N},0) \\ \frac{\mathcal{N}}{2}, \frac{\mathcal{N}_\pi-\mathcal{N}_\nu}{2}, \frac{\mathcal{N}}{3} \end{matrix} \quad \begin{matrix} (0,1) \\ \frac{1}{2}, -\frac{1}{2}, -\frac{1}{3} \end{matrix} \;\middle|\; \begin{matrix} (\mathcal{N}-1-2i,i) \\ \frac{\mathcal{N}-1-2i}{2}, \frac{\mathcal{N}_\pi-\mathcal{N}_\nu-1}{2}, \frac{\mathcal{N}-1}{3} \end{matrix} \right\rangle \\
&\quad \times \langle (\mathcal{N}-1,0); \alpha' \| T^{(0,1)} \| (\mathcal{N},0); \alpha \rangle_{SU(3)} \\
&= \left\langle \begin{matrix} (\mathcal{N},0) \\ \frac{\mathcal{N}}{2}, \frac{\mathcal{N}}{3} \end{matrix} \quad \begin{matrix} (0,1) \\ \frac{1}{2}, -\frac{1}{3} \end{matrix} \;\middle|\; \begin{matrix} (\mathcal{N}-1-2i,i) \\ \frac{\mathcal{N}-1}{2}, \frac{\mathcal{N}-1}{3} \end{matrix} \right\rangle \left\langle \tfrac{\mathcal{N}}{2}, \tfrac{\mathcal{N}_\pi-\mathcal{N}_\nu}{2}, \tfrac{1}{2}, -\tfrac{1}{2} \middle| \tfrac{\mathcal{N}-1-2i}{2}, \tfrac{\mathcal{N}_\pi-\mathcal{N}_\nu-1}{2} \right\rangle \\
&\quad \times \langle (\mathcal{N}-1,0); \alpha' \| T^{(0,1)} \| (\mathcal{N},0); \alpha \rangle_{SU(3)}
\end{aligned}$$

$$C_1 = \sqrt{\frac{\mathcal{N}_\pi}{\mathcal{N}+2}} \langle (\mathcal{N}-1,0);\alpha' \| T^{(0,1)} \| (\mathcal{N},0);\alpha \rangle$$

and similar to

$$|^{195}\mathrm{Pt}\rangle_{\mathrm{on}} \quad \overset{a_\pi^\dagger \widetilde{b}_\pi}{\longrightarrow} \quad |^{196}\mathrm{Au}\rangle_{\mathrm{oo}}$$

$$C_2 \quad = \quad (-1)^L \sqrt{\frac{2L+1}{4}} \sqrt{\frac{\mathcal{N}_\pi}{\mathcal{N}+2}} \langle (\mathcal{N}-1,0);\alpha' \| T^{(0,1)} \| (\mathcal{N},0);\alpha \rangle$$

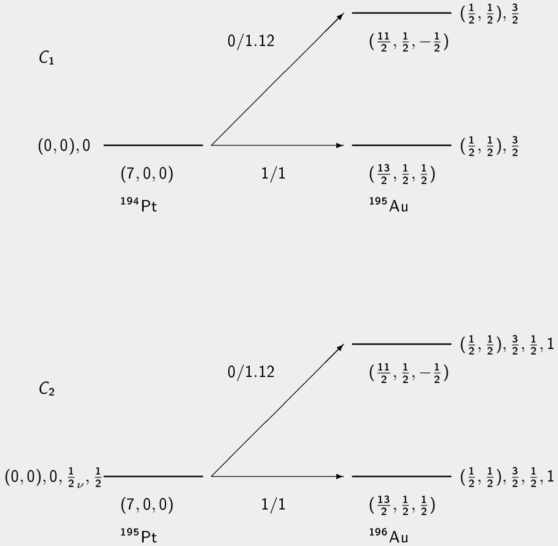
whence

$$\frac{C_2}{ISF_2} = \frac{C_1}{ISF_1}$$

whereby we have the correlation

$$C_2 \quad = \quad (-1)^L \sqrt{\frac{2L+1}{4}} C_1$$

Example


C_1
0/1.12
$(\frac{1}{2},\frac{1}{2}),\frac{3}{2}$
$(\frac{11}{2},\frac{1}{2},-\frac{1}{2})$
$(0,0),0$
$(7,0,0)$
^{194}Pt
1/1
$(\frac{1}{2},\frac{1}{2}),\frac{3}{2}$
$(\frac{13}{2},\frac{1}{2},\frac{1}{2})$
^{195}Au
C_2
0/1.12
$(\frac{1}{2},\frac{1}{2}),\frac{3}{2},\frac{1}{2},1$
$(\frac{11}{2},\frac{1}{2},-\frac{1}{2})$
$(0,0),0,\frac{1}{2}_{\nu},\frac{1}{2}$
$(7,0,0)$
^{195}Pt
1/1
$(\frac{1}{2},\frac{1}{2}),\frac{3}{2},\frac{1}{2},1$
$(\frac{13}{2},\frac{1}{2},\frac{1}{2})$
^{196}Au

Two nucleon transfer

Operators of two nucleon transfer are represented by the product of two operators of one nucleon

Two nucleon tranfer "Indirect form"

$$
\begin{array}{ccccccc}
 & & & a^{\dagger}_{j_\nu}\, b_\nu & & & \\
 & & & \longrightarrow & & & \\
 & & & \longleftarrow & & & \\
 & {}^{195}_{79}\mathrm{Au}_{116} & & b^{\dagger}_{j_\nu}\, a_\nu & & {}^{196}_{79}\mathrm{Au}_{117} & \\
a^{\dagger}_{j_\pi}\, b_\pi & \uparrow & C_4 & \nwarrow \quad \nearrow & C_3 & \uparrow & a^{\dagger}_{j_\pi}\, b_\pi \\
 & {}^{194}_{78}\mathrm{Pt}_{116} & & & & {}^{195}_{78}\mathrm{Pt}_{117} &
\end{array}
$$

$$T^{(\lambda)} = [T_\nu^{(j_\nu)} \times T_\pi^{(j_\pi)}]^{(\lambda)}$$

$$C_3 = {}_{\mathbf{oo}}\langle {}^{196}Au \| T^{(\lambda)} \| {}^{194}Pt \rangle_{\mathbf{ee}} \qquad C_4 = {}_{\mathbf{op}}\langle {}^{195}Au \| T^{(\lambda)'} \| {}^{195}Pt \rangle_{\mathbf{on}}$$

$$C_3 = \Sigma_i \, {}_{\mathbf{oo}}\langle {}^{196}Au \| T_\nu^{(j_\nu)} \| {}^{195}Au \rangle_{\mathbf{op}} \, {}_{\mathbf{op_i}}\langle {}^{195}Au \| T_\pi^{(j_\pi)} \| {}^{194}Pt \rangle_{\mathbf{ee}}$$

$$C_3 = \Sigma_i \, {}_{oo}\langle {}^{196}Au \| T_{\nu}^{(j_\nu)} \| {}^{195}Au \rangle_{op\ op_i} \langle {}^{195}Au \| T_{\pi}^{(j_\pi)} \| {}^{194}Pt \rangle_{ee}$$

We assume as the initial state the ground state of $\left|{}^{195}\mathrm{Au}\right\rangle_{gs}$ and the symmetric state $\left|{}^{194}\mathrm{Pt}\right\rangle$

$$\begin{aligned}
C_3 \quad = \quad & (-1)^{1+2j_\pi+j_\nu+l_\nu+\frac{1}{2}} \frac{\sqrt{(2\lambda+1)(2j_\nu+1)}}{\sqrt{(2j_\pi+1)}} \left\{ \begin{array}{ccc} j_\nu & \lambda & j_\pi \\ J & l_\nu & \frac{1}{2} \end{array} \right\} \sum_{\sigma''} \\
& \left\langle \begin{array}{cc|c} (\mathcal{N}-1,0) & (1,1) & (\mathcal{N}-1-2j,j) \\ \frac{\mathcal{N}-1}{2}, \frac{\mathcal{N}-1}{3} & \frac{1}{2}, -1 & \frac{\mathcal{N}-2}{2}, \frac{\mathcal{N}-4}{3} \end{array} \right\rangle \\
& \langle \tfrac{\mathcal{N}-1}{2}, \tfrac{\mathcal{N}_\pi-\mathcal{N}_\nu-1}{2}, \tfrac{1}{2}, \tfrac{1}{2} | \tfrac{\mathcal{N}-2}{2}, \tfrac{\mathcal{N}_\pi-\mathcal{N}_\nu}{2} \rangle \\
& \times \langle (\mathcal{N}-1-2j,j); \alpha \| T^{(1,1)} \| (\mathcal{N}-1,0); \alpha'' \rangle_{SU(3)} \\
& \left\langle \begin{array}{cc|c} (\mathcal{N},0) & (0,1) & (\mathcal{N}-1,0) \\ \frac{\mathcal{N}}{2}, \frac{\mathcal{N}}{3} & \frac{1}{2}, -\frac{1}{3} & \frac{\mathcal{N}-1}{2}, \frac{\mathcal{N}-1}{3} \end{array} \right\rangle \\
& \langle \tfrac{\mathcal{N}}{2}, \tfrac{\mathcal{N}_\pi-\mathcal{N}_\nu}{2}, \tfrac{1}{2}, -\tfrac{1}{2} | \tfrac{\mathcal{N}-1}{2}, \tfrac{\mathcal{N}_\pi-\mathcal{N}_\nu-1}{2} \rangle \\
& \times \langle (\mathcal{N}-1,0); \alpha'' \| T^{(0,1)} \| (\mathcal{N},0); \alpha' \rangle_{SU(3)}
\end{aligned}$$

We can express the two nucleon transfer C_4 in terms of C_3 multiplied by extra isoescalar factors and Clesbsch-Gordon coefficients, ISF_3 and ISF_4

$$\sum_{\sigma''} \langle \mathcal{N}-1-2j, j); \alpha \| T^{(1,1)} \| (\mathcal{N}-1, 0); \alpha'' \rangle_{SU(3)} \\ \langle (\mathcal{N}-1, 0); \alpha'' \| T^{(0,1)} \| (\mathcal{N}, 0); \alpha' \rangle_{SU(3)} = \frac{C_3}{ISF_3}$$

and similar

$$\sum_{\sigma''} \langle (\mathcal{N}-1-2j, j); \alpha \| T^{(1,1)} \| (\mathcal{N}-1, 0); \alpha'' \rangle_{SU(3)} \\ \langle (\mathcal{N}-1, 0); \alpha'' \| T^{(0,1)} \| (\mathcal{N}, 0); \alpha' \rangle_{SU(3)} = \frac{C_4}{ISF_4}$$

whence

$$\frac{C_4}{ISF_4} = \frac{C_3}{ISF_3}$$

whereby we get the correlation

$$C_4 = \frac{ISF_4}{ISF_3} C_3 = \begin{cases} -\gamma & C_3 & j = 0 \\ \gamma & \sqrt{\frac{\mathcal{N}_\pi - 1}{\mathcal{N}_\nu(\mathcal{N}-2)}}\, C_3 & j = 1 \end{cases}$$

where

$$\gamma = (-1)^{2J} \frac{\begin{Bmatrix} j_\nu & \frac{3}{2} & \lambda \\ J & \frac{1}{2} & l_\nu \end{Bmatrix}}{\begin{Bmatrix} j_\nu & l_\nu & \frac{1}{2} \\ J & \lambda & \frac{3}{2} \end{Bmatrix}}$$

Now we are working on two nucleon transfers with only one operator

Two nucleon transfer "Direct form"

$$\begin{array}{ccc} {}^{195}_{79}\mathrm{Au}_{116} & & {}^{196}_{79}\mathrm{Au}_{117} \\ & D_4 \nwarrow \ \nearrow D_3 & \\ {}^{194}_{78}\mathrm{Pt}_{116} & & {}^{195}_{78}\mathrm{Pt}_{117} \end{array}$$

this approach provide no intermediate states. We introduce the operator

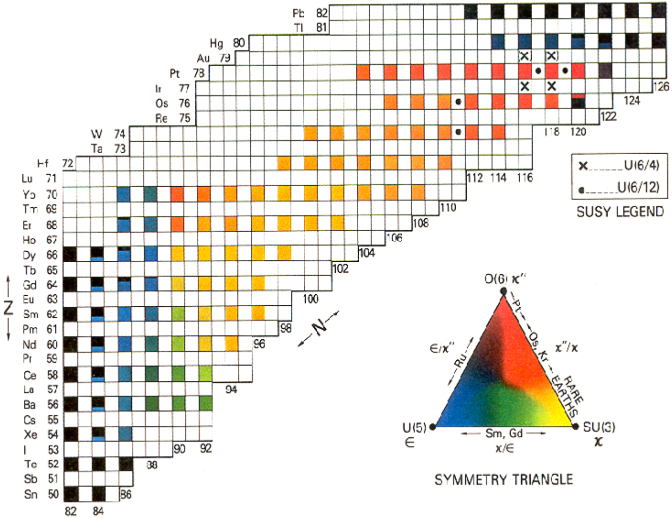
$$T^{(\lambda)} = [(a^{\dagger}_{j_\nu}\tilde{b}_{\nu_{l_\nu}})^{L_\nu} \times (a^{\dagger}_{j_\pi}\tilde{b}_{\pi_{l_\pi}})^{L_\pi}]^{(\lambda)}$$

$$D_3 = \langle {}^{196}\mathrm{Au}|a^{\dagger}_{\nu}b_{\nu}a^{\dagger}_{\pi}b_{\pi}|{}^{194}\mathrm{Pt}\rangle \propto \langle {}^{196}\mathrm{Au}|\, T^{(1,2)}_{1,0,-\frac{4}{3}}\,|{}^{194}\mathrm{Pt}\rangle$$

and similar to the another reaction

$$D_4 = \langle {}^{195}\mathrm{Au}|b^{\dagger}_{\nu}a_{\nu}a^{\dagger}_{\pi}b_{\pi}|{}^{195}\mathrm{Pt}\rangle \propto \langle {}^{195}\mathrm{Au}|\, T^{(1,2)}_{1,-1,\frac{2}{3}}\,|{}^{195}\mathrm{Pt}\rangle$$

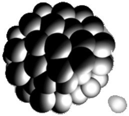
Summary and Conclusions



- There are different symmetries in Nuclear physics which help us study different nuclei

Summary and Conclusions

- We developed the GF-spin applied to write nuclear reactions in terms of matrix elements of $SU(3)$.
- We have determined some correlations between different nucleon transfer reactions like **one** and **two** nucleons transfer.



- We found experimental data of the spectrum of ^{196}Au with ^{195}Pt(^{3}He,d) which open the possibility to continue the research of the computation of the spectroscopic strenghts of the associated correlated reaction.

Summary and Conclusions

- The GF-spin also could be used in different models described in terms of couplings of three different symmmetric representations of $U(6)$.
  - IBM3 (IBM for light nuclei)

$$
\begin{array}{ccccccc}
U(18) & \supset & U(6) & \supset & \ldots & SO(3) & \\
\downarrow & & \downarrow & & & \downarrow & \\
[N] & & [N_1, N_2, N_3] & & & L & \\
& \otimes & U(3) & \supset & SU(3) & \supset & SO(3) \\
& & \downarrow & & \downarrow & & \downarrow \\
& & [N] & & (\lambda, \mu) & & T
\end{array}
$$

J. P. Elliott *Prog. Part. Nucl. Phys* **25**, 325 (1990)

  - Neutron Skin

$$
\begin{array}{ccccc}
U(6) & \otimes & U(6) & \otimes & U(6) \\
\downarrow & & \downarrow & & \downarrow \\
[N_\pi] & & [N_{\nu_c}] & & [N_{\nu_s}]
\end{array}
$$

Warner D D and Van Isacker P *Phys. Lett* B **395** 145, (1997).

- There are some challenges to determine all the isocalar factors involved in our approach.

- Would be interesting try to generalize this method to different symmetries:
  - Partial dynamical $SU(3)$symmetry
  - Generalized Partial Dynamical Symmetry in Nuclei

A. Leviatan and I. Sinai Phys. Rev. C **60** (1999)
A. Leviatan and P. Van Isacker Phys. Rev. Lett. **89** 22 (2002)

- And also think about possible phase transitions between dynamical symmetries.

F. Iachello, Phys. Rev. Lett. 85, 3580 (2000).

Acknowledgments

Thanks to organizers: **Neelima G. Kelkar and Fernando Cristancho** and the following institutions:

Universidad Juárez Autónoma de Tabasco

Universidad Nacional Autónoma de México

División Académica de Ciencias Básicas

Instituto de Ciencias Nucleares

Universidad de los Andes, Colombia

Universidad Nacional de Colombia

Thank you for your attention.